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# Liquidity, leverage, and Lehman: A structural analysis of financial institutions in crisis ☆

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ABSTRACT

This paper presents a flexible, lattice-based structural credit risk model that uses equity market information and a detailed depiction of a financial institution's liability structure to analyze default risk. The model is applied to examine the term structure of default probabilities for Lehman Brothers prior to its demise. The results indicate, as early as March, that the firm would likely lose access to external capital within two years. The model can be used as both a diagnostic tool for the early detection of financial distress and a prescriptive tool for addressing the sources of risk in large, complex financial institutions.


## 1. Introduction

The recent financial crisis has demonstrated the pressing need for new tools to both measure and manage the risks of financial institutions. Perhaps no single event better illustrates the lapses in risk management and financial oversight than the dramatic failure of Lehman Brothers in September 2008. In only a few months, Lehman went from a leading and respected bulge bracket investment bank to a firm struggling to find external financing, and ultimately to a firm in throes of bankruptcy. This paper introduces a structural credit risk model to examine the interrelated and endogenous factors that served as the main catalysts of Lehman's default and bankruptcy: (1) excessive leverage, (2) over-reliance on short-term debt, (3) under-collateralization, and (4) inability to raise capital. The flexible, lattice-based model makes use of equity market information along with a detailed depiction of Lehman Brothers' liability structure to analyze the evolution of the firm's default probabilities on a month-by-month basis throughout 2008. The model allows for the identification of the early warning

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signs of rapidly escalating default risk. These warning signs may be useful to regulators and risk managers as a diagnostic tool to preemptively identify at-risk financial institutions (such as Lehman Brothers) that may be in need of intervention, before it is too late.

In response to the financial crisis, there has been increased attention on the importance of the liability structure of financial institutions (Kashyap et al., 2008; Squam Lake Working Group, 2009). An overreliance on short-term debt makes financial institutions more vulnerable to liquidity shocks; not necessarily in the sense of traditional depositor bank runs (as in Diamond and Dybvig, 1983), but instead, as shown by Gorton and Metrick (2012), as runs on other short-term debt instruments (i.e., repos). This paper focuses on economic default, or insolvency that arises from the value of distressed assets being insufficient to support long-term illiquid liabilities, rather than liquidity-driven failure. The underlying reason for the inability of a financial institution to renew repo contracts and the decision by lending institutions to curtail financing is thus the leverage and credit quality.

Economic default is defined endogenously in our model as the point at which the financial institution can no longer raise capital (debt or equity) in a perfect market. Of course, this definition abstracts real-world market imperfections and frictions that exacerbate the insolvency problems. Nevertheless, even using this strict definition of economic default, the model still provides substantial insight into how the specific makeup of a financial institution's balance sheet – from the liability structure in terms of both maturity and seniority to the liquidity of both assets and liabilities – amplifies the risk of distress resulting from their high degree of leverage.

Our analysis also highlights the importance of distinguishing between different quality assets on a financial institution's balance sheet. Berger and Bouwman (2009) argue that the assets of a financial institution, in addition to its liabilities, can also be classified into three liquidity class – liquid, semi-liquid, and illiquid. Since the value of the liquid assets is independent of the credit risk of the financial institution, it would be incorrect to include them in the estimation. Our model assumes that the most liquid assets can be sold at book value which can then be used to cover the most liquid liabilities. What remains is a form of *Net Debt* made up primarily of the least liquid, publicly-traded debt.

Our model incorporates the complex nature of the illiquid liabilities of financial institutions as well. The impact of the detailed liability structure on financial institution risk can only been seen in a multi-period model and not in a single period model such as the Black–Scholes–Merton model (see Black and Scholes, 1973; Merton, 1974) or the popular KMV implementation (see Crosbie et al., 2003). In a single period, there is no place for the modeler to allow debt to rollover or reduce debt through de-leveraging. Our model, which builds upon this option-theoretical framework presented by Geske (1979) and Leland (1994), allows for refinancing of debt and provides an endogenous and structural analysis of credit risk. Our model includes a flexible refinancing parameter which allows for different assumptions about how the bank funds its maturing debt, nesting both the Leland (1994) and Geske (1977) approaches. The Leland (1994) model, and its extensions, assumes that the firm replaces the maturing debt with an identical new debt, which results in a constant level of debt over time. Such models, which assume that firms *rollover* debt, capture some elements of the real world in which financial institutions continue to operate with high degrees of leverage. Alternatively, models based on the Geske (1977) compound option pricing model assume that the firm issues equity to fund maturing debt and implicitly de-leverages over time.

We apply our model to the case of Lehman Brothers to demonstrate both its diagnostic ability and corrective potential for regulators and risk managers. To this end we utilize a comprehensive data set of all publicly traded bonds ever issued by Lehman Brothers. The bond data, collected from FactSet, is supplemented with data from financial statements and regulatory filings; thus we create a detailed picture of the liability structure at the end of each month from December 2007 to August 2008.[4] We also use the market value of equity at month end and equity volatility as inputs in the model to estimate the market value of assets and asset volatility.

We implement a debt refinancing strategy that is intermediate between rollover of all debt and the paydown of debt with the issuance of new equity. Our attempt is to capture Lehman Brothers' (or any financial institution's) funding strategy in times of crisis, by choosing an intermediate value for the flexible refinancing parameter. We find that this generates a reasonable term structure of default probabilities, whose evolution we are able to study on a monthly basis leading up to the firm's bankruptcy filing in September 2008. From the term structure we are able to compute the *forward* default probabilities, which we find contains considerable information about future economic distress. The analysis of Lehman Brothers indicates as early as March 2008 that Lehman Brothers would likely lose access to external capital (debt or equity) within the next year.

We argue that the model's default probability estimates can be a very useful prognostic tool for regulators and risk managers. Our results demonstrate that markets clearly anticipated the financial crisis at Lehman Brothers well before the firm actually failed. The ex-ante increase in the forward default probability can be used to flag at-risk institutions. Our results confirm that the meager capital infusion in the Spring of 2008 was not sufficient to reduce Lehman Brothers' default risk to acceptable levels.

The rest of the paper is as follows: Section 2 reviews the relevant literature. Section 3 provides an overview of the model framework and uses a numerical example to help develop the intuition. Section 4 contains our analysis of Lehman Brothers. We first discuss the institutional background of Lehman Brothers including a timeline of the events during 2008, details on the balance sheet – assets, liabilities, and *net debt* – as well as some preliminary analysis of the firm's financial condition. We then present the model results and comprehensive analysis of Lehman Brothers' default risk; we show that our model is able to predict the distress that Lehman would encounter and demonstrates the need for equity capital; we also estimate the collateral that should have been required by senior creditors. Section 5 concludes.

## 2. Related literature

Our paper is related to three strands of literature: (1) research on the financial crisis, specifically papers that deal with the link between leverage and liquidity; (2) the role of market information in supervising and regulating financial institutions; and (3) the structural credit risk literature with applications to the analysis of banking firms. We review each of these in turn.

### 2.1. The financial crisis

Since our analysis examines the Financial Crisis – and specifically the failure of Lehman Brothers – from a credit risk perspective, we focus on recent research that seeks to provide a better understanding of the tension between leverage and liquidity in large, complex

[4] Our initial analysis was completed in 2009. In the few years that have since passed, previously-confidential documents have been made public and we now know a lot more about Lehman Brothers' final months and weeks (see, e.g., Valukas (2010) and Financial Crisis Inquiry Commission (2011)). Our results are consistent with many of the bankruptcy examiner's findings (Valukas (2010)) which we believe strengthens our argument that market information plays a valuable role in supervising and regulating large financial institutions. Furthermore, we were able to provide a more complete analysis of Lehman Brothers' liability structure by hand collecting data on private debts, secured financing, and collateral positions from these sources.

financial institutions during this time. Brunnermeier (2009) gives a very thorough treatment of liquidity during the Crisis, tying to both the theoretical models and empirical evidence. Shin (2009) profiles one of the first victims of the crisis, the UK based Northern Rock Bank in September of 2007. He describes a new type of bank run; one that is driven by traditional liquidity issues and maturity mismatch and further exacerbated by high leverage. Excess leverage, illiquidity, and credit risk went on to affect nearly every financial institution. Duffie (2010) describes how large non-depository financial institutions were brought down by runs from capital market creditors and counterparties. He further shows that the overreliance on short-term, unstable sources of funding – i.e., repurchase agreements, or repos – accelerated the process. Krishnamurthy (2010) reviews the mechanics and dislocations of the debt markets during the Financial Crisis. Both of these papers motivate the need for a model such as ours in terms of looking at the liability structure of large, complex financial institutions and quantifying the impact of collateral, repo haircuts, off-balance-sheet transactions, and access to capital on their overall risk.

Our characterization of Lehman Brothers in terms of repo transactions and off-balance-sheet financing fits with the description of "securitized banks" in Gorton and Metrick (2012). Their paper also provides good context for our model and specifically our analysis of Lehman. The application of our model to estimating collateral requirements is loosely related to the significant theoretical contributions of Geanakoplos (2010) and He and Xiong (2012a). Lastly, Adrian and Shin (2010) is an important paper documenting how financial institutions manage their leverage and the dynamics of the relationship between leverage and liquidity in the market. Our analysis actually complements theirs as we try to understand how the relationship manifested itself and perhaps broke down during the Crisis.

### 2.2. The use of market information

For years, some researchers have called for the use of market information in evaluating the extent to which banks take excessive risks. Flannery (1998) surveys the literature to evaluate how well market participants are able to assess the financial condition of banks. He concludes that bank supervisors should regularly incorporate market information in an effort to provide the most comprehensive oversight system. Krainer and Lopez (2004) specifically advocate the use of equity market information in this capacity. More recently, the use of market information in financial institution risk analysis has gained traction with measures such as the Marginal Expected Shortfall (MES) and SRISK which are computed on a real-time basis at the NYU-Stern Volatility Laboratory (V-Lab).[5] In Acharya et al. (2012b), the authors argue for the use of these market-based measures as applied to capital adequacy and systemic risk. Their work is motivated in part by the fallout after Lehman Brothers failed. An alternative market-based measure of systemic risk is proposed by Huang et al. (2012), where the authors use CDS market information to extract expectations about individual bank default probabilities and loss-given-defaults. A recent paper by Flannery (2013) uses a structural credit risk model (the version of the Black–Scholes–Merton model applied to banks in Ronn and Verma (1986) and Ronn and Verma (1989)) to examine the capital adequacy of banks. He finds that market-implied capital ratios are more informative than the book-value ratios typically used by regulators, and that the default probabilities computed by the model exceed the maximum targeted by Basel.

The use of market information should be approached with the caveat that it could induce the "self-fulfilling prophecy" problem.[6] If the market thinks that regulators will step in, then this could artificially keep asset prices high, which in turn may signal to the regulators that they need not step in, and so on and so forth (see Bond et al. (2010) for a general model of this phenomenon with an application to bank supervision and regulation). Based on this argument, we further suggest that market information can be a useful supplement to existing regulatory but not a replacement.

### 2.3. Structural credit risk models

Structural credit risk models began with Black and Scholes (1973), in their seminal option pricing paper, where they noted that when a firm has debt in its capital structure, equity is like a call option on the firm's unlevered assets. This idea was later formalized by Merton (1974). If the value of the firm's assets is not greater than the face value of the debt then shareholders choose to let the call option expire (i.e., default) and bondholders do not receive their promised payment but rather take ownership of the assets. In the Black–Scholes–Merton (BSM) framework, the probability that the call option is not exercised at maturity is thus a measure of default probability. Leverage, which is essentially the moneyness of the option, is clearly a key determinant of the default risk in this framework. However, the optionality-component introduces nonlinearities that are not picked up by a simple leverage ratio. Structural models use market information, specifically the market value of equity and equity volatility, as key inputs to arrive at default probability.

The KMV model, as described by Crosbie et al. (2003), is a modified version of the original BSM model that is very popular in practice. In order to adhere to the single-period European option framework, the KMV approach reduces the entire liability structure of a firm to a linear combination of two points: short-term and long-term debt. This approach thus assumes an exogenously specified simplistic default condition. Many extensions have been developed looking to incorporate more realistic features of debt. One class of extensions, commonly referred to as "barrier" structural models, was pioneered by Black and Cox (1976). Default probabilities are given by the first passage time density to the barrier. Longstaff and Schwartz (1995) and Madan and Unal (2000) develop models with stochastic interest rates and a flat exogenous barrier and Collin-Dufresne and Goldstein (2001) incorporate mean-reverting leverage ratios. Leland (1994) derives a barrier structural model with *endogenous default*. The model solves for both the optimal capital structure and the price of risky debt in the presence of taxes and bankruptcy costs. Leland and Toft (1996) extend the Leland (1994) model to take into account debt maturity.

Distinct from the barrier models is the compound option model of Geske (1977).[7] In the Geske model, shareholders own a *compound* option on the firm's unlevered assets. Default probabilities are found by calculating the probability that the compound option is not going to be exercised at a particular future cash flow time. This allows for the calculation of both the conditional and unconditional default probabilities and actually results in a term structure of default probabilities. Default is determined endogenously as a function of the liability structure and the associated promised cash flows.

