Classifying Factor Velocity with Swarm Intelligence: Market Pricing of Fast- and Slow-Moving Factors

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ABSTRACT

Utilizing a dataset of 190 factor portfolios spanning over three decades, we apply a swarm-based classification model to estimate factor velocity and analyze its implications for asset pricing. Our results show that slower-moving factors generate higher abnormal returns than their faster-moving counterparts, underscoring the critical role of price adjustment speed in market dynamics. Furthermore, our results suggest that trading frictions impede the rapid assimilation of information, contributing to the observed return patterns. This research offers new insights into return predictability and demonstrates the potential of swarm intelligence as a powerful tool for financial modeling.

KEYWORDS

swarm intelligence, pricing factors, Fama-French factors

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I Introduction

Momentum investments have been the centerpiece of investments in stocks for the past 2 decades. Momentum investing refers to buying stocks that have had high returns in the past and selling those that have had poor returns over the same period. Researchers have identified persistent momentum trends in stock markets as far back as the Victorian era (c. 1830s to 1900). Richard Driehaus is sometimes considered the father of momentum investing. He once said "far more money is made buying high and selling at even higher prices." This strategy reportedly delivered compound annual returns of 30% for Driehaus Capital Management in the 12 years after it was set up in the 1980s.[1]

Momentum supporters believe that there is a fundamental reason for momentum phenomenon to persistently exist, although momentum in stocks totally lacks economic theory.[2] Besides momentum, we believe that all those technical indicators, although not supported by any economy theory, are supported by psychological behaviors of market participants – herding.

While herding itself cannot be easily observed due to the fact that individual trading data are not publicly (or even legally) available, the consequences are. In other words, although we cannot measure how investors herd in buying/selling stocks, we can observe how stock prices (returns) move as a result of herding. When herding exists, certain stocks will be chased (bought) or avoided (sold). These result in winner stocks and loser stocks. As herding behaviors change, winners can become losers and vice versa.

Past studies demonstrate the diverse applications of swarm intelligence algorithms in financial contexts, including portfolio optimization and stock price prediction (see, e.g., Zhu and Wang, (2010), Hegazy, Soliman and Salam (2014), Liu, Wei and Xu (2022), Kaucic et al. (2023), and Huang et al. (2024)). These studies leverage swarm intelligence to decision-making processes by

[1] This is taken from Richard Driehaus and Momentum Investing. For the original source please see Beattie (2007), Marek (2015), and Schwager (1992).
[2] The pioneering academic research on momentum investing can be traced back to Jegadeesh and Titman (1993). They documented how strategies of buying recent stock winners and selling recent losers generated significantly higher near-term returns than the U.S. market overall from 1965 to 1989. They established the basic time frame for momentum-investing success as a three-to-12-month window on either side. Since then, it has been booming into one of the largest research areas in finance. Recently, they (Jegadeesh and Titman (2023)) wrote an excellent review piece of the past 30 years of momentum research.

simulating collective agent behaviors observed in nature, such as those of birds, fish, and insects. However, to our knowledge, no existing research has specifically examined how stock market prices influence or interact with the velocity of agents in a swarm system. This study aims to bridge that gap by exploring the market pricing of velocity in financial swarm dynamics.

Velocity is a crucial component of system intelligence in swarm-based models. In natural and artificial swarm systems, velocity reflects the rate of change in an agent's position, capturing both speed and directional movement. It serves as a fundamental mechanism by which information is propagated and decisions are adjusted dynamically within the swarm. In financial markets, velocity can be viewed as a proxy for the responsiveness of stocks to new information, as well as a key determinant of trend formation and reversals. Stocks that exhibit higher velocity within the swarm framework may represent "leaders" entities that drive market sentiment, while lower-velocity stocks may act as "followers," adjusting their movements based on lagged signals from faster-moving assets.

Incorporating velocity into swarm intelligence models of financial markets enhances our understanding of price formation, momentum effects, and return predictability. A system that lacks velocity-awareness may fail to capture the hierarchical influence of stocks, where certain assets dictate the movement patterns of others. By analyzing how velocity distributes across financial agents, we can gain deeper insights into price discovery mechanisms and the differentiation between trend-driving and trend-reactive assets.

This study, therefore, introduces a velocity-driven perspective on swarm intelligence in financial markets. By modeling returns of factor portfolios formed based on stylized return predictors through a swarm-based system that accounts for velocity differences, we aim to uncover new dimensions of market behavior and pricing dynamics that have previously been overlooked in the literature.

II SWARM INTELLIGENCE

Swarm intelligence is a powerful artificial intelligence tool to model animal behaviors, as well as solving high-dimensional optimization problems. The basic idea of swarm intelligence is derived from those animals (such as birds, ants, bees, and fish) that rely on group effort to achieve their basic survival needs – seek food and avoid prey. The intelligence behind this collective behavior is how they communicate among one another.

There are two broadly classified categories in swarm intelligence – boids and particle swarm optimization (PSO). While both share the fundamental theory, they vary in the following manner. Boids is mainly for behaviors of the swarm and PSO can be used for optimization. Roughly speaking, PSO is used when there is a loss function (i.e. a landscape) and boids without. We should note that although they have been recognized two separate models, they can be easily combined if needed. For example, although our estimation seems to follow particle swarm optimization, there is no optimization. If needed, we can estimate alignment and cohesion (which reflect another version of swarm speed) along with leader-following. We briefly sketch each model in this section.

A. Boids (without a leader)

Reynolds (1987)[3] was the first to "artificialize" such natural intelligence and create a computer algorithm named Boids (for bird-oid object). Reynold's algorithm is amazingly simple. For any given bird, Reynold devises a set of linear equations (vectors) combining which determines how the bird should fly to its next destination.

