homework #2 Fitting Yield Curve with the Vasicek Model

1. Assume certain parameter values (e.g. $\alpha = 0.9$, $\mu = 0.05$, $\sigma = 0.1$, $\lambda = 0$) and use the Vasicek model (equation (14.2) on page 300) to plot a yield curve.
   - Let *t* be current time 0 and *T* be maturity time from 1 ~ 30 and compute $P(t,T)$ using (14.2)
   - Compute $y(t,T) = -\ln P(t,T)/(T-t)$
   - Draw the yield curve
2. Collect a yield curve (e.g. CMT rates) of any given day and fit it with the Vasicek model (i.e. compute α, μ, σ, and let $\lambda = 0$).
   - What are the values?
   - Draw the raw yield data (dots) and fitted curve (line)

Bonus

1. Do above using the CIR model (equation (14.4))
2. Do above using the two-factor Vasicek model (explained below)

Multi-factor Vasicek model

As in the lecture notes, equation (14.2) is derived via

$$P(t,T) = E\left[\exp\left(-\int_t^T r(u)du\right)\right]$$

Now let the interest rate be decomposed into K factors as follows:

$r(t) = \sum_{k=1}^{K} y_k(t)$ where factors are independent

Then

$$\begin{aligned} P(t,T) &= E\left[\exp\left(-\int_t^T r(u)du\right)\right] \\ &= E\left[\exp\left(-\left(\int_t^T y_1(u)du + \int y_2(u)du + \cdots\right)\right)\right] \\ &= E\left[\exp\left(-\int_t^T y_1(u)du\right)\right] E\left[\exp\left(-\int_t^T y_1(u)du\right)\right]\cdots \\ &= P_1(t,T)P_2(t,T)\cdots \end{aligned}$$

where each $P_i(t,T)$ is equation (14.2). Hence, we have K times as many parameters as in the one-factor model.

due on 4/1 (paper submission only)