Several papers present empirical studies on the performance of structural models in computing default probabilities and predicting default. Bharath and Shumway (2008) conclude that the KMV version of the BSM structural model cannot produce a sufficient statistic for predicting default. However, in their analysis they did not consider structural models that include a more complete

[5] http://vlab.stern.nyu.edu/.

[6] We thank the referee for bringing up this point.

[7] The Geske compound option model is also derived in a no-arbitrage setting in Geske (1979), where a leverage effect is shown to result in non-constant volatilities. The original Geske (1977) model was modified by Geske and Johnson (1984) to properly account for the seniority structure of debt.

representation of the liability structure or models with endogenous default. Leland (2004) looks at default probabilities for industrial firms, calculated using two structural models – the *endogenous barrier* model of Leland and Toft (1996) and the *exogenous barrier* model of Longstaff and Schwartz (1995). Delianedis and Geske (2003) compute risk-neutral default probabilities (RNDP's) using the original BSM model and the Geske model.

Our model, which is a lattice-based model with endogenous default, nests a direct extension of the Geske compound option model with a discrete cash flow version of the Leland model.[8] Our model allows for a full specification of the financial institution's liability structure and does not require calibration. This makes it very attractive for the analysis of financial institutions. Leland (2009) examines the performance of structural credit risk models during the Financial Crisis; specifically looking at Goldman Sachs and JP Morgan from 2006 to 2009. He notes that, relative to industrial firms, it is more difficult to calibrate structural models for financial firms due to the high degree of leverage and the reliance on disproportionate amounts of short-term debt, including repos and deposits. As we show using Lehman Brothers as an illustration, our model is especially well-suited for measuring default risk in financial institutions. Prior to our paper, there have been very few advances in structural modeling of banking firms. A notable exception is Liao et al. (2009) who study agency conflicts and asymmetric information within banks using a structural credit risk model.

## 3. Model framework

In this section we discuss our dynamic lattice based structural model for estimating default risk in financial institutions. Structural models are especially well-suited for managing and monitoring credit risk, either internally or externally, as they use the most recent inputs from financial statements and market data. We first review the role of capital structure assumptions in structural models and next present our lattice model.

### *3.1. Endogenous default and capital structure policy*

Black and Scholes (1973) and Merton (1974) pioneered the notion that when a firm has risky debt outstanding, the equity is very much like a call option where shareholders are faced with the decision to exercise when payment is due to debtholders. Upon maturity of the debt, shareholders can choose to not make the payment, thereby letting the call option expire unexercised, and default. In the Black–Scholes–Merton (BSM) approach, the liabilities are modeled as a fixed point barrier with only one future date in which the exercise decision is made. The firm has the option to default only at one point – on the final maturity date of the debt.

Financial institutions have many different debt contracts outstanding at any given time and, therefore, the fixed point barrier has limited application. The basic BSM model has been extended to accommodate more complex liability structures consisting of debt with different maturities, seniorities, coupon payments, and covenants (see Geske, 1977; Black and Cox, 1976; Leland, 1994; Leland and Toft, 1996) and allow for an endogenous default barrier. To allow for endogenous default, the survival condition must be that the firm has the capability to raise capital. Such a condition can be shown to be identical to requiring that the asset value be no less than the total debt value, which is dependent upon the default barrier.

Let the firm's assets evolve according to a diffusion process with dynamics described by the Stochastic Differential Equation:

$$\frac{dA(t)}{A(t)} = rdt + \sigma_A dW(t) \tag{1}$$

where $A_t$ represent the value of the firm's assets at time $t$, $r$ represents the risk-free rate, $\sigma_A$ represents the volatility of the firm's assets, and $dW_t$ is the Wiener process under the risk neutral measure.

Define $E$ as the equity value of the firm. Then, both Geske (Eq. 1 on page 66, 1979) and Leland (Eq. 2 on page 1218, 1994) show that the partial differential equation for the equity (that is the same as that of the Black–Scholes) is as follows:

$$rE = \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2 E}{\partial A^2} + rA\frac{\partial E}{\partial A} + \frac{\partial E}{\partial t} \tag{2}$$

Also define $D$ as the market value of debt for the firm; then from the market-value equivalent of the balance sheet identity it must be that

$$D(t) = A(t) - E(t), \tag{3}$$

for all $t$. The capital structure policy of the firm is formulated as follows. The firm has a collection of debts with various coupons and maturities. These debts have associated with them a sequence of cash obligations defined as $K_1, K_2, \dots, K_k$ that are paid at times $T_1, T_2, \dots, T_k$. On the date of the final contractual cash flow at terminal time $T_k$, the firm liquidates. Now define $K_k^* = K_k + \lambda K_{k-1}^*$ where $\lambda \in [0,1]$ is our "refinancing parameter". We discuss the implications of this parameter later in the section.

The value to equity holders and the optimal exercise decision is then given by,

$$E(T_k) = \max\{A(T_k) - K_k^*, 0\} \tag{4}$$

where $E(T_k)$ is the equity value at the terminal time $T_k$ , $A(T_k)$ is the asset value at the terminal time $T_k$, and $K_k^* = K_k + \lambda K_{k-1}^*$ for $k > 0$ and $K_1^* = K_1$ is the first debt cash flow.

At time $T_{k-1}$, when cash flow $K_{k-1}^*$ is due, the equity holders must decide if such a payment is worthwhile. If the equity holders decide rationally that it is, then the payment is made and the firm survives; otherwise the firm defaults. Formally, we can specify the equity value as

$$E(T_{k-1}) = \begin{cases} \mathbb{E}_{T_{k-1}}\left[e^{-r(T_k - T_{k-1})}E(T_k)\right] - K_{k-1}^* & \text{if} \mathbb{E}_{T_{k-1}}\left[e^{-r(T_k - T_{k-1})}E(T_k)\right] > K_{k-1}^* \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $\mathbb{E}_u[\cdot]$ represents the risk-neutral expectation conditional on time $u$.[9] Continue to move backwards and we obtain the current equity value as:

$$E(t) = \max\left\{\mathbb{E}_t\left[e^{-r(T_1 - t)}E(T_1)\right] - K_0, 0\right\} \tag{6}$$

where $K_0 = 0$. As demonstrated by Geske (1977), $E(t)$ involves multidimensional integrals thereby making it very difficult to implement. Leland (1994) and Leland and Toft (1996) derive closed-form solutions with the simplification that cash flows are paid continuously and are level over time, giving rise to a flat default barrier.

In general, the default barrier, the level at which the asset value must stay above in order for the firm to survive, can only be solved endogenously in a recursive manner as $\mathbb{E}_{T_i}\left[e^{-r(T_i - T_{i-1})}E(T_i)\right] = K_{i-1}^*$ for $i = 1, \dots, k$. It is apparent that this endogenous default barrier, which we denote as $\bar{A}(T_i)$, is the asset value that sets the expected equity value, as a function of assets, exactly equal to the next cash flow obligation. This is the point at which existing shareholders will walk away as the firm will no longer be able to raise capital (by rolling-over debt and/or raising new equity).

[8] Jabbour et al. (2010) use a similar lattice approach, noting the flexibility of the model and application to firms with complex liability structures. They do not, however, specifically look at financial institutions nor do they emphasize the refinancing assumptions embedded in various structural credit risk models.

[9] Note that later on in our model such formulation is cast in a discrete-time, lattice framework and the expectation is taken at the lattice time steps, not the cash flow time steps. Yet the valuation described by the equation is still valid.

In modeling a financial institution, we must consider the scenario that a regulator may decide to intervene before the firm's asset value falls to the level of the default barrier. We incorporate a "safety factor" in the default boundary to account for regulatory intervention in the form of prompt corrective action or early resolution, before conditions deteriorate too far.[10] This safety factor is incorporated into the model by scaling the default barrier up by a factor of $(1+\alpha)$, where $\alpha$ is the desired point at which the regulator will intervene before the financial institution reaches the default threshold, and either force the institution to raise capital (prompt corrective action) or begin to unwind and dissolve it (early resolution). The inclusion of the safety factor results in an effective increase in default probabilities, since the regulator will step in earlier than otherwise. That is, once the asset value falls to $\overline{A}(1+\alpha)$, the threshold condition is triggered; therefore, the model indicates the likelihood of default due to regulatory restructuring. In the empirical implementation, we set $\alpha = 0.02$ which implies that intervention occurs when the asset value falls within 2% of the default barrier. However, we note that regulators or risk managers could use any value for $\alpha$ including $\alpha = 0$ if they assume no prompt corrective action and/or resolution. Default probabilities are monotonically increasing in $\alpha$. Another potentially interesting feature about incorporating a scale factor such as $\alpha$ is that one may chose to randomize its value adding another source of uncertainty. This is similar to the approach taken by Duffie and Lando (2001) to model imperfect information about a firm's assets and liabilities.

While modeling the endogenous default barrier is similar in both the Geske and Leland approaches, how maturing debt is financed differs substantially. The boundary conditions for Eq. (2) are different for the Geske and Leland approaches, which leads to different closed-form solutions. In the Geske model, maturing debt is funded by new equity, i.e., the firm goes through a de-leveraging process and we refer to this as the *de-leveraging* assumption. In the Leland and Leland–Toft models, an identical new debt is issued to pay for the maturing debt, i.e., the firm rolls-over its debt and we refer to this as the *rollover* assumption. The different assumptions essentially imply different capital structure policies for the financial institution. The Geske approach implies that the firm de-leverages over time and the Leland approach assumes that the firm maintains a constant amount of debt (in face value terms) over time. Indeed, the Geske and Leland approaches represent only two possible ways of modeling capital structure policy and firms clearly can follow intermediate policies, wherein they replace a part of their debt with equity and rollover the rest, as well. Our approach allows for a range of capital structure assumptions. To model this, we have to let the cash flow at time $T_j$ be a function of the capital structure policy and refinancing decision of the firm. If all maturing debt is replaced by new equity then $K_j^*$ is the raw cash flow (coupon and/or face value due); if maturing debt is financed by new debt then $K_j^*$ represents the cumulative sum of all previous cash flows. We parameterize this funding decision with $\lambda$: when $\lambda = 0$ the firm does not rollover any of its debt but rather raises new equity to fund its imminent obligations, and when $\lambda = 1$ the firm rolls-over all of its debt to the next period (i.e., funds the imminent obligations by issuing new debt). In practice financial institutions rarely, if ever, adhere to one of the extreme strategies, but more likely rollover debt when they can and raise equity when they need to in some proportion. In support of this, Adrian and Shin (2010) provide empirical evidence of financial institutions actively managing their leverage which, in our model, would be represented by the firm strategically choosing a value of $\lambda$ to maximize some objective function. Presumably $\lambda$ would be time-varying as well, but both of these items are left for future research, along with tying $\lambda$ to liquidity conditions, as they are nontrivial extensions of the model and do not serve an immediate need for our analysis here. For now, we assume some $\lambda \in [0,1]$ exogenous to the model. The Geske compound option model and the Leland–Toft flat barrier model, loosely represent two cases nested within our model, when $\lambda = 0$ and $\lambda = 1$, respectively. We also note that it is possible to choose a random $\lambda$ so as to achieve mean-reverting leverage akin to the Collin-Dufresne and Goldstein (2001) model, which does not have endogenous default and therefore does not have the insight we are looking for in terms of modeling when the financial institution would lose access to capital.

We also note that there is a known relationship between the volatility of assets and the volatility of equity.[11] It is:

$$\sigma_E = \frac{A}{E}\sigma_A \frac{\partial E}{\partial A} \tag{7}$$

where,

$$\frac{\partial E}{\partial A} = \mathbb{E}_{T_i}[A(T_i)\mathbb{I}] \tag{8}$$

where, $\mathbb{I}$ is equal to 1 if $A(T_i) > K_i^*$ or 0 otherwise for all $i$.

In implementing our model, we use the following iterative procedure. We start with initial guesses for the market value of assets $A(T_0)$ and the asset volatility $\sigma_A$. We solve the model for the value of equity $E(T_0)$ and $\sigma_E$ corresponding to these initial guesses for $A(T_0)$ and $\sigma_A$. We compare the realized values for $E(T_0)$ and $\sigma_E$ to the observed values, and update the estimate for $A(T_0)$ and $E(T_0)$ using a bisection search algorithm.

Our model is a discrete time, lattice implementation and directly solves for the endogenous default barrier without the need for the expensive iterative calculations. Shareholders of the firm are modeled as having several sequential exercise decisions to make. For each debt in the capital structure, we note the expiration date and cash flow, taking into account the debt rollover policy. At each of the debt maturity dates, a cash flow payment is due and the shareholders have an exercise decision to make. The shareholders can default on the debt by choosing not to pay the amount due and turn over the assets to the debt holders. The decision to exercise the option to default fully takes into account the relative value of the assets and the debt of the firm. At each node in the lattice the default condition is evaluated. Recall, the default barrier refers to the asset value below which the shareholders will exercise the option to default. This is traced out across the lattice at each point in time and, therefore, the default boundary represents the "cut off" asset value: nodes above are survival states and those below represent the firm being in default at time $t$.