The factors that determine how various vectors are combined are: separation, alignment, and cohesion. As their names suggest, "separation" is to avoid collision with other birds, "alignment" decides how a particular bird should fly in a direction by referencing to its fellow birds, and "cohesion" decides how fast (speed) a particular bird should fly to the center of its fellow birds.

There are countless versions of Boids.[4] One can add obstacles. One can add an objective destination (swim to target). One can do Boids in a maze. The basic Boids as described in Figure 1 can be described by the following algorithm.

[Figure 1 here]

In the swarm model, let $i = 1, \cdots, m$ be firm (bird) and $j = 1, \cdots, n$ be stock (dimension). At each given point in time, a map of the locations of bird is taken. The map is a 15-dimensional hyper-cube image with each axis bounded between 0 and 1.

[3] Reynold created Boid in 1986: "Boids is an artificial life program, developed by Craig Reynolds in 1986, which simulates the flocking behaviour of birds."

[4] For example, see Google Scholar: https://scholar.google.com/scholar?q=boids+flocking+algorithm&hl=en&as_sdt=0&as_vis=1&oi=scholart

In the swarm model, let $\vec{x}_t^{(i)}$ be a vector of n coordinates (dimensions) where $x_{j,t}^{(i)}$ and $j = 1,\cdots,n$ is an element. In other words, $\vec{x}_t^{(i)}$ is the position of bird i at time t. For the sake of completeness, define F as a flock of birds $\{f^{(1)},\cdots,f^{(m)}\}$ whose positions at time t are collected in set $X_t = \{\vec{x}_t^{(1)},\cdots,\vec{x}_t^{(m)}\}$. In other words, X_t is an $n \times m$ matrix. The velocity of each bird is defined as a weighted average of various forces.

$$(1) \qquad \vec{v}_t^{(i)} = w_A \vec{v}_{A,t}^{(i)} + w_C \vec{v}_{C,t}^{(i)} + w_S \vec{v}_{S,t}^{(i)}$$

where weights sum to 1 and

$$(2) \qquad \begin{aligned} \vec{v}_{A,t}^{(i)} &= \operatorname{avg}\left(\vec{v}_{t-1}^{(j\neq i)} \mid f^{(j)} \in F\right) - \vec{v}_{t-1}^{(i)} \\ \vec{v}_{C,t}^{(i)} &= \operatorname{avg}\left(\vec{x}_{t-1}^{(j\neq i)} \mid f^{(j)} \in F\right) - \vec{x}_{t-1}^{(i)} \\ \vec{v}_{S,t}^{(i)} &= \max\left\{\left|\vec{x}_t^{(i)} - \vec{x}_{t-1}^{(i)}\right|, \varepsilon\right\} \end{aligned}$$

representing alignment, cohesion, and separation respectively.

The update of position is:

$$(3) \qquad \vec{x}_t^{(i)} = \vec{x}_{t-1}^{(i)} + \vec{v}_t^{(i)}$$

In terms of data, each position is a snapshot of the locations of all birds at a given time. From one snapshot to another, it represents a migration (i.e. velocity).

As it can be easily seen, the main purpose of the swarm here is to describe how birds move. For example, if cohesion is dominant, then eventually all birds will line up in a straight line. Similarly, if alignment is dominant, then they all want to move in the same direction parallelly. Finally if separation is dominant, then they all move randomly. Since there is no stopping time, these birds will keep moving indefinitely.

B. Particle Swarm Optimization (*with a leader*)

Particle swarm optimization (PSO), from its name, is an optimization tool using swarm.[5] In PSO, an objective function (or penalty function) is given so birds can all reach the optimal location. The communication mechanism among the birds is the same as Boids and yet how they move is different.

[5] See Eberhart and Kennedy (1995) and Shi and Eberhart (1998).

One can think of PSO is swarm with a landscape (i.e. the objective function). Birds now will try to reach either the peak of bottom (i.e. global optimum) of the landscape. In this case, they will stop moving once their objective is met. A typical PSO can be demonstrated as Figure 2.

[Figure 2 here]

In order to achieve convergence, the velocity of a PSO is given as:

(4) $$\vec{v}_t^{(i)} = w_t \vec{v}_{t-1}^{(i)} + c_1 r_1 (\vec{p}_{t-1}^{(i)} - \vec{x}_{t-1}^{(i)}) + c_2 r_2 (\vec{g}_{t-1} - \vec{x}_{t-1}^{(i)})$$

where $w_t < 1$ is a decaying weight (e.g. we can let $w_t = \delta^t$ and $\delta < 1$), c_1 and c_2 are two constants, r_1 and r_2 are two random variables, and

(5) $$\vec{p}_t^{(i)} = \left\{ \vec{x}_{\tau \le t}^{(i)} \mid \max_{\tau} \phi(\vec{x}_{\tau}^{(i)}) \right\}$$

is the personal best and $\vec{g}_t = \max_i \vec{p}_t^{(i)}$ is the global best and $\phi(\cdot)$ is the fitness function.

In (4), c_1 and c_2 are learning rates. These are standard parameters in artificial intelligence to attain most effective learning. The two random variables r_1 and r_2 are "exploration" meaning that the birds do not follow what they "learn" exactly. This is key to artificial intelligence to avoid local optima. Finally, birds follow the instruction from $\vec{g}_{t-1} - \vec{x}_{t-1}^{(i)}$ and $\vec{p}_{t-1}^{(i)} - \vec{x}_{t-1}^{(i)}$ which are regarded as "exploitation" since the they would like to learn from given information.