Our lattice model has $k$ cash flows and a total number of $n$ periods, denoted as $L_1, L_2, \ldots, L_n$. The cash flows are due at times $T_1, T_2, \ldots, T_k$ that are not necessarily equally spaced. In between any two cash flows, the lattice is further partitioned into $m_1, m_2, \ldots, m_k$ periods respectively. In other words, $m_i$ represents the number of steps between $T_i$ and $T_{i-1}$ and $\sum_{i=1}^{k} m_i = n$ (see Fig. 1 below). Note that these cash flows may contain interest and principal from various debts. In our model, $T_n$ is the liquidation date of the firm. Certainly, $T_n \geqslant T_k$ as the firm should not liquidate prior to its last cash obligation.

We estimate the default boundary, debt value, and equity value by backward recursion in the tree. The refinancing assumptions translate to the structure of the liabilities at each node of the tree. A Leland model implementation, for example, would required that the liability structure at each node be identical to the liability structure at the first node (subject to discretization approximations). A Geske model implementation, on the other hand, would only incorporate debt maturing at the node and debt maturing at

[10] We thank the referee for suggesting this addition to the model.

[11] This follows directly from Ito's formula.

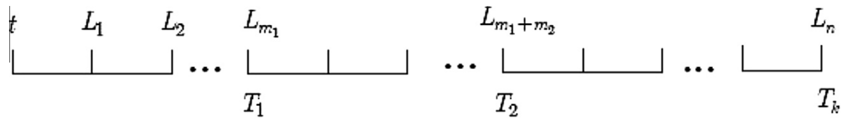


**Fig. 1.** Timeline of cash flows in the lattice.

all other nodes further down the tree. In our model, the $\lambda$ parameter governs the liability structure in a flexible, yet rigorous way.

While the model emphasizes the role of a financial institution's liability structure in driving default risk, the assets of the bank are also an important factor. One of the benefits of our structural approach to modeling bank default risk is that it allows for an effective way to include both sides of the balance sheet when assessing a bank's risk. To extend the model for any arbitrary $n$ asset classes, we re-define the SDE in Eq. (1) to

$$\frac{dA_i(t)}{A_i(t)} = rdt + \sigma_{A_i} dW_i(t) \tag{9}$$

where $i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} A_i(t) = A(t)$ is the total assets of the firm at time $t$. The firm's equity $E(t)$ continues to be a call option on these assets and the PDE in Eq. (2) holds. We note that when each asset follows a lognormal process, the sum of all assets will not follow a lognormal process, making a closed-form solution virtually impossible. However, our lattice implementation can easily accommodate this extension.

For the sake of exposition and without any loss of generality, we let $n = 2$. The first may represent liquid assets and the second represents illiquid assets. We can assume that the market value of the liquid asset is the same as the book value, whereas the market value of the illiquid asset could be valued below its book value in times of crisis. In an extreme (and most conservative) case where market liquidity entirely disappears, $A_2 = 0$ and therefore total asset value approaches $A_1$.

We believe our approach is especially well-suited for analyzing default in financial institutions for three reasons. First, it is important to incorporate an endogenous default boundary that takes into account market conditions and the ability to raise capital in evaluating economic default or insolvency.[12] Second, our model takes all of the debts and associated cash flows into account along with alternative financing assumptions which provides a more accurate and robust depiction of a financial institution's liability structure. Third, we allow for the decomposition of the assets into multiple categories.

Next, we provide some examples to help illustrate how our model works and the insights that can be obtained.

### 3.2. A three-cashflow, six-period lattice example

Consider a 6-period binomial lattice model where three cash flows are paid. Hence, $n = 6$ and $k = 3$. We can assume all periods are evenly spaced, although this need not be the case. Following the notation in Fig. 1 above, in periods $L_2$ $(T_1)$, $L_4$ $(T_2)$, and $L_6$ $(T_3)$, cash flows $K_1, K_2$, and $K_3$ are paid.

Along the lattice, the equity value is computed as the risk neutral expectation of the values in the next period, that is $E(L_i) = \mathbb{E}_{L_i}\left[e^{-r\Delta t}E(L_{i+1})\right]$ for any $t$ strictly in between two cash flows. At a time of the cash flow, however, the following calculation is executed: $E(T_j) = E(L_i) = \max\left\{\mathbb{E}_{L_i}\left[e^{-r\Delta t}E(L_{i+1})\right] - K_j, 0\right\}$. The above two steps can be used to calculate the values at all other nodes in the lattice.

This lattice model permits us to easily track important values for the model. First, is the default barrier value at each cash flow time $T_j$, symbolized as $\bar{A}(T_j)$. The default barrier represents the critical values such that the firm is solvent if the asset values are above these critical values. In a numerical demonstration later, one can see how the barrier values can be observed and captured.

Second, we track the survival probability at each cash flow period, $T_j$. The survival probability is the joint probability that the asset value stays above the default boundary for all $t$ before and including $T_j$. That is,

$$Q(t, T_j) = \Pr(A(T_1) > \bar{A}(T_1) \cap A(T_2) > \bar{A}(T_2) \cap \cdots \cap A(T_j) > \bar{A}(T_j)) \tag{10}$$

This joint probability is easy to compute within the lattice. We simply trace each path in the lattice and count only those that survive.

Third, we can track the default probabilities. The spot default probability between any two cash flow periods is simply the incremental change in survival probability. Specifically,

$$p(t, T_{j-1}, T_j) = Q(t, T_{j-1}) - Q(t, T_j) \tag{11}$$

This means that to compute the $j$th default probability, the firm must survive until time $T_{j-1}$ and then default at time $T_j$. Similar to the computation of the survival probability, we trace defaults along the lattice.

We also want to compute the cumulative default probabilities, which will give us our default probability term structure. Given the spot default probabilities, the cumulative probability of defaulting at or before time $T_j$ is just the sum of the spot default probabilities up to that time, which is easily shown to be

$$DP(t, T_j) = 1 - Q(t, T_j). \tag{12}$$

Therefore, at any given time $t$, we can compute the cumulative default probabilities for increasing $T_j$'s and obtain a full default probability term structure.

From the default probability term structure, we can extract *forward* default probabilities, which later we suggest would serve as a good "early warning signal" for identifying distressed financial institutions in times of crisis. This is computed as

$$f(t, T_j, T_{j+h}) = \frac{(Q(t, T_j) - Q(t, T_{j+h}))}{Q(t, T_j)} \tag{13}$$

which shows the probability of defaulting between $T_j$ and $T_{j+h}$ standing at time $t$ now, conditional upon surviving up to time $T_j$.

Once we have the default and survival probabilities we can compute the market values of equity and debt as a function of the asset values and cash flows along the lattice. Equity volatility is computed as a function of asset volatility, the hedge ratio, and the market values of debt and equity as indicated by Eqs. (7) and (8).

Lastly, we can use the model to compute the recovery on default, a quantity that will be very important in our proposed measure for default-risk-based haircuts on secured debt.[13] Let us suppose, for simplicity, that the first cash flow, $K_1$, is the most senior

[12] Davydenko (2012) studies the distinction between default due to insolvency, which we refer to as "economic default", and default due to illiquidity within the context of structural credit risk models and the default boundary. He and Xiong (2012b) explicitly model liquidity shocks in the debt markets and the impact that it has on the default condition within a Leland framework. Neither of these papers deal specifically with financial institutions.

[13] Stulz and Johnson (1985) develop a structural model of secured debt.

and the last cash flow, $K_3$, is the most junior. Then, in our model, the recovery on default for senior debt would be

$$R_1 = \min\{K_1, A(\tau)\}, \tag{14}$$

for the intermediate debt class it is

$$R_2 = \min\{K_2, A(\tau) - K_1\}, \tag{15}$$

and for the junior debt class it is

$$R_3 = \min\{K_3, A(\tau) - K_1 - K_2\}. \tag{16}$$

Note that in the above equations, the asset value upon default, $A(\tau)$ is a random variable that is not known at time $t < \tau$. However, we do know that $A(\tau) \leqslant \overline{A}(T_1)$. In order to estimate the recovery on default we could substitute $\overline{A}(T_1)$ into the equations to get an upper bound on what each debt class could expect to recover upon default at time $\tau$ or we could compute the path-wise expectation $\mathbb{E}_t[A(\tau)]$.

### 3.3. Numerical example

In this section we present a numerical example of our lattice model and our approach to solving for the market value of illiquid assets and asset volatility given equity value and volatility.

Let the equity value and equity volatility observed in the market for our hypothetical firm be \$27.4 and 78.5%, respectively. These represent the input market parameters needed by our model. Next assume that our firm faces three cash flow obligations, i.e., $k = 3$, to be paid at $T_1, T_2$, and $T_3$. The cash obligations are $K_1 = 10, K_2 = 20$, and $K_3 = 275$ respectively. We show below how we can implement the lattice model and infer the market value of assets and asset volatility.

For this example, we will construct and use a 6-period equally spaced binomial lattice with $\Delta t = 0.5$ year. That is, $n = 6, L_2 = 1, \ldots, L_6 = 3$ years. The timing of the three cash flows in the tree is therefore, $T_1 = L_2 = 1$, $T_2 = L_4 = 2$, and $T_3 = L_6 = 3$. Let the risk free rate be 3%.

We begin with an assumed value for the market value of assets and an assumption for the asset volatility. Let the market value of the assets of the firm be 300 and let asset volatility, $\sigma$, be 10%. Using the standard binomial model of Cox et al. (1979), we have: $u = e^{\sigma\sqrt{\Delta t}} = 1.0733$ and $d = e^{-\sigma\sqrt{\Delta t}} = 0.9317$. The risk-neutral probabilities are $q = \frac{e^{r\Delta t} - d}{u - d} = 0.5891$ and $1 - q = 0.4109$. Fig. 2 shows the asset value binomial tree beginning at $A(0) = 300$, probabilities of each state occurring are given below the asset value in parentheses. The asset value at period $i$ in state $j$ is $A_i(j) = A_0 u^j d^{i-j}$ for $i \leqslant n$ and $j \leqslant i$ is the number of up steps. The probability of ending up at node $(i,j)$ is given by the binomial distribution formula. Note that over the three years ($i = 6$), the two extreme asset values are $A(j = 6) = \$458.5$ with a 4.2% probability of occurring and $A(j = 0) = \$196.30$ with a 0.5% probability of occurring. That is there is a 4.2% chance of a +53% asset return and a 0.5% of a −35% asset return over the three year period. Such extreme values are consistent with the assumption of a discretized Geometric Brownian Motion (multiplicative binomial approximation to lognormal distribution).

We next show how the lattice can be used to compute default probabilities as well as estimates of equity value and volatility given the firm's liability structure.

#### 3.3.1. De-leveraging ($\lambda = 0$)

As discussed before, we need to make assumptions about the policy that the firm will implement with respect to refinancing maturing debt. In this section, we illustrate the case where the firm de-leverages as in the Geske (1977) model. The Geske assumption implies that the cash flow amount that is due at each time $t$ is equal to the amount of debt maturing at time $t$ as there is no rollover of debt. This corresponds to our model implemented with $\lambda = 0$. Therefore, the cash flows that the firm has to pay are \$10, \$20, and \$275 at 1, 2, and 3 years respectively.

We begin by determining the optimal default decision that equity holders will implement given the asset value tree and the liability structure given above. Fig. 3A, B and C, respectively, show the equity values (assuming the tree of asset values in Fig. 2) in the lattice over the three time periods, working backwards from the last time step. It follows a "waterfall" structure, where the cash flows are paid sequentially and, in those states where it is optimal, equity is raised to make the debt payment. The equity value also takes into account the optimal decision to not make the debt payment and declare bankruptcy, which is shown as zero values in the lattice. Everything is weighted by the probability of the particular state occurring (shown in Fig. 2 at each node). We note that we are using a small number of binomial steps in our illustration, which reduces the precision of the numerical values in the example due to lack of granularity. The granularity of the lattice and precision

| | | T=1 | | T=2 | | T=3 |
|---|---|---|---|---|---|---|
| | | | | | | 458.5 |
| | | | | | 427.2 | (0.042) |
| | | | | 398.1 | (0.071) | 398.1 |
| | | | 370.9 | (0.12) | 370.9 | (0.17) |
| | | 345.6 | (0.20) | 345.6 | (0.25) | 345.6 |
| | 322.0 | (0.35) | 322.0 | (0.34) | 322.0 | (0.31) |
| $A_0$=300 | (0.59) | 300.0 | (0.43) | 300.0 | (0.35) | 300.0 |
| | 279.5 | (0.48) | 279.5 | (0.35) | 279.5 | (0.28) |
| | (0.41) | 260.4 | (0.30) | 260.4 | (0.24) | 260.4 |
| | | (0.17) | 242.7 | (0.16) | 242.7 | (0.15) |
| | | | (0.07) | 226.1 | (0.08) | 226.1 |
| | | | | (0.03) | 210.7 | (0.04) |
| | | | | | (0.01) | 196.3 |
| | | | | | | (0.005) |

**Fig. 2.** Numerical Example – 3-period Asset Binomial Tree. Fig. 2 presents the binomial tree used in the numerical example described in Section 3.3. The binomial tree is constructer for the following parameters: Current asset value of $A = 300, \sigma = 10\%$ and $r = 3\%$. Using the standard binomial model of Cox et al. (1979), we have: $u = e^{\sigma\sqrt{\Delta t}} = 1.0733$ and $d = e^{-\sigma\sqrt{\Delta t}} = 0.9317$. The risk-neutral probabilities are $q = \frac{e^{r\Delta t} - d}{u - d} = 0.5891$ and $1 - q = 0.4109$.