The update of position is same as (3) and repeated here:

(6) $$\vec{x}_t^{(i)} = \vec{x}_{t-1}^{(i)} + \vec{v}_t^{(i)}$$

Note that swarm is very flexible and we can put exploration separately (i.e. not mingled with exploitation). In this paper, we alter (4) as follows:

(4a) $$\vec{v}_t^{(i)} = w_L \vec{v}_{L,t}^{(i)}$$

where

(7) $$\vec{v}_{L,t}^{(i)} = \alpha_L (\vec{g}_{t-1} - \vec{x}_{t-1}^{(i)}) \quad \text{or} \quad = \alpha_L^{(i)} (\vec{g}_{t-1} - \vec{x}_{t-1}^{(i)})$$

Note that in this paper, we do not have a landscape. The global best (which is the best position of the landscape) is replaced by a "leader" who is the one with the position $\vec{g}_t$. As a result, there is no convergence or stopping time in our empirical work.

III EMPIRICAL METHODOLOGY

We can estimate the parameters of swarm (equation (1)) using data. Empirical (i.e. data) positions can be labeled as $\vec{\xi}_t^{(i)}$ and velocity as $\vec{\nu}_t^{(i)}$ (to substitute for $\vec{x}_t^{(i)}$ and $\vec{v}_t^{(i)}$ in (3)). Hence, similar to (3):

(8) $$\vec{\nu}_t^{(i)} = \vec{\xi}_t^{(i)} - \vec{\xi}_{t-1}^{(i)}$$

Our objective function is to minimize the sum of squared errors between $\vec{\nu}_t^{(i)}$ and $\vec{v}_t^{(i)}$:

(9) $$\min_{\alpha_{P,t}^{(i)}} (\vec{v}_t^{(i)} - \vec{\nu}_t^{(i)})'(\vec{v}_t^{(i)} - \vec{\nu}_t^{(i)}) = \min_{\alpha_{P,t}^{(i)}} \sum_{j=1}^{n} (v_{j,t}^{(i)} - \nu_{j,t}^{(i)})^2$$

Taking partial derivative and setting it to 0:

(10) $$\sum (v_{j,t}^{(i)} - \nu_{j,t}^{(i)}) \frac{\partial v_{j,t}^{(i)}}{\partial \theta} = 0$$

where θ is a chosen parameter.

Clearly, (10) in general has no closed-form solution and needs to be solved numerically. However, if we focus on one parameter at a time (i.e. holding other parameters constants), then there is a closed-form solution, which is what we implement in the empirical section.

In our empirical work, we only interested in leader-following. Hence,

(11) $$\vec{v}_{L,t}^{(i)} = \alpha_{L,t}^{(i)}(\vec{g}_{t-1} - \vec{x}_{t-1}^{(i)})$$

where the former is each firm has its own $\alpha_{L,t}^{(i)}$ and the latter is all firms share the same $\alpha_{L,t}$. The solution (see the Appendix) is:

(12) $$\alpha_{L,t}^{(i)} = \frac{\sum_{j=1}^{n} \nu_{j,t}^{(i)}(g_{j,t} - x_{j,t}^{(i)})}{\sum_{j=1}^{n} (g_{j,t} - x_{j,t}^{(i)})^2}$$

In this paper, we do not have a landscape. What we do is to loop through each portfolio as the leader and computes the speed of its followers. If this portfolio is not a leader, then the other portfolios will not follow it and the speeds are therefore small. If this portfolio is indeed the leader, then the other portfolios will follow it closely and the speeds will be large.

In other words, each portfolio's position (i.e. past 12 months of returns) when it is assumed to be the leader, is

(13) $$g_{j,t} = x_{j,t}^{(i)}$$

and as a result, $\alpha_{L,t}^{(i)} = 0$. The average across individual brokers/dealers is:

(14) $$\begin{aligned} \overline{\alpha}_{L,t}^{(i)} &= \frac{1}{m-1}\sum_{i=1}^{m-1} \alpha_{L,t}^{(i)} \\ &= \frac{1}{m-1}\sum_{i=1}^{m-1} \frac{\sum_{j=1}^{n} \nu_{j,t}^{(i)}(g_{j,t} - x_{j,t}^{(i)})}{\sum_{j=1}^{n} (g_{j,t} - x_{j,t}^{(i)})^2} \end{aligned}$$

A more generalized optimization can be found in Chen, Miller and Toh (2023a,b).

IV SWARM-RELATED LITERATURE IN FINANCE

There are two branches of finance literature that discusses the investors' behavior: people following people and stocks following stocks. The former behavior is herding and the latter is lead-lag.

A Herding

Herding is the behavior of individuals in a group acting collectively without centralized direction. Herding is originally observed in animals in herds, packs, bird flocks, fish schools, and so on, as well as in humans. Shiller (1984) seems to be the first author who studied herding in the financial market. He describes investors influenced by their peers in making investment decisions as follows:

> "Investing in speculative assets is a social activity. Investors spend a substantial part of their leisure time discussing investments, reading about investments, or gossiping about others' successes or failures in investing. It is thus plausible that investors' behavior (and hence, prices of speculative assets) would be influenced by social movements."

As we can all agree, although humans are highly intelligent, they can behave foolishly when they are in a crowd. Hence, with no surprise, the literature has predominantly regarded herding as an irrational behavior. This is because herding does not generate better returns (see Mavruk (2022) for a review of such results). The literature documents the following reasons for herding, all of which demonstrate a certain behavioral bias (See Mavruk for further details):

- fads
- fear
- greed
- reputation
- noise

All of the above causes can easily result in market disturbances and destabilization. Hence, herding can move the market away from its fundamentals. The conclusion that herding is irrational is consistent with the common wisdom raised by Shiller.