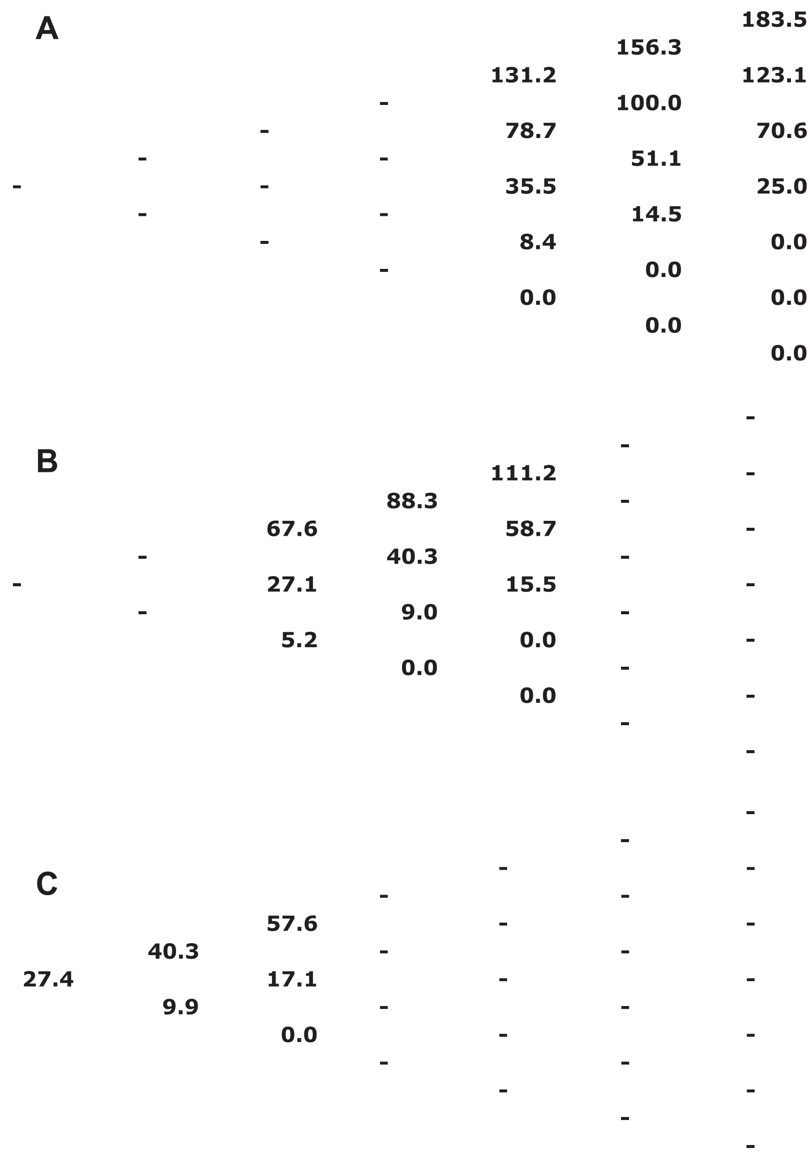


**Fig. 3.** Numerical Example: Equity Values in a Discrete 3-period Binomial Tree. Fig. 3 shows the equity value at the different nodes of the asset value binomial tree shown in Fig. 2. The liability structure of the firm is assumed to have debt maturing each year for three years. The face value of the debt maturing in years 1, 2, and 3 are $K_1 = \$10, K_2 = \$20$, and $K_3 = \$275$, respectively. In this illustration, the firm is assumed to replace debt with equity and de-leverage, i.e., the Geske model. Parameter values of $A = 300$ and $\sigma_a = 10\%$ result in the value of equity equal of \$27.4.

of the numerical values become less of an issue in any real world application that uses a large number of binomial steps.

The default boundary is obtained by tracing the lattice and identifying the states at which it is optimal for equity holders to not make a payment. Using Eqs. (4)–(6), it is clear that these are the states corresponding to zero values in Fig. 3C, B and A. For example, in Year 1, default occurs in the bottom state, i.e., the state with the lowest asset value. Hence, the default boundary must lie between 300 and 260.44. We assume that the firm defaults at the average of the asset values where the firm just survives and the firm just defaults, e.g., the bottom two nodes in Year 1. Of course, in models with a higher number of steps and a smaller time period per step, the gap will narrow and converge on the asset value that sets equity just equals to the debt cash flow. In Year 2, the bottom two states default and top three states survive. Hence, the default boundary falls between 260.44 (the node in Fig. 2 which corresponds to zero equity value in Fig. 3B) and 300 (the node in Fig. 2 which corresponds to the adjacent positive equity value in Fig. 3B). In Year 3, we know for sure the default boundary is 275; the last debt payment. The default boundary curve is therefore: 280.22 (the average of 300 and 260.44), 280.22, and 275 at 1, 2, and 3 years, respectively.

To compute the survival probability, we trace survival through the lattice, i.e., wherever the value of equity is greater than 0. For the first cash flow, survival occurs at the asset values of 322.0 and 279.5, which correspond to equity values of 57.6 and 17.1 in Fig. 3C. Recall that $q = 0.5891$, so the survival probability is equal to $q^2 + 2q(1 - q)$ or 83.1% in this case. Similarly, in the second year the survival probability is 74.9% and 70.0% in the third year. The default probabilities are therefore 16.9%, 8.2%, and 4.9% in Years 1, 2, and 3, respectively.

We see from Fig. 3, the value of equity at $t = 0$ is equal to \$27.4 and the equity value at $T_3$ $(t = 3)$ can be as high as \$183.5. The change in equity value arises from two factors. First, the initial value of equity at $t = 0$ increases as the firm experiences a positive shock represented by the up factor, $u$, which is a function of volatility. The higher market volatility is, the more dispersion in the state-contingent equity values coming from the structural model.

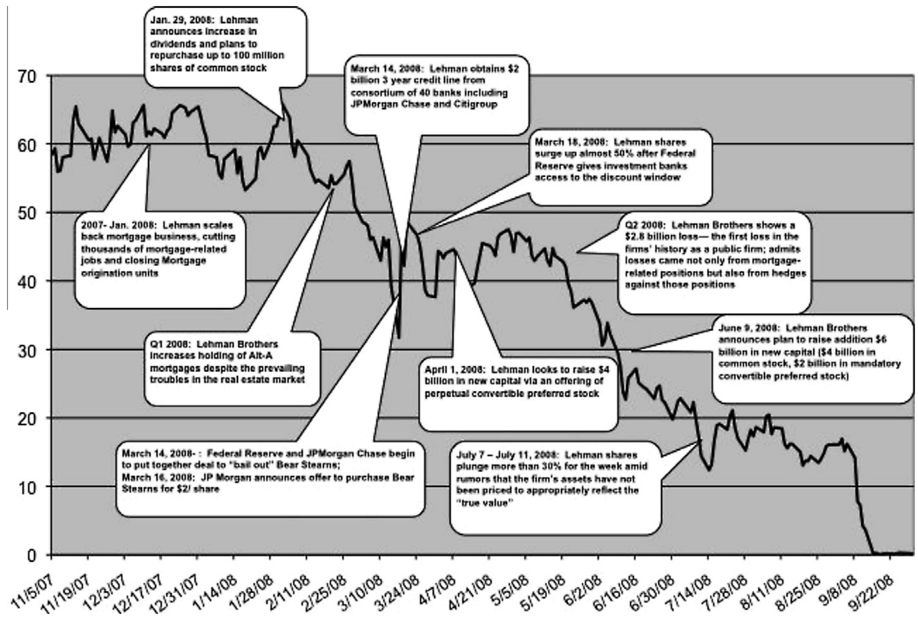


**Fig. 4.** Lehman Brothers time line.

Second, the de-leveraging assumption ($\lambda = 0$) implies that the firm issues new equity to retire debt and make each cash flow. Each cash flow payment results in an increase in the equity base which is reflected in all future equity values. This can be seen, for instance, going from the highest state at $T_2$ in Fig. 3B where equity value is \$111.2 to the same state in Fig. 3A where equity value is \$131.2, reflecting the \$20 debt payment and simultaneous raise in equity.

The model also gives the market value of the firm's debt. Using the accounting identity, the value of debt at $t = 0$, is \$272.6 (300.0–27.4). Finally, Ito's Lemma gives the equity volatility at time $t = 0$ as a function of the asset volatility. Applying the transformation, $\sigma_E = \frac{A}{E}\sigma_A\frac{E}{A}$, we find the equity volatility to be 78.43%. Thus, the asset value of \$300 and asset volatility of 10% are consistent with equity value and volatility we specified at the beginning of the section. In practice, it is the equity value and equity volatility that are the observed data points and, therefore, we must solve for the asset value ($A(0)$) and asset volatility ($\sigma_A$) that sets the model outputs equal to the observed equity value and equity volatility. This is done using an iterative procedure; given an initial guess for the asset value and asset volatility, a search algorithm updates the values until the model outputs come within some specified margin of the observed data.[14]

*3.3.2. Debt rollover ($\lambda = 1$)*

In this section, we present the calculations for the case if the firm were to rollover debt into similar maturity debt. This is parameterized in the model with $\lambda = 1$. The rollover assumption implies that the cash flow due at each time $T_j$ is paid for by a new debt due in the next cash flow payment time, i.e., $T_{j+1}$. As a result, the firm faces cash flow needs of $K_1 = \$10, K_2 = \$30$ (i.e., 10 + 20), and $K_3 = \$305$ (i.e., 30 + 275), respectively. Note, however, that the firm issues new debt to retire debt, so that there is no exchange of debt for equity in the tree. Using the same asset value tree in Fig. 2, the value of equity under this assumption is \$8 and the value of the firm's debt is \$292. The default probability at $T_1$ ($t = 1$) is 65.3%. This shows the additional default risk from the rollover of debts rather than de-leveraging over time.

The liability structure is such that the firm has bonds maturing at $t = (1, 2, \text{ and } 3)$ with face values 275, 10, and 20, respectively. The rollover assumption implies that the cash flows $K_1, K_2$, and $K_3$ are 275, 285, and 305, respectively. The value of equity under this assumption is 0 and the value of the firm's debt is 300 (assuming no bankruptcy cost). The default probability at $T_1$ ($t = 1$) is 100%. The example indicates the importance of the debt maturity structure and the ability of our lattice model to incorporate alternate capital structure policies and debt refinancing assumptions.

## 4. Analysis of Lehman Brothers

On September 15, 2008, Lehman Brothers Holding Inc. (Lehman hereafter) filed the largest bankruptcy in the U.S. history with gross assets over \$600 billion. Lehman was the 4th largest investment bank in the USA behind Goldman Sachs, Morgan Stanley, and Merrill Lynch prior to its bankruptcy. The C.E.O. of Lehman was Richard (Dick) Fuld who began his career with Lehman Brothers in 1969 and has been characterized as a stereotypical Wall Street investment banker: aggressive, ruthless, and ambitious. At the time of its demise, Lehman was highly leveraged and used a large amount of short-term repurchase transactions (i.e., *repos*). The high leverage and reliance on short-term financing was rumored to have led to difficulties in Lehman being able to renew the contracts and banks refused to lend, leading to Lehman's demise. Lehman's downfall represented an inflection point in the crisis and set off the worst economic recession after World War II.

Whether Lehman's failure, and indeed the entire crisis, was a liquidity crisis or a credit crisis is an ongoing debate. Irrespective of the reason, we are able to use our model to better understand the dynamic interaction between Lehman's excessive use of leverage, passivity in raising capital, and ultimately the loss of confidence of their creditors and counterparties. So while it may seems that immediate cause of Lehman's demise was the inability to

[14] For example, suppose we observed equity value of \$21.1 rather than \$27.4 and estimated equity volatility to be 61.52%, then clearly our assumption of asset value equal to \$300 and asset volatility of 10% would be incorrect. However, it is not difficult to find that the correct asset value and volatility would be \$300 and 5%, respectively. Similarly, if we observed equity value of \$35.5 and estimated equity volatility to be 85.26% then the right asset value and volatility would be \$300 and 15%, respectively. We discuss the implications of this in our empirical analysis later in the paper.

renew the repo contracts, ultimately the decision by the banks to curtail financing was driven by their evaluation of Lehman's leverage and credit quality.

Fig. 4 shows a time line of events at Lehman Brothers. Lehman reported earnings of $489 million for the first quarter of 2008, and was able to raise $4 billion equity capital in April. However, Lehman subsequently reported large losses, $2.8 billion in the second quarter of 2008, as more subprime investment failures unfolded. We show that our model predicts the substantial increase in default risk for Lehman Brothers over the first few months of 2008. We also undertake a hypothetical exercise on the amount of equity Lehman should have raised in order to reduce default risk to acceptable levels, and we examine whether capital infusions alone could have prevented bankruptcy. Finally, we use the model to compute a default-risk-based measure of haircuts on short-term secured debt (i.e., repos) to estimate how much collateral senior creditors should have required if the amount was a function of Lehman Brothers' increasing risk.