As a result, it can be expected that herding is not persistent. It happens sometimes, ceases to happen sometimes, and furthermore contradicts itself sometimes. This is extremely similar to the performances of technical analyses that herding is time-varying and situation-dependent. For this reason, a major effort in the literature is to identify the determinants of herding. Summarized nicely by Rahayu et al. (2020), herding is stronger (Please see the citations of relevant papers in Rahayu):

- when volatility is higher
- in a crisis
- for small stocks
- when there are large price movements
- in a declining market
- when rates rise
- in a poor information environment

It is obvious that such situations are not sustainable situations. They happen only temporarily and are naturally not persistent.

Herding is in general classified as stock herding or investor herding. As their names suggest, the former refers to how investors rush in and out of a stock, and the latter refers to how investors follow their peers. Understandably, the methodology to study both types of herding is the same, while the difference is in data. For stock herding, the data are usu-ally more price related (i.e., in order to measure trading profits) and more frequent, while for investor herding, the data are more shares related (i.e., investors' holdings) and less frequent.

The measures of herding vary. Yet they can be traced back to Lakonishok, Shleifer, and Vishney (1992), who define herding as the difference between the actual change in "purchase intensity" and the expected change of "purchase intensity" (For other measures of herding, see Bikhchandani and Sharma (2001) and Mavruk (2015) for their surveys):

(15) $\mid p_{t+1} - p_t \mid - \mathbb{E}[\mid p_{t+1} - p_t \mid]$

where p_t is purchase intensity. While this definition reflects common wisdom, finding a good proxy for purchase intensity is a challenge. Various authors, given data availability and research focus, use different proxies. Lakonishok, Shleifer, and Vishney (1992) use changes in share holdings. While this is a sensible proxy for purchase intensity, such data are only available at the institutional level and only four times a year (i.e., quarterly). As a result, not only does it fail to measure more frequent herding, which is how herding can meaningfully impact trading profits, it also fails to measure retail trading, which is believed to be more interesting (because it is more tightly related to trading profits). In other words, their results are limited to fund managers and are in low frequency. Strictly speaking, it is a result of how information is transferred among fund managers, not herding, which is more understood as trading related.

The other extreme is to use very frequent data (daily), and yet the quality of the proxy is lowered. For example, Ukpong et al. (2021) use excess returns (the difference between a stock's return and the market return) as the proxy. Contrary to Lakonishok, Shleifer, and Vishny (1992), they capture the dynamics of herding more perfectly and yet the measurement errors in their results are larger. Using swarm, we are free of the above criticisms in that we directly measure how individual investors follow or not their peers.

A small portion of the literature argues that herding can be rational, mainly caused by information asymmetry. Some individuals in a group have superior information than others. As one can understand, such evidence can only exist in investor herding (not stock herding) and is only limited to institutional investors. These institutions mimic one an-other due to information asymmetry, which is distinctly different from and hard to reconcile with the aforementioned reasoning of herding. In swarm intelligence, however, these two types of herding can be easily integrated. The former is the usual notion of swarm via separation, alignment, and cohesion (see the next section for details), and the latter is leader following (either via a landscape or a predefined leader or a group of leaders).

Evidence on herding also indicates anti-herding, known as contrarian. Similar to the standard contrarian notion, anti-herding refers to a situation where certain investors tend to diverge from the rest of the group. Again, this can be captured more faithfully by swarm intelligence via alignment, cohesion, and separation.

Some researchers relate herding to momentum (The pioneering academic research on momentum investing can be traced back to Jegadeesh and Titman (1993). They documented how strategies of buying recent stock winners and selling recent losers generated significantly higher near-term returns than the U.S. market overall from 1965 to 1989. They established the basic time frame for momentum-investing success as a 3-to-12-month window on either side. Since then, it has been booming into one of the largest research areas in finance. Recently, they (Jegadeesh and Titman (2023)) wrote an excellent review piece of the past 30 years of momentum research). As the first authors systemically documenting herding, Grinblatt, Titman, and Wermers (1995) define herding as how a group of investors move in and out of the market simultaneously—like a herd. They do not study why they herd. Recently, Demirer, Lien, and Zhang (2015) evaluate the impact of industry herding on return momentum. They find that the profitability of industry momentum strategies depends on the level of herding in an industry. Lin, Wu, and Zhang (2023) investigate the impact of herding behavior on the momentum effect. Using a new firm-level herding measurement, they find that investors require higher returns in high herding stocks, and they require even higher returns in high herding stocks among previous losers, indicating that investors herd against the previous losers while they herd toward the winners. Chen (2020), using intra-day volume data of 2016 over 62 countries, finds that uninformed country-level herding is highly related to momentum (Note that although the data used by Chen (2020) are intra-day, he aggregates information to daily).

Finally, herding is observed internationally. Besides Chen's work on 62 countries, Rahayu et al. (2020) provide an excellent review of literature. They have documented strong evidence of herding internationally (over 20 nations).

B Lead-Lag

Asset returns that move faster, referred to as leaders in our study, could be driven by several factors. One possibility is that they exhibit higher liquidity, allowing investors to incorporate value-relevant information more quickly (Amihud and Mendelson (1986), Pastor and Stambaugh (2003)). Another potential explanation is that these assets attract heightened investor attention, often due to media coverage or prevailing market sentiment (Barber and Odean (2008), Tetlock (2007)). When assets gain excessive attention, their prices may deviate from fundamentals, leading to a temporary overvaluation that eventually corrects over time. If investor-driven overreaction propels fast-moving assets, we would expect them to underperform subsequently, as measured by alphas relative to standard asset pricing models such as the Fama-French factor models (Fama and French (1993, 2015)) or the Hou-Xue-Zhang factor models (Hou, Xue, and Zhang (2015)). In other words, the overvaluation fueled by heightened investor attention should lead to lower subsequent alphas as prices revert to their intrinsic values. However, if the primary driver of rapid price movement is superior liquidity conditions, then these assets would likely incorporate value-relevant information efficiently, leaving no room for abnormal performance and ensuring fair pricing.

Conversely, assets that move slower, referred to as laggards, may do so for several reasons. One possibility is that they operate in more mature sectors where the information flow is relatively stable, leading to fewer price-adjusting events (Fama and French (1995)). As a result, the absence of significant informational updates causes slow price movements. Another potential explanation is that these assets face higher trading frictions, such as lower liquidity, wider bid-ask spreads, and higher transaction costs (Acharya and Pedersen (2005), Easley and O'Hara (2004)). In this case, market participants take longer to process and incorporate new information, leading to a slower diffusion of price-relevant signals. If slow movement results from operating in a stable industry with predictable fundamentals, there may be no systematic impact on future returns. However, if frictions and impediments to trading hinder the incorporation of information, the direction of future abnormal returns depends on whether the market predominantly underreacts to good news or bad news. If slow-moving assets experience an underreaction to positive information, they could generate positive alphas, whereas an underreaction to negative information could lead to negative alphas. This remains an empirical question, and we rely on data-driven analysis to uncover the relation between asset velocity and return predictability.

IV EMPIRICAL FINDINGS

In this section, we briefly introduce the data we use, how a swarm model is applied in order to estimate the speed of following the leader, and investigate market pricing of slow- and fast-moving portfolios.

A Data

The test assets are monthly Hou-Xue-Zhang 190-factor portfolios over 23 years obtained from the Q-factor data library.[6] It contains portfolios in 6 categories with different periods:

- frictions (February 1990 ~ December 2022) 10 factors
- intangibles (January 1990 ~ December 2022) 31 factors
- investments (January 1973 ~ December 2022) 32 factors
- momentum (July 1979 ~ December 2022) 42 factors
- profitability (January 1985 ~ December 2022) 44 factors
- value vs. growth (January 1985 ~ December 2022) 31 factors

In order to run our swarm model, we adopt the common period from February 1990 till December 2022 – a total of 395 months.

We use lag 12 months as features. That is, the universe of the swarm is a 12-dimensional space. In other words, on each month, a portfolio (a particle, or a fish) is positioned in a 12-dimensional space according to its past 12 months of returns. As each month passes by, the portfolio (or fish) moves (or swims) to the next 12 months of returns.

However, to help visualize how we fit a swarm model via data, we only use lag 2 months as a demonstration in Figure 3. Figure 3 plots Frictions' 10 portfolios for 4 consecutive months: April through July of 1990. Each panel contains positions (i.e. returns) of the past two months. For example, panel (A) plots 10 portfolios' February returns on the x-axis and March returns on the y-axis.

[Figure 3 here]

We pick two portfolios as an example. Portfolio beta_1 is marked red and portfolio tv_1 is marked yellow. From Figure 3, we can clearly observe how they move from month to month. As explained earlier, on each month, the position of a portfolio is its past two months of returns. The returns of the two portfolios are given below.

Data	beta_1	tv_1
2/1990	3.5051	1.4045
3/1990	7.4907	1.6897
4/1990	0.3085	-1.5450
5/1990	8.5011	2.3310
6/1990	-0.9011	-0.7728

[6] https://global-q.org/index.html.

In panel (A), beta_1's position is (3.5051, 7.4907) and tv_1's position is (1.4045, 1.6897). In panel (B), beta_1's position is (7.4907, 0.3085) and tv_1's position is (1.6897, -1.5450). From month to month, we can now see the migration of portfolios. We then use these positions to estimate the swarm speed. Hence, the positions are:

position	beta_1		tv_1	
	X	y	x	Y
4/1990	3.5051	7.4907	1.4045	1.6897
5/1990	7.4907	0.3085	1.6897	-1.5450
6/1990	0.3085	8.5011	-1.5450	2.3310
7/1990	8.5011	-0.9011	2.3310	-0.7728

Then velocity, which is the difference between two consecutive positions, can be calculated as follows:

velocity	beta_1		tv_1	
	X	y	x	Y
5/1990	3.9856	-7.1822	0.2852	-3.2347
6/1990	-7.1822	8.1926	-3.2347	3.8760
7/1990	8.1926	-9.4022	3.8760	-3.1038

These numerical results are then fed to equation (14) to calculate the speed parameter. In our empirical work, 12 lags are used. Hence, $m = 190$ and $n = 12$.

B Speed Result

The speed results are plotted in Figure 4 and Figure 5. Figure 4 presents a distribution of average speeds across portfolios. Each observation in this distribution is an average speed of a portfolio over the entire sample period. It can be seen that (1) all speeds are positive, demonstrating a definite swarm behavior (somewhat surprising!) and (2) the mode of the distribution is slightly less than 0.5 with the next highest peak to be slightly higher than 0.5.

Figure 5 presents the time series of average speed across portfolios. As we can see, the variability over speed over time is not noteworthy and the mean and median are very close to each other, demonstrating no significant skewness. Figure 5 also plots 25th and 75th percentiles and they indicate a roughly symmetrical distribution across portfolios.