### 4.1. The balance sheet

We begin with a discussion of the evolution of Lehman's financial condition before and during the crisis period. Table 1 provides three balance sheets for Lehman Brothers: 11/30/2007, 02/29/2008, and 05/31/2008.[15] Panel A depicts the assets and Panel B depicts the liabilities. Both the assets and the liabilities have been categorized by their liquidity following the categorization scheme used in Berger and Bouwman (2009). The most liquid assets and liabilities are italicized, those with intermediate liquidity are in standard font, and those that are illiquid are bold-faced. We begin with a discussion of Lehman's assets.

Lehman's total assets were $691.1B on 11/30/2007, $786.0B on 02/29/2008 and $639.4B on 05/31/2008. The percentage of assets in each of the three liquidity categories changes subtly over time. The percentage of liquid assets was 52.5%, 53.8%, and 53.2%; assets with intermediate levels of liquidity was 42.4%, 41.6%, and 41.4%; and illiquid assets was 5.1%, 4.6%, and 5.4%, on 11/30/2007, 02/29/2008, and 05/31/2008, respectively. Liquid assets include cash and equivalents, cash and securities segregated for regulatory purposes, government/agency debt, corporate debt, corporate equities, commercial paper and money market instruments, reverse repos, FX related contracts, other fixed income derivatives, and OTC and exchange-traded equity derivatives. Assets with intermediate liquidity include receivables, mortgages and mortgage backed securities, securities borrowed, and interest rate/credit derivatives. Lastly, illiquid assets include property, plant, and equipment, intangible assets and goodwill, other real estate, and "other assets".

Table 1 also shows the liquidity classification of Lehman's liabilities following Berger and Bouwman (2009). The most liquid assets and liabilities are color-coded blue, those with intermediate liquidity are color-coded green, and those that are illiquid are color-coded red. The percentage of liquid liabilities was 52.8%, 54.6%, and 48.4%; liabilities with intermediate levels of liquidity was 26.3%, 26.1%, and 27.2%; and illiquid liabilities was 20.9%, 19.3%, and 24.3%, on 11/30/2007, 02/29/2008, and 05/31/2008, respectively.

We next explore various facets of Lehman's assets and liability structure that are important in determining Lehman's default risk. In particular, a more complete depiction of the liability structure is a key input in our model, and so we look at the liabilities along three dimensions: liquidity, maturity structure and seniority structure.

[15] The asset figures were obtained from the relevant 10-K and 10-Q filings along with their associated footnotes.

#### 4.1.1. Liquidity and Net Debt[16]

A key innovation in our empirical implementation of the structural credit risk model highlights the important distinction among different asset qualities on a financial institution's balance sheet. A natural way to make the distinction is in terms of the liquidity of various asset classes. We argue that the market value of liquid assets does not play a role in the economic default of the financial institution, and therefore should not be used in estimating the default risk. Indeed, liquid assets can be readily sold in the market and the proceeds used to offset liquid liabilities that are due. This is consistent with our model and our definition of economic default. We, therefore, use the liquidity classification in Table 1, and presented graphically in Fig. 5, to make the argument that it is essentially the *Net Debt* – i.e., the level of debt after netting out liquid assets and liabilities – that matters when assessing the default risk for financial institutions. The residual illiquid asset values compared to the remaining net debt will be the central driver of economic default or the ability of the financial institution to raise capital in the future (after selling off the most liquid assets and using the proceeds to satisfy the most liquid liabilities). We can then use our model to estimate default probabilities as the likelihood of encountering economic default states in the future after this netting has been done. Our methodology, therefore, takes a unique approach to understanding how both sides of the balance sheet contributed to the failure of Lehman Brothers, and we believe that it is likely to achieve similar success for identifying potential future problems for similar large, complex financial institutions in crisis.

Net Debt is commonly used in valuation models and is defined as (see, for instance, Berk et al. (2010), page 393):

$$\begin{aligned}\text{Net Debt} = {} & \text{Short-Term Debt} + \text{Long-Term Debt} - (\text{Excess Cash} \\ & + \text{Marketable Securities}).\end{aligned}$$

Our methodology emphasizes the role of Net Debt in analyzing the default risk of Lehman.[17] We see from Table 1 and Fig. 5, that the level of liquid and semi-liquid liabilities of Lehman Brothers is completely offset by the amount of liquid and semi-liquid assets. We therefore, as a first approximation, use the level of publicly traded bonds as a proxy of the level of Net Debt of Lehman. Table 2 provides details on the publicly traded bonds that Lehman Brothers had outstanding as of January 2008. Table 3 expands this and shows the debt maturity for each month in our sample period. The Total Public Debt amounts at the bottom of Table 3 are between 23% to 30% of Total Liabilities over our sample period. It is no coincidence that Fig. 5 shows that the illiquid liabilities make up 24% of the Total Liabilities in the second quarter of 2008, since these are comprised of, for the most part, the publicly traded debt.

We note two areas where the appropriateness of netting out liquid assets and liabilities are particularly evident in the analysis of Lehman Brothers: repos and security lending. In the repo market it is common for a firm to simultaneously enter into repurchase agreements and reverse repurchase agreements. From a default risk perspective one might argue that these positions offset each other, and thus what matters is the *net* repo position; specifically,

[16] We thank the referee for highlighting this feature of our empirical analysis.

[17] The use of Net Debt has been addressed in the previous academic literature within the context of credit risk and risk management. For instance, Acharya et al. (2007) study the relationship between cash/equivalents and debt within the context of corporate risk management and hedging. They examine the substitutability between cash and short-term debt, and find that financially constrained firms tend to prefer holding excess cash including cash equivalents, while financially unconstrained firms use cash to pay down debt. Acharya et al. (2012a) study the relationship between cash holdings and credit risk. They consider the endogeneity between cash holdings and credit risk, as riskier firms may endogenously choose to hold more cash. Once controlling for this endogeneity, they find that cash reduces default probability in the short term. We are the first, however, to explicitly build this into the structural credit risk model and the analysis of financial institution default.

**Table 1**
Lehman Brothers balance sheet for 2007 and 2008 ($ Millions).

| | 31-May-08 | 29-Feb-08 | 30-Nov-07 |
|---|---|---|---|
| ***Panel A: Assets*** | | | |
| *Cash and equivalents* | *6513* | *7564* | *7286* |
| *Cash and securities segregated for regulatory purposes* | *13,031* | *16,569* | *12,743* |
| Receivables-brokers, dealers, and clearing organizations | 16,701 | 11,915 | 11,005 |
| Customer receivables | 20,784 | 37,298 | 29,622 |
| Other receivables | 4236 | 3186 | 2650 |
| **Net property, plant, and equipment** | **4278** | **4189** | **3861** |
| **Intangible assets and goodwill, Net** | **4101** | **4112** | **4127** |
| Mortgages and MBS | 72,461 | 84,609 | 89,106 |
| *Gov't/Agency debt* | *26,988* | *44,574* | *40,892* |
| *Corporate debt* | *49,999* | *59,750* | *54,098* |
| *Corporate equities* | *47,549* | *56,118* | *58,521* |
| **Real estate held for sale** | **20,664** | **22,562** | **21,917** |
| *CP and money market instruments* | *4757* | *3433* | *4000* |
| *Reverse repo* | *169,684* | *210,166* | *162,635* |
| Securities borrowed | 124,842 | 158,515 | 138,599 |
| Interest rate, credit derivatives | 25,648 | 31,082 | 22,028 |
| *FX forward contracts and options* | *2383* | *3087* | *2479* |
| *Other fixed income derivatives* | *10,341* | *11,856* | *8450* |
| *Equity derivatives (OTC)* | *6022* | *6330* | *8357* |
| *Equity derivatives (exchange traded)* | *2597* | *3257* | *3281* |
| **Other assets** | *5853* | *5863* | *5406* |
| ***Total assets*** | ***639,432*** | ***786,035*** | ***691,063*** |
| ***Panel B: Liabilities*** | | | |
| *Commercial paper* | *7948* | *7751* | *3101* |
| Other short-term borrowings | 5703 | 7753 | 7645 |
| **Current portion of LTD** | **20,991** | **18,510** | **16,801** |
| *Repurchase agreements* | *127,846* | *197,128* | *181,732* |
| Short-term secured borrowing | 660 | 510 | 519 |
| Other secured borrowing | 24,656 | 24,539 | 22,992 |
| *MBS (sold)* | *351* | *552* | *332* |
| *Gov't debt (sold)* | *63,731* | *108,759* | *71,813* |
| *CP & money market instruments (sold)* | *12* | *12* | *12* |
| *Corporate debt (sold)* | *8344* | *8738* | *6759* |
| *Corporate equity (sold)* | *43,184* | *41,035* | *39,080* |
| Loaned securities | 55,420 | 54,847 | 53,307 |
| Interest rate, credit derivatives | 9733 | 15,248 | 10,915 |
| *FX forward contracts and options* | *2270* | *3679* | *2888* |
| *Other fixed income derivatives* | *5692* | *7827* | *6024* |
| *Equity derivatives (OTC)* | *6391* | *9309* | *9279* |
| *Equity derivatives (exchange traded)* | *1799* | *1744* | *2515* |
| *Time-deposits at US bank subsidiaries* | *10,530* | *12,591* | *16,189* |
| *Time-deposits at non-US bank subsidiaries* | *16,854* | *14,052* | *10,974* |
| *Savings-deposits at US bank subsidiaries* | *1427* | *1824* | *1556* |
| *Savings-deposits at non-US bank subsidiaries* | *544* | *362* | *644* |
| Broker payables | 3835 | 11,717 | 3101 |
| Customers payables | 57,251 | 72,835 | 61,206 |
| Accrued expenses | 9802 | 11,596 | 16,039 |
| **Junior subordinated notes** | **5004** | **4976** | **4977** |
| **Subordinated debentures** | **12,625** | **11,181** | **9259** |
| **Senior notes** | **110,553** | **112,128** | **108,914** |
| ***Total liabilities*** | ***613,156*** | ***761,203*** | ***668,573*** |
| ***Total shareholders' equity*** | ***26,276*** | ***24,832*** | ***22,490*** |
| ***Total liabilities and shareholders' equity*** | ***639,432*** | ***786,035*** | ***691,063*** |

Panel A: This panel shows Lehman Brothers' quarterly balance sheet with assets categorized by liquidity. Italic font denotes liquid assets, regular font denotes semi-liquid assets, and bold font denotes illiquid assets, generally following the classification in Berger and Bouwman (2009). Data comes from 10-Q and 10-K filings and footnotes. Amounts are in millions of dollars.
Panel B: This panel shows Lehman Brothers' quarterly balance sheet with liabilities categorized by liquidity. Italic font denotes liquid liabilities, regular font denotes semi-liquid liabilities, and bold font denotes illiquid liabilities, generally following the classification in Berger and Bouwman (2009). Data comes from 10-Q and 10-K filings and footnotes. Amounts are in millions of dollars.

the dollar difference between repos (liabilities) and reverse repos (assets). The practice of offsetting repo and reverse repo positions is called "matched-book trading" and is utilized to reduce the bank's short-term funding risk.