[Figure 4 here]

[Figure 5 here]

C. Market Pricing of Factor Velocity

To investigate how the market prices factors with different velocity measures derived from swarm intelligence, we conduct portfolio-level analysis. Each month, we sort 190 factors from the Q-factor data library into quintiles based on their swarm velocity (SV). We then compute the equal-weighted returns for each quintile and compare the return differences between the top and bottom quintiles. Table 1 presents the results. Column 1 reports the average characteristics of the sorting variable, Column 2 provides the mean equal-weighted returns within each quintile, and Columns 3 to 8 display alphas relative to various factor models. These models include: 1) the Fama-French five-factor model (FF5), which includes market (MKT), size (SMB), book-to-market (HML), investment (CMA), and profitability (RMW) factors (Fama & French, 2015), with the resulting intercept term labeled as FF5 alpha; 2) the Fama-French six-factor model (FF6), which adds the momentum (UMD) factor to FF5, producing FF6 alpha (Carhart, 1997); 3) the Fama-French-Carhart six-factor model augmented with the Pastor-Stambaugh liquidity factor (Pastor & Stambaugh, 2003), generating FF6PS alpha; 4) the Hou-Xue-Zhang four-factor model (Q-factor model), which includes R_MKT, R_ME, investment (R_IA), and profitability (R_ROE) (Hou, Xue, & Zhang, 2015), with the corresponding Q alpha; and 5) the Hou-Mo-Xue-Zhang five-factor model, which extends the Q-factor model with the growth (R_EG) factor, producing Q5 alpha. The last row of Table 2 reports the differences in mean returns and alphas between the top and bottom quintiles, with Newey-West adjusted t-statistics (Newey & West, 1987) accounting for serial correlation. Our sample period spans March 1991 to December 2022 to ensure all 190 factors are available each month.

Table 1 reveals that factors in the lowest SV quintile earn significantly higher abnormal returns than those in the highest SV quintile, with an alpha difference ranging from 17 to 22 basis points per month. Importantly, this return differential is primarily driven by the superior performance of slower-moving factors.

We further examine whether swarm speed exhibits return predictability beyond a one-month horizon. To test this, we relate the swarm velocity measure to future returns of the previously formed quintile factor portfolios over horizons extending from two months to 12 months ahead. Table 2 presents the Hou-MO-Xue-Zhang five-factor alphas (Q5) for these horizons. The results show that the return spread between the top and bottom SV quintile portfolios remains negative across all horizons, ranging from 5 to 17 basis points per month. However, only the two-month-ahead alpha spread is statistically significant, driven by the continued outperformance of slow-moving factors.

Next, we explore the characteristics of fast-moving versus slow-moving factors. Panel A of Table 3 lists the top 10 slowest-moving factors (left panel) and the top 10 fastest-moving factors (right panel). For comparability, values in the table are demeaned. The slowest-moving factors exhibit an average velocity below the mean of all 190 factors, with a probability exceeding 50% of falling into the slow-moving quintile and a 20% lower likelihood of appearing in the fast-moving quintile. Conversely, the fastest-moving factors exhibit significantly higher velocity scores, with probabilities more than 50% above the mean for appearing in the fast-mover quintile and approximately 20% lower for falling into the slow-mover quintile.

To improve interpretability and identify underlying drivers of return differentials, we follow Hou, Xue, and Zhang (2015) and categorize factor-level results by their respective themes, including frictions, intangibles, investments, momentum, profitability, and value-vs-growth. Panel B of Table 3 shows that friction-related factors are disproportionately overrepresented in the slow-mover portfolio, appearing at more than twice their unconditional share. In contrast, investment-related factors are significantly underrepresented, comprising less than 50% of their unconditional share. Within the fast-mover portfolio, momentum-related factors dominate, accounting for 36.20% of the portfolio versus their unconditional share of 25.37%, while intangibles and value-vs-growth factors are notably underrepresented.

These findings, combined with our portfolio return results, support the hypothesis that trading frictions impede the swift incorporation of value-relevant information, leading to subsequent outperformance of slow-moving factors. On the other hand, the statistically insignificant alphas of future returns of fast movers is inconsistent with the idea that heightened investor attention results in temporary overvaluation of fast movers, leading to subsequent underperformance.

V CONCLUSIONS

This study presents a novel application of swarm intelligence to classify factor velocity and investigate its role in asset pricing. Swarm is an ideal methodology to model the herding behavior of people trading in the investment world. Given that individual's trading data are not available, it is common then to follow portfolios (or factors) as a proxy of investors herding.

By leveraging a swarm-based approach, we systematically identify factor portfolios that act as market leaders and followers based on their return movement speed. Our results indicate that slow-moving factors generate higher one-month-ahead returns than fast-moving ones, a finding that holds across multiple asset pricing models. This return differential is likely due to trading frictions or delayed information processing by market participants.

These findings open new avenues for understanding return predictability and market efficiency, emphasizing how velocity classification can be integrated into existing asset pricing frameworks. Future research could enhance these models by incorporating alternative swarm intelligence techniques and exploring different test assets. Our study underscores the potential of artificial intelligence in finance, offering a fresh perspective on how collective behaviors shape stock returns.

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Table 1. Univariate Portfolio Sorts on Swarm Velocity

Each month, we sort the 190 factors into quintiles based on their swarm velocity (SV). Column 1 presents the average characteristic of the sorting variable. Column 2 reports the mean equal-weighted returns across factors within each quintile. Columns 3 to 8 present the alphas for each quintile portfolio relative to various factor models, including the Fama-French five-factor model, the Fama-French-Carhart six-factor model, the Fama-French-Carhart six-factor model augmented with the Pastor-Stambaugh liquidity factor, the Hou-Xue-Zhang four-factor model, and the Hou-Mo-Xue-Zhang five-factor model. The last row reports the differences in mean returns and alphas between the top and bottom quintiles. Newey-West adjusted *t*-statistics with six lags are reported in parentheses. The sample period spans from March 1991 to December 2022.

	SV	Mean	FF5	FF6	FF6PS	Q	Q5
Low	0.21	0.43	0.41	0.25	0.25	0.31	0.24
		(4.85)	(3.94)	(3.81)	(3.71)	(3.52)	(2.57)
2	0.37	0.23	0.16	0.13	0.14	0.12	0.06
		(4.43)	(3.18)	(2.52)	(2.52)	(2.36)	(1.20)
3	0.47	0.07	0.06	0.06	0.07	0.04	0.00
		(1.48)	(1.27)	(1.15)	(1.33)	(0.79)	(-0.04)
4	0.58	0.20	0.17	0.10	0.10	0.11	0.07
		(3.46)	(3.27)	(2.40)	(2.38)	(2.37)	(1.55)
High	0.73	0.17	0.16	0.07	0.07	0.09	0.01
		(2.99)	(3.26)	(1.58)	(1.60)	(1.76)	(0.11)
High-Low	0.52	-0.26	-0.25	-0.18	-0.17	-0.22	-0.21
		(-3.39)	(-2.86)	(-2.52)	(-2.42)	(-2.61)	(-2.63)

Table 2. Long-term Return Predictability

This table presents the Hou-Mo-Xue-Zhang five-factor alphas for the top and bottom quintile portfolios sorted by swarm velocity (SV), along with their alpha differentials across horizons ranging from two to 12 months ahead. Newey-West adjusted *t*-statistics with six lags are reported in parentheses. The sample period spans from March 1991 to December 2022.

	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11	t+12
Low	0.18	0.17	0.18	0.16	0.21	0.21	0.18	0.17	0.18	0.16	0.16
	(2.05)	(1.88)	(1.90)	(1.53)	(2.27)	(2.28)	(1.82)	(1.78)	(1.99)	(1.81)	(1.60)
High	0.01	0.06	0.05	0.04	0.05	0.03	0.03	0.05	0.01	0.04	0.08
	(0.20)	(1.10)	(0.87)	(0.80)	(0.87)	(0.46)	(0.51)	(0.91)	(0.21)	(0.75)	(1.51)
High-Low	-0.17	-0.11	-0.10	-0.11	-0.11	-0.13	-0.11	-0.09	-0.13	-0.12	-0.05
	(-2.18)	(-1.38)	(-1.29)	(-1.08)	(-1.32)	(-1.64)	(-1.25)	(-1.15)	(-1.58)	(-1.41)	(-0.67)

Table 3. Characteristics of Slow-mover and Fast-mover Portfolios

Panel A presents the top 10 slow-moving factors, as measured by swarm velocity (SV), along with their probabilities of appearing in the slow-mover portfolio (bottom SV quintile) and the fast-mover portfolio (top SV quintile). To facilitate comparison, the values reported in the panel are mean-adjusted. In Panel B, Column 1 presents the percentage of factors within each category relative to the total set of 190 factors. Columns 2 and 3 report the representation of each category in the slow-mover and fast-mover portfolios, respectively.

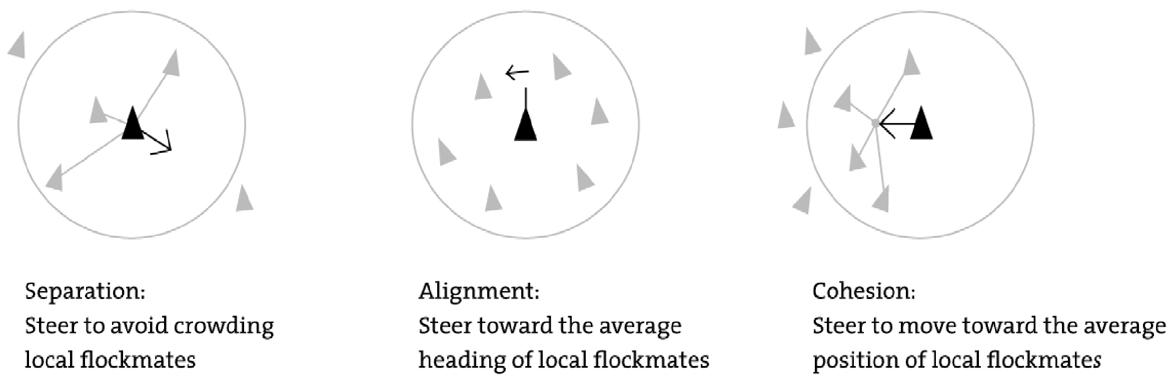
Panel A. Top slow-mover and fast-mover factors

Top 10 Slow-movers				Top 10 Fast-movers			
Characteristics	SV	Presence in slow-mover portfolio	Presence in fast-mover portfolio	Characteristics	SV	Presence in slow-mover portfolio	Presence in fast-mover portfolio
beta_1	-0.28	0.69	-0.20	cm_12	0.37	-0.20	0.78
tv_1	-0.29	0.68	-0.20	sim_12	0.33	-0.20	0.77
r11_1	-0.31	0.67	-0.20	abr_12	0.33	-0.20	0.76
ivff_1	-0.24	0.61	-0.20	ilr_12	0.32	-0.20	0.73
r1n	-0.29	0.57	-0.20	sm_12	0.34	-0.20	0.72
r11_6	-0.25	0.57	-0.20	droe_12	0.28	-0.20	0.70
p52w_6	-0.26	0.56	-0.20	cm_6	0.27	-0.19	0.67
r6_1	-0.26	0.54	-0.19	resid6_12	0.25	-0.20	0.64
ivq_1	-0.22	0.50	-0.20	droe_6	0.23	-0.20	0.57
fp_6	-0.20	0.46	-0.20	tbiq_12	0.25	-0.20	0.55