The first few lines of Table 4 show that from 2007 to 2008 the net repo position was reduced: it was initially positive and then became negative. The negative net repo position implies that the firm appears to have more of these short-term assets (reverse repos) than liabilities (repos) and thus should have less default risk. For example, by the end of May of 2008, Lehman had $128B in repos and almost $170B in reverse repos, resulting in a net repo position of -$42B. The natural interpretation is that if Lehman had to liquidate all its repo positions, it would result in a $42B cash position. This logic can be extended to many if not all of the short-term, liquid liabilities on Lehman Brothers' balance sheet. Securities lending is another mechanism for collateralized financing for large, non-commercial bank financial institutions (see Adrian et al. (2012)). Here, the broker–dealer lends out securities

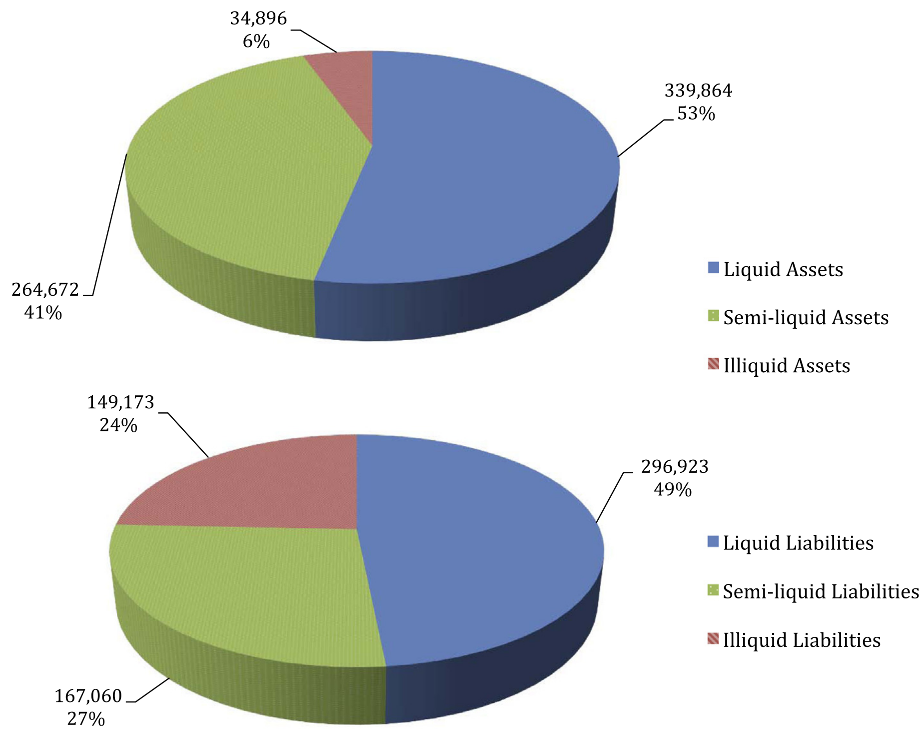


**Fig. 5.** Balance Sheet Liquidity for Lehman Brothers (2008:Q2). Fig. 5 shows a graphical representation of the liquidity structure of Lehman Brothers' balance sheet as of the second quarter of 2008. The liquidity classification mirrors Table 1 following the approach in Berger and Bouwman (2009). Data comes from 10-Q and 10-K filings and footnotes. Amounts are in millions of dollars with the percentage of Total Assets and Total Liabilities appearing below.

and in return collects cash collateral on which it pays interest. This represents a liability for the firm. However, on the other side of the balance sheet, the broker–dealer borrows securities which also requires the posting of cash collateral, and is essentially lending cash with the borrowed securities as a form of collateral. Loaned securities and securities borrowed appear next in Table 4. As with repos, Lehman Brothers borrowed more securities than they loaned in all three reported quarters. This means that Lehman was a net lender of cash collateral in their securities lending/borrowing business. For example, at the end of May 2008, Lehman had loaned $53B worth of securities and borrowed over $138B of securities. This results in a net position of -$85B. Similar logic can be applied to most of the liquid and semi-liquid derivative transactions where netting is permitted and practiced for both accounting and operational purposes. When the most liquid assets and liabilities are netted out, we are left with a few remaining classes of liabilities in Net Debt, comprised mostly of publicly traded debt. However, to be as complete as possible we must also consider private and off-balance-sheet debt.

#### 4.1.2. Private and off-balance-sheet debt

Our focus on Net Debt and the use of publicly traded debt ignores the use of off-balance-sheet liabilities. We therefore supplement this data on publicly traded debt with hand-collected data on private debts and off-balance-sheet obligations from the firm's financial statements and regulatory filings (e.g., 10-K, 10-Q).[18] We note that regulators can demand access to these data and can easily incorporate the true maturity structure of liabilities in any real-time analysis using our model. Indeed, one advantage of our model is the ease with which additional cash-flows can be incorporated into the analysis when such cash-flow obligations are deemed to be relevant by the regulators.

One proxy for off-balance-sheet financing for which we have detailed data is the notorious "Repo105". As was revealed in the financial press and further documented by the official bankruptcy examiner's report (Valukas, 2010), on a daily basis Lehman Brothers engaged in these Repo105 transactions. Without getting into the technical details of the Repo105, they were overnight sale-and-repurchase agreements much like a standard repo; however, they were not booked as repos, but instead they were booked as asset sales. Repo105's were a way for Lehman to finance its positions using the standard practice of pledging securities as collateral, but rather than being recorded as a short-term collateralized loan, the assets were taken off the balance sheet thereby reducing Lehman Brothers' effective leverage. In the bankruptcy examiner's report, Lehman executives and traders admit that Repo105 was a form of "window dressing" used to reduce the leverage ratio that was disclosed to investors; the bankruptcy examiner's analysis found that the use of Repo105 was highest at month-end and quarter-end (Valukas, 2010). Table 5 shows the month-end Repo105 figures for 2008. The quarter-end values are shown in italics to illustrate the extent to which Lehman used the Repo105 transactions to hide its leverage for financial statement reporting. On average, Lehman moved more than $40B of assets off of its balance sheet daily and used these funds to finance their trading positions without any increase in liabilities, even though the master agreement for the Repo105 transactions indicated that the assets would be re-purchased at a spread. This represents an off-balance-sheet liability that did not appear in the financial statements, but increased the default risk in the same way a naked repo would (repo without the offsetting reverse repo). To that end, we use the monthly Repo105 data (from Valukas (2010)) to proxy off-balance-sheet liabilities and include it in our 1-year maturity bucket in computing the term structure of default probabilities.

[18] More recently, we were able to update our analysis with much more detailed data, hand-collected from the exhibits accompanying the Final Reports of both the Financial Crisis Inquiry Commission (Financial Crisis Inquiry Commission, 2011) and the court-appointed bankruptcy examiner (Valukas, 2010).

**Table 2**
Lehman Brothers Debts as of January 2008 ($ Millions).

| Maturity | Notional |
|---|---|
| 2008 | 19,172 |
| 2009 | 22,138 |
| 2010 | 15,792 |
| 2011 | 17,577 |
| 2012 | 17,385 |
| 2013 | 15,560 |
| 2014 | 9815 |
| 2015 | 10,685 |
| 2016 | 5893 |
| 2017 | 8999 |
| 2018 | 716 |
| 2019 | 1778 |
| 2020 | 493 |
| 2021 | 305 |
| 2022 | 1233 |
| 2023 | 441 |
| 2024 | 29 |
| 2025 | 13 |
| 2026 | 427 |
| 2027 | 1136 |
| 2028 | 308 |
| 2029 | 279 |
| 2030 | 215 |
| 2031 | 151 |
| 2032 | 823 |
| 2033 | 61 |
| 2034 | 422 |
| 2035 | 1388 |
| 2036 | 4211 |
| 2037 | 7831 |
| 2038+ | 4996 |
| Grand Total | 170,271 |

| Coupon Type | Notional | Embedded Option | Notional | Percent of Total (%) |
|---|---|---|---|---|
| Combination | 2,135 | Callable | 28,171 | 16.54 |
| Combo – Fixed/Floating | 5,549 | Putable | 1,819 | 1.07 |
| Combo – Floating/Fixed | 2,600 | Convertible | 271 | 0.16 |
| Extendible reset | 428 | | | |
| Fixed listing | 85,296 | | | |
| Floating rate | 73,774 | | | |
| Graduated rate | 210 | | | |
| Step-Up/Down | 182 | | | |
| N/A | 96 | | | |

This table shows a detailed breakdown of Lehman Brothers' publicly traded debt issues outstanding as of January 2008. Data is from FactSet. Amounts are in millions of dollars.

#### 4.1.3. Debt maturity structure

To compute the default probability term structure for Lehman Brothers, we collect detailed data on its publicly traded debt and study the debt's maturity structure. In addition to our Net Debt argument, the use of publicly traded debt in our model has the implication that the forward default probabilities contain information about Lehman Brothers' ability to raise funds in the *capital markets* in the future (either debt or equity, depending on the refinancing assumption). We compile a comprehensive dataset of every single note and bond issued by Lehman Brothers from the Fixed Income Explorer in FactSet, which pulls data from various sources including TRACE, Mergent, FISD, etc. The dataset includes basic information of the bonds in FactSet such as the CUSIP number, the total face value of the issue, the issue date, the coupon rate, and the maturity date. In addition, it also includes other detailed information such as the ''Status'' (matured/redeemed/active), embedded options (callable/putable/convertible), the redemption date (where applicable), the ratings as per Moody's and S&P, the coupon type (fixed/floating/etc.), and the seniority. After downloading the complete dataset, we ran a filter to look at the outstanding debt securities as of any given day. We then collate the debt of Lehman by calendar year and estimate the dollar value of Lehman's debt in each calendar year. Table 2 details the notional amount of the debts maturing in each calendar year as of January 2008. As seen in the table, Lehman has substantial amounts of debt maturing in the short term in 2008 and 2009. The table also shows other details of the outstanding publicly-traded debt, including coupon type and embedded options.

Table 3 presents the maturity structure of Lehman Brothers' publicly traded debt on a monthly basis over our sample period, from December 2007 until September 2008. For each month, we sum up the face value of the outstanding debt maturing each year for the next 19 years. Bonds maturing in 20 years or more are aggregated together for ease of presentation. In running the model, we use 30 periods and therefore have an even finer granularity on the longer end of the maturity spectrum and are able to compute the cumulative default probabilities from 1 to 30 years out. However, it is clear to see that the amount of debt maturing each year thins out significantly after 10 years. The last three columns of the table contain summary statistics for each maturity bucket over our sample period. At the bottom of each monthly column we show the total amount of public debt that was outstanding as of that month.

The pattern of Lehman Brothers' debt maturity structure is not surprising for a financial institution. The largest proportion of debts mature within two years and then, on average, declines each year from 4 to 10. There is a brief increase in the average debt outstanding for the 10-year maturity bucket, but then it drops off significantly thereafter. The apparent spike in the 20+ year maturity bucket represents the fact that it aggregates all of the longer dated Lehman bonds along with non-standard debt securities including perpetual debt and trust preferred securities as well as 30–40 year Mortgage Backed Securities.

Another interesting pattern in the data is that there is substantial variation in the debt outstanding in some of the shorter-term maturity buckets. For instance, the average amount of debt outstanding maturing within one year is about $13.6B but the standard deviation is almost $4.5B. Meanwhile, the average amount of debt outstanding maturing in two years is $26.8B with a standard deviation of $2.7B. The trend seems to be that, as each month passes, the amount of debt due within one year decreases while the amount of debt due in two years increases. This indicates that short-term debts were being rolled forward. This is actually picked up by our model and is indicated by the rising one- and two-year *forward* default probabilities. So while it may appear that Lehman was retiring the most immediate debts, the total public debt outstanding increased over 2008 from $170B at the beginning of the year to almost $180B in the summer. Debt maturing in 2, 3, and 4 years increased during this time period. Other models would not be able to incorporate this information, since the models lump all debts together. However, the debt maturity structure is a critical determinant of not only the term structure of default probabilities, but also the shape of the default barrier, and is reflected in the forward default probabilities – our ''early warning signal'' for future bank distress. The fact that Lehman Brothers increased the use of these shorter-term debt maturities in its capital structure suggests that Lehman was creating a potentially dangerous situation where it would have to refinance these short-term obligations in the near future. One of the key points to our analysis is that it is not only the total amount of debt outstanding, but when these debts are due and how they are going to be paid down that impacts the default risk of large, complex financial institutions.

#### 4.1.4. Seniority structure of debt

To provide another dimension to our analysis of the liability structure, we also study the seniority of Lehman Brothers' debts. To obtain a more accurate representation of the seniority structure

**Table 3**
Lehman Brothers Debt Maturity ($ Million).

| Years to Maturity | December 2007 | January 2008 | February 2008 | March 2008 | April 2008 | May 2008 | June 2008 | July 2008 | August 2008 | September 2008 | Mean | Median | Standard deviation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 20,628 | 19,172 | 16,893 | 16,271 | 14,389 | 11,453 | 10,695 | 10,316 | 8597 | 8069 | 13,648 | 12,921 | 4453 |
| 2 | 21,202 | 22,138 | 27,527 | 27,678 | 27,839 | 28,173 | 28,315 | 28,306 | 28,308 | 28,308 | 26,779 | 28,006 | 2717 |
| 3 | 15,478 | 15,792 | 16,030 | 16,193 | 16,333 | 16,549 | 16,677 | 16,695 | 16,695 | 16,695 | 16,313 | 16,441 | 433 |
| 4 | 17,508 | 17,577 | 17,662 | 17,812 | 17,817 | 17,859 | 20,021 | 20,029 | 20,030 | 20,030 | 18,634 | 17,838 | 1203 |
| 5 | 17,408 | 17,385 | 17,473 | 17,587 | 17,570 | 17,599 | 17,605 | 17,608 | 17,610 | 17,610 | 17,545 | 17,593 | 88 |
| 6 | 11,532 | 15,560 | 15,817 | 16,078 | 16,152 | 16,208 | 16,353 | 16,702 | 16,788 | 16,798 | 15,798 | 16,180 | 1553 |
| 7 | 9896 | 9815 | 9833 | 9916 | 9871 | 9880 | 9911 | 9984 | 9991 | 9993 | 9909 | 9903 | 63 |
| 8 | 10,665 | 10,685 | 10,675 | 10,721 | 10,673 | 10,687 | 10,688 | 10,696 | 10,696 | 10,696 | 10,688 | 10,687 | 15 |
| 9 | 5886 | 5893 | 5971 | 5971 | 5970 | 5975 | 5975 | 5978 | 5978 | 5978 | 5958 | 5973 | 36 |
| 10 | 9003 | 8999 | 8999 | 9022 | 9020 | 9020 | 9066 | 9066 | 9066 | 9066 | 9033 | 9021 | 29 |
| 11 | 548 | 716 | 736 | 867 | 3411 | 4427 | 4471 | 4471 | 4480 | 4480 | 2861 | 3919 | 1874 |
| 12 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 1778 | 0 |
| 13 | 465 | 493 | 510 | 514 | 545 | 545 | 561 | 577 | 577 | 577 | 536 | 545 | 39 |
| 14 | 269 | 305 | 348 | 348 | 348 | 348 | 348 | 348 | 348 | 348 | 335 | 348 | 27 |
| 15 | 1281 | 1233 | 1233 | 1173 | 1152 | 1142 | 1133 | 1128 | 1128 | 1128 | 1173 | 1147 | 55 |
| 16 | 287 | 441 | 669 | 884 | 950 | 971 | 1016 | 1017 | 1017 | 1017 | 826 | 960 | 268 |
| 17 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 0 |
| 18 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 0 |
| 19 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 427 | 0 |
| 20+ | 21,769 | 21,820 | 22,635 | 22,404 | 22,469 | 24,311 | 24,311 | 24,309 | 24,309 | 24,309 | 23,264 | 23,472 | 1133 |
| Total Public Debt | 166,072 | 170,271 | 175,258 | 175,686 | 176,756 | 177,394 | 179,393 | 179,477 | 177,865 | 177,349 | | | |

**Table 4**
Lehman Brothers collateralized transactions.

| | 2007:Q4 | 2008:Q1 | 2008:Q2 |
|---|---|---|---|
| Repo | $181,732 | $197,128 | $127,846 |
| Reverse repo | $162,635 | $210,166 | $169,684 |
| Net | $19,097 | ($13,038) | ($41,838) |
| Loaned securities | $55,420.0 | $54,847.0 | $53,307 |
| Securities borrowed | $124,842.0 | $158,515.0 | $138,599.0 |
| Net | ($69,422.0) | ($103,668.0) | ($85,292) |
| Own collateral pledged | $150,000 | $155,000 | $123,031 |
| Collateral permitted to re-pledge | $798,000 | $929,000 | $518,000 |
| Collateral actually re-pledged | $725,000 | $852,000 | $427,000 |
| Percentage re-pledged | 90.85% | 91.71% | 82.43% |

This table presents data related to Lehman Brothers' collateralized transactions including repurchase agreements and reverse repurchase agreements, loaned securities and borrowed securities (cash collateral borrowed and lent, respectively), as well as the amount and sources of collateral pledged. Amounts are in millions of dollars. Data are from Lehman Brothers' 10-K and 10-Q filings and the accompanying footnotes.

**Table 5**
Lehman Brothers Repo105 Transactions.

| Date | Repo105 Amount |
|---|---|
| 29-Jan-08 | $28,884 |
| *29-Feb-08* | *$49,102* |
| 28-Mar-08 | $24,597 |
| 30-Apr-08 | $24,709 |
| *30-May-08* | *$50,383* |
| 29-Jul-08 | $14,548 |
| 29-Aug-08 | $26,383 |

This table presents the month-end Repo105 transactions used by Lehman Brothers to hide their true leverage. These were repurchase agreements that were classified as asset sales and therefore moved off balance sheet. The quarter-end values are shown in italics to illustrate the extent to which Lehman ramped up their usage for financial statement reporting purposes. We add these monthly amounts to our dataset for the maturity structure of Lehman's debt to proxy for short-term, off-balance-sheet liabilities. Data was obtained from Valukas (2010). Amounts are in millions of dollars.

we re-visited Lehman Brothers' financial statements and regulatory filings as well as previously-confidential documents that have been made public as part of the examinations conducted by the Financial Crisis Inquiry Commission (Financial Crisis Inquiry Commission, 2011) and the court-appointed bankruptcy examiner (Valukas, 2010). The frequency of the data is on a quarterly basis which is more coarse than the monthly frequency employed in our analysis of the maturity structure; however, it provides an interesting second perspective from which to study the impact of the liability structure on Lehman Brothers' distress.

To look at the seniority structure, we take all of the debts (public and private) and classify them as follows (in order of decreasing seniority): short-term secured, short-term unsecured, other short-term/intermediate-term debt, senior long-term debt/debentures, junior long-term debt/debentures. The first class is constructed as the sum of the outstanding short-term collateralized financing agreements including repos (gross) and Repo105, the tactical accounting maneuver discussed above that allowed Lehman to book repurchase agreements as asset sales. Note that, unlike in our analysis of the maturity structure, here in the seniority analysis we are *not* netting repo and reverse repo positions. Rather, we utilize the gross amount of the collateralized financing arrangements. The rationale behind this has to do with the common practice of re-pledging collateral or "asset rehypothecation".

It is typical in a repurchase agreement that the firm posts collateral against the borrowings. However, it turns out Lehman Brothers – and indeed many Wall Street firms – engaged in the practice of rehypothecation of assets; that is, the re-pledging of assets that have already been pledged by one of their counterparties. Table 4 shows that Lehman Brothers only pledged $150B of their own assets as collateral, which is not enough to cover the gross repo positions in any given quarter. In fact, in the final quarter of 2007 and the first quarter of 2008 there were $32B and $42B deficiencies, respectively, between the outstanding gross repos and the value of Lehman's own collateral pledged. From where does the additional collateral come? Lehman was holding $800B of assets in trust as collateral that was pledged to them from counterparties and clients. Of that, Lehman *re-pledged* 90%. This not only represents a risk to Lehman's repo counterparties, but also a risk to the entire financial system. Because this is common practice, the failure of any one participant would trigger a domino effect, resulting in multiple claims to the same asset. The significance to our analysis is that even though Lehman may have had declining net repo positions over 2008, the amount of their own assets pledged as collateral was always less than the amount owed via repurchase agreements. We will show later on that Lehman was indeed under-collateralized, which played a role in their failure.

Thus, in our seniority analysis we want to compute the expected recovery and implied haircut on these collateralized transactions assuming that creditors and counterparties would not be able to recover their collateral immediately because of the rehypothecation and deficiency in assets pledged. This class represents the senior-most tranche of the seniority structure of Lehman Brothers' debt. They are short-term, collateralized obligations that should have the least amount of risk in the event of a default. The next class is also made up of short-term obligations, but they represent less senior claims since they are unsecured. The second tranche is comprised of short-term, unsecured debt. This includes commercial paper, which because of its unsecured status and slightly longer average maturity, is subordinated to the collateralized counterparts. The second tranche also contains revolving lines of credit, short-term portion of long-term debentures, and other short-term unsecured debt. The last two tranches contain the long-term debt in Lehman's liability structure. This follows the assumption that long-term debt is effectively junior to short-term debt found in corporate finance papers (see, e.g., Ho and Singer, 1982; Diamond, 1993) as well as in the credit risk literature (Geske and Johnson, 1984). However, we further make the distinction between senior and subordinated debentures, which make up the third and fourth tranches, respectively. We can then use this seniority structure in our model to compute default probabilities and expected recoveries for each of the tranches. Furthermore, assuming the most dire of circumstances – where there is zero recovery – we can estimate a default probability based measure of the haircut that should be applied to the collateralized tranche so as to minimize risk of loss.

### 4.2. Market value inputs

Our model uses two market-determined inputs for calculating Lehman Brothers' default probability. The first is the market value of Lehman Brothers' equity and the second is the volatility of

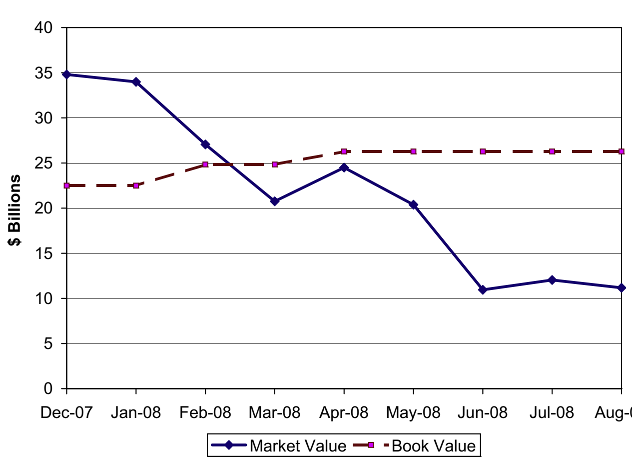


**Fig. 6.** Lehman Brothers Equity Value by Month. Fig. 6 shows the book value and the market value of equity for Lehman Brothers at the end of each month from December 2007 to August 2008. Market value of equity is calculated as the product of the stock price multiplied by the number of shares outstanding and is represented by the solid line. Book value of equity is as reported in COMPUSTAT. The market value of equity is one of the market determined inputs for our model.

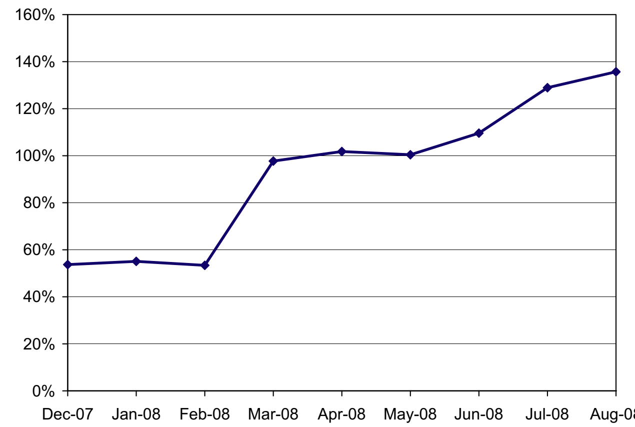


**Fig. 7.** Lehman Brothers Volatility. Fig. 7 shows the historical 120 day volatility for each month from December 2008 to August 2008. Historical volatility is calculated by annualizing the standard deviation of daily returns over the prior 3-month period using prices from CRSP. The data on volatility is the second market determined input for our model.

Lehman's stock returns. Fig. 6 shows the book value and the market value of Lehman Brothers' equity over the period December 2007 to August 2008. The market value of equity is calculated as the product of the closing stock price on the estimation day and the non-diluted number of shares outstanding on that day. The book value of equity is as reported by COMPUSTAT. As the figure shows, the market value of equity had a precipitous decline in 2008. Specifically, in February, the equity value dropped from $35.1 billion to $26.8 billion: a drop of 23.6%. The initial signs of the pending financial crisis are reflected in this dramatic drop in equity value, which led Lehman to raise additional funds in March 2008. Lehman obtained a $2 billion, 3 year credit line from a consortium of 40 banks, including JPMorgan Chase and Citigroup. They also raised $4 billion in perpetual convertible preferred stock at the beginning of April 2008. The infusion stabilized Lehman over the next two months; however a reported loss of $2.8 billion in May 2008 led to a further stock price decline of 27.9% in May.

The second input to the model is the volatility of Lehman's stock returns. Fig. 7 shows the historical 120-day stock return volatility for Lehman. We calculate the volatility as the annualized standard deviation of Lehman's daily stock returns over the prior 120-day period. We use the most recent period so that we can focus on the most recently available market information of Lehman's stock returns. As Fig. 7 shows, the volatility estimate spiked in March 2008 reflecting the increasing uncertainty in the markets surrounding the implosion of Bear Stearns.

Volatility is of course not directly observable and has to be estimated. Our use of a 120-day window and daily returns balances the need for more recent information, thereby capturing the latest market signal and a longer window for reducing estimation error. As volatility is a crucial parameter, any practical implementation of our model could well involve alternate ways of estimating volatility.[19]

### 4.3. Model implementation

In this section, we describe the lattice structure that we implement for the structural model described in the previous section, for determining the term structure of default probabilities for Lehman

[19] As previously discussed in the context of our numerical example (Section 3.3), higher equity volatility estimates can lead to the lattice model having very large and very small future values for Lehman's assets, and using a longer returns horizon could dampen some of the daily fluctuations.

Brothers. We begin with developing a binomial model for the evolution of market values for Lehman. Two inputs are needed for constructing the binomial tree: the market value of assets today and the asset volatility. As these are unknown, we start with an initial guess and construct the binomial tree based on this guess. The total number of binomial steps is determined by the number of maturity buckets for Lehman's liabilities and the granularity of time steps between the maturities of the liability buckets. We choose annual maturity buckets over a 30 year window and use 8 time steps a year, for a total of 240 time steps in the binomial tree. Our construction ensures that the time-to-maturity of Lehman's debt falls on the nodes in the binomial tree, and not between the nodes. This allows for the evaluation of the refinancing decision for all of Lehman's debt and mitigating one potential source of estimation error. The granularity of time steps balances speed of execution and accuracy.

We solve for the value of Lehman's equity using backward recursion. At each node where Lehman's debt matures, we determine economic default by comparing the value of the assets with the value of the debt maturing at that node. If Lehman is solvent, then the value of the equity is simply the probability-weighted present value of next period's equity values in the binomial lattice; otherwise, it is zero. This process allows us to estimate a value for Lehman's equity. We can now use the estimated value of Lehman's equity and the assumed values of assets and asset volatility to estimate Lehman's equity volatility.

We next compare the estimated values and observed values of Lehman's equity and equity volatility and update our initial guesses for asset values and asset volatility using simple linear interpolation. The relationship between the parameters for the asset and the parameters for equity is non-linear and therefore the process is iterative. Once we converge on a solution, the binomial tree gives us a detailed picture of the value of debt at each of the nodes in the tree and the states in which Lehman is in default. We use this information to compute the term structure of default probabilities using the binomial probability structure and estimate recovery rates for debt.