Panel B. Presence in slow-mover and fast-mover portfolios by category

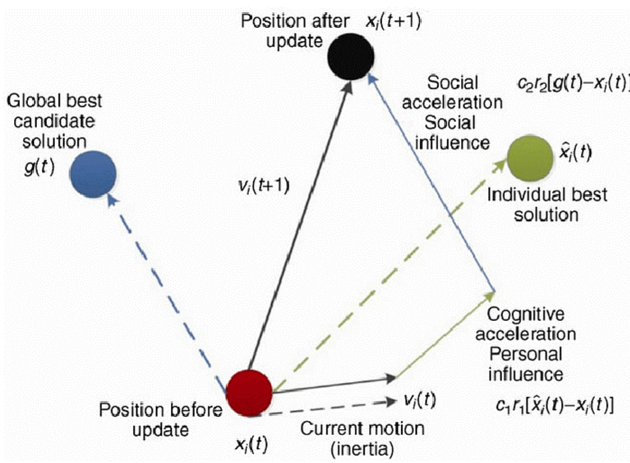
Category	% of factors	% in slow-mover portfolio	% in fast-mover portfolio
Frictions	5.26	11.55	4.64
Intangibles	16.32	17.48	7.71
Investments	16.84	8.31	20.32
Momentum	22.11	25.37	36.20
Profitability	23.16	17.03	24.36
Value-vs-Growth	16.32	20.26	6.77
Sum (%)	100	100	100

Figure 1 Swam Intelligence – boids



source:
https://people.engr.tamu.edu/sueda/courses/CSCE450/2023F/projects/Frank_Martinez/index.html

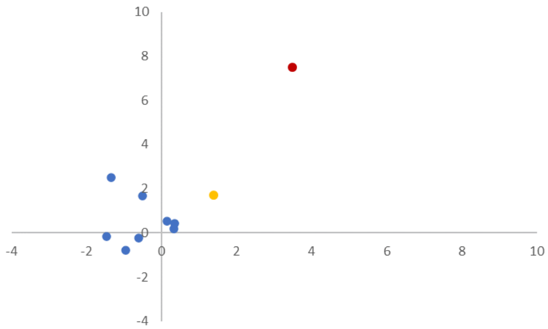
Figure 2 Particle Swam Optimization



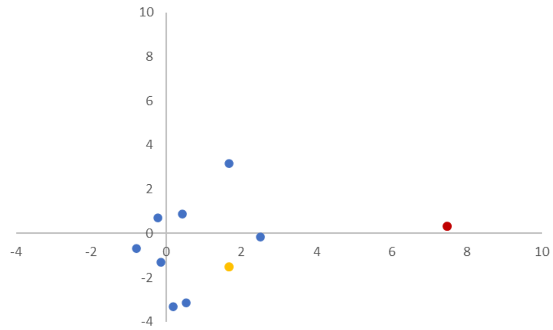
Sources: https://medium.com/analytics-vidhya/implementing-particle-swarm-optimization-pso-algorithm-in-python-9efc2eb179a6

Figure 3 Frictions Portfolios (lag 2 months)

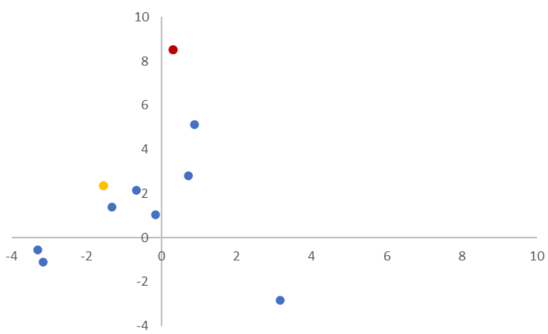
(A) April 1990



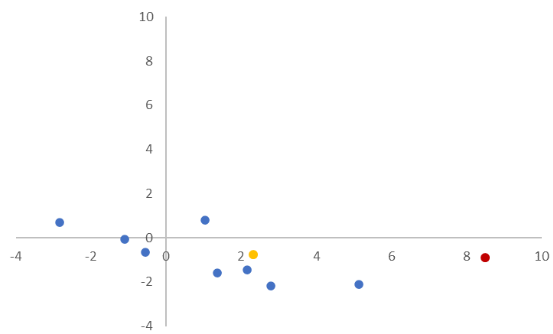
(B) May 1990



(C) June 1990



(D) July 1990



The red dot is beta_1 and the yellow dot is tv_1.

Figure 4 Average Speed Distribution

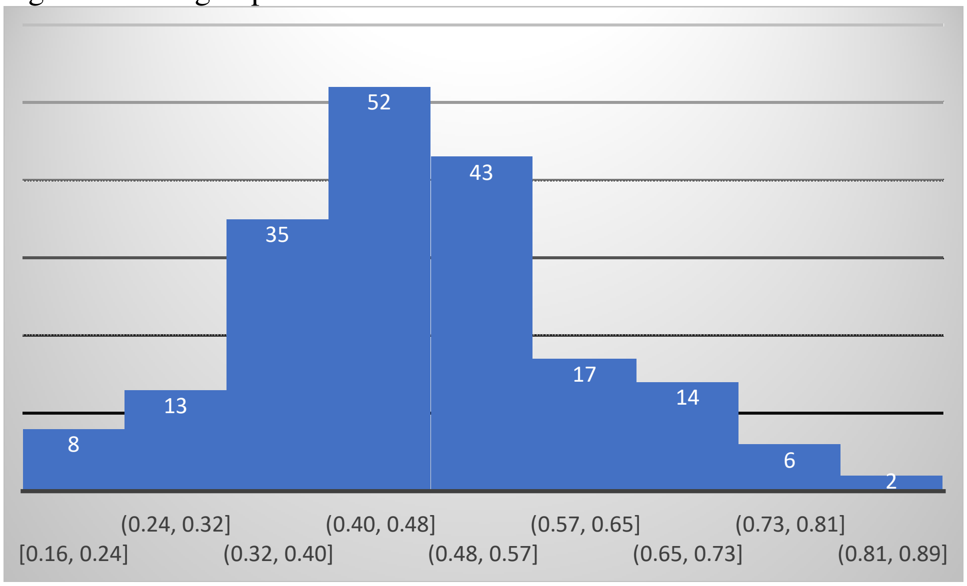


Figure 5 Average Speed over time