We implement the above algorithm and develop results for three possible strategies Lehman can use for dealing with debt maturing at any node. These three strategies vary in the amount of debt that is reissued and reflect models used in literature. In the de-leveraging approach, $\lambda = 0$, and we assume that Lehman will replace the debt with equity as in Geske (1977). In this specification, Lehman effectively de-leverages over time. In the rollover approach, $\lambda = 1$, and we assume that Lehman will replace its existing debt with new debt of the same maturity as in Leland and Toft (1996). In this specification, the capital structure will remain stable over time. Both of these approaches reflect extremes in how Lehman will deal with debt and are perhaps not practical for a firm that would like to maintain a high leverage but which faces financing constraints. We therefore implement a hybrid strategy in our third specification, where $\lambda = 0.5$.

For example, on January 1, 2008, the FactSet data indicates that Lehman had total publicly traded debt with a face value of $170.2 billion maturing over the next 30 years. The market value of Lehman's equity is $33.98 billion and historical equity volatility is 55.07%. Using our approach, we find that the model-implied market value of Lehman Brothers' assets have a value of $202.55 billion and an asset volatility of 13.94%. Recall, that since our methodology involves netting out the most liquid assets and liabilities, this is the implied market value of the illiquid ["bad"] assets on Lehman Brothers' books at the beginning of 2008. This additional insight is expanded upon below. Since the market value balance sheet identity must hold, we can easily calculate the model-implied market value of the publicly traded debt to be $168.57 billion.

### 4.4. Model results

We use data on the liability structure, the equity market value, and the 120-day historical equity volatility in our structural model to generate several metrics that are useful for evaluating the risk of large financial institutions, especially in times of crisis. Recall, our model gives rise to a complete term structure of default probabilities, so we begin by computing these default probabilities for Lehman Brothers on a monthly basis from December 2007 to September 2008. It turns out that not just the level, but also the shape of this term structure, contains important information for risk management and regulators. Our model yields an "early warning signal" in terms of the forward default probabilities which indicate the likelihood that the particular institution will encounter difficulties raising capital in the future, conditional on having survived to that point. The model can also be used in a stress test capacity to estimate how much capital a distressed financial institution would need to raise in order to bring its default probability down to a specified level.

In addition, the model uses equity market information and the firm's full liability structure, all of which are observable, to compute an implied market value of a financial institution's assets. We use this implied market value of assets to track Lehman Brothers' implied market value leverage ratio over 2008. We then compare these implied leverage ratios with the book value leverage ratios, and obtain some interesting insight on how severe the downward spiral was for Lehman; especially at the end. Lastly, we use our data on the seniority structure to estimate recovery rates on all classes of debt and, in addition, default-risk-adjusted haircuts on repos. We should note, before presenting our results, that our model yields risk-neutral default probabilities, which combine both a risk premium and the true default probability. The relationship between the risk-neutral and true default probability is non-linear, but it can be shown that the risk-neutral default probabilities will always be less than the true default probabilities under the physical measure under certain conditions.[20]

#### 4.4.1. Default probabilities

Fig. 8 presents the evolution of the term structure of default probabilities for the dates 1/31/2008, 3/31/2008, 4/30/2008, 6/30/2008, and 8/31/2008 for $\lambda = 0.5$. The shape of term structures (slope and degree of concavity) contains valuable information about the interaction between market conditions and liability structure, which both contribute to the potential for financial distress. Each of the five months' default probability term structures are monotonically increasing and concave over time, which is what one would expect [see Leland, 2004]; however, their initial slopes and degrees of concavity vary substantially. In January 2008, the one year default probability was about 33%, with the cumulative default probabilities rising a bit over the next two to three years and then flattening out at around 37%. Thus, although the overall default risk was relatively high, given the information available about Lehman at the time, the market did not expect a massive increase in the conditional default probability over time. This is in stark contrast to March 2008, where the one year default probability is about 73%. The term structure curve for March 2008 is steeper and more concave indicating that cumulative default probabilities exhibit larger increases in the near future but take longer to level off. Over the next month there was a downward shift in the entire default probability term structure; not surprisingly, since there was a large $4 billion capital infusion in April of 2008, the one-year default probability is a substantially lower 52%. However the cumulative default probabilities rapidly

[20] Most notable is the condition that there is a positive risk premium, and therefore the drift under the physical measure is greater than the drift under the risk-neutral measure, which is the risk-free rate; i.e., $\mu > r$.

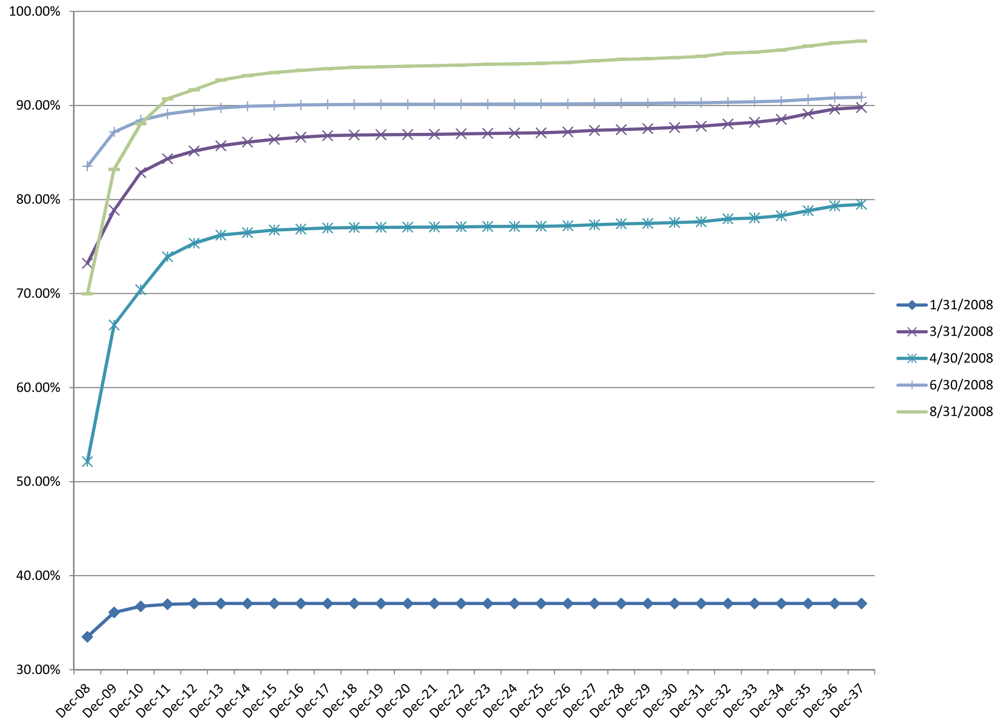


**Fig. 8.** Lehman Brothers Default Probability Term Structure. Fig. 8 shows the cumulative default probabilities, or the term structure of default probabilities, for Lehman Brothers as of January 2008, March 2008, April 2008, June 2008, and August 2008. Each curve depicts the probability, at that date, of Lehman Brothers defaulting between then and the end every year from 2008 to 2032.

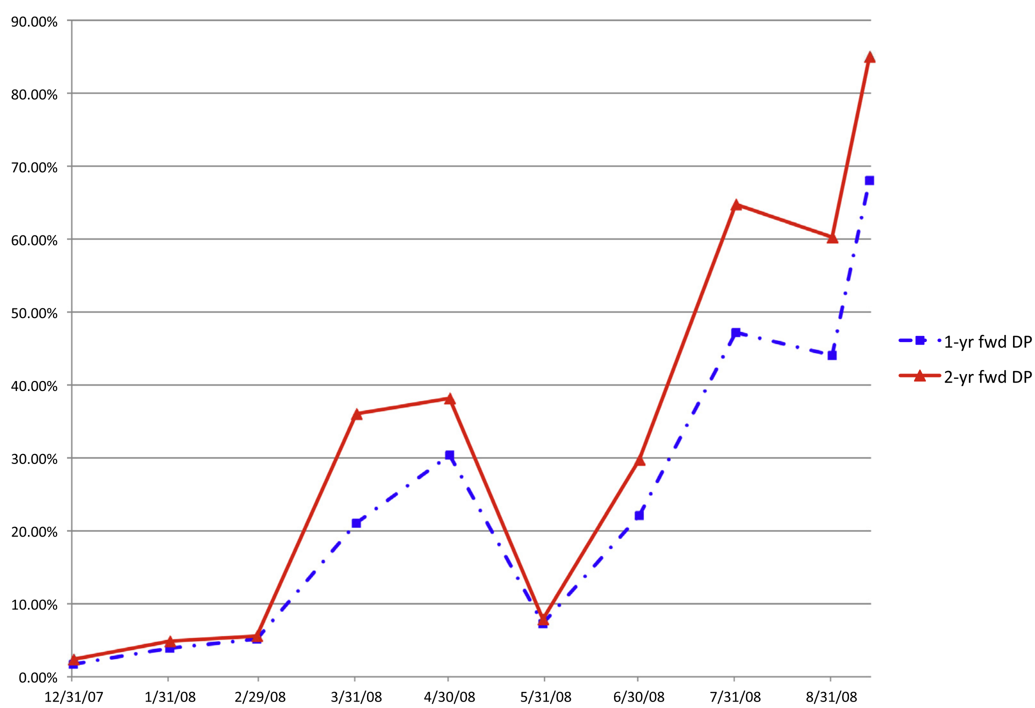


**Fig. 9.** Lehman Brothers Forward Default Probabilities. Fig. 9 shows the evolution of the one- and two-year *forward* default probabilities for each month from January 2008 to September 2008. This is extracted from the default probability term structure and represents the likelihood that Lehman Brothers would default over the next one (blue dashed line) to two (red solid line) years assuming that they survive until then. These probabilities reflect the refinancing parameter set to $\lambda = 0.5$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

jumps to over 70% in two to three years, and then steadily rises to over 79%. Clearly, the market felt that the capital infusion was insufficient to reduce Lehmans risk in the *longer term*. The curve for June 2008 begins with an initial one-year default probability of 83% and in less than ten years rises to over 90%. Lastly, the curve for August 2008 begins with an initial one-year default probability of just under 70%, but then rockets to over 80% in the next year, and continues to climb, ultimately surpassing 95%.

The shapes of these term structures contain valuable information about the interaction between market conditions and liability structure, which both contribute to the potential for financial distress and economic default. One important piece of information that we can extract from these term structures is the expected future short-term default probabilities, which are the *forward* default probabilities. As shown in Delianedis and Geske (2003), these forward default probabilities have considerable predictive power in terms of ratings downgrades. Recall, that the default condition in our model is the point at which the firm will no longer be able to raise capital (under the specified refinancing assumption). Therefore, these 1-year and 2-year forward default probabilities convey the market's expectation of Lehman's ability to raise capital within the next one or two years, conditional on surviving that long.

Fig. 9 plots the evolution of these forward default probabilities on a monthly basis over 2008 for $\lambda = 0.5$. In January and February, the one and two-year forward default probabilities tracked each other closely, both increasing gradually from 2% at the end of December 2007 to 5% at the end of February 2008. During the month of March, however, the one-year and two-year forward default probabilities diverge. They both spike dramatically, but the one-year forward probability quadruples from 5% to 21% while the two-year forward probability septuples from 5% to 36%. The increase in the two forward default probabilities continues through April, with the one-year forward default probability jumping to 30% and the two-year increasing to 40%. These extremely high forward default probabilities indicate that the market had serious concerns about the likelihood that Lehman would survive even for one year. In May 2008 both of the two forward default probabilities shift downward to below 9%, possibly in response to Lehman's successful issuance of $4B perpetual convertible preferred stock in an attempt to recapitalize and de-lever as well as gaining access to the Fed's discount window. Beginning in June and continuing thereafter, the one-year and two-year forward default probabilities begin to skyrocket, taking values of 22% and 30%, respectively, at the end of June and 68% and 85%, respectively, at the end of August. This has interesting implications, since perhaps if Lehman Brothers de-levered more aggressively in the summer of 2008, either by raising equity capital and/or selling off assets, then their chances of surviving the next one or two years would have been better. These spiking forward default probabilities are an alarm to regulators that Lehman Brothers was in deep trouble as early as March and that the capital infusion that arrived in April was insufficient.

Having illustrated the potential of the one-year and two-year forward default probabilities generated by our model for use as early warning signals to help regulators and risk managers foresee Lehman Brothers dire predicament in the Spring of 2008, our next goal is to use the model in a stress test capacity to determine how much capital needed to be raised.

### 4.4.2. Equity infusion

We next examine how default risk changes when a financial institution adds equity to the balance sheet. We use March 2008 as an example, as default risk increases substantially in that month. Several additional assumptions have to be made in order to analyze the impact of an equity infusion. First, we need to model how the firm uses the additional capital raised. We assume that capital raised during a crisis is retained as cash or invested in riskless assets and the additional capital serves as a cushion to absorb further losses and reduce default risk. Assets therefore are assumed to grow by the same amount as the amount of equity raised.

Second, we note that we could either allow for debt to be rolled-over or require that the firm should further de-leverage going forward after the equity infusion. In a crisis, we argue that it is more appropriate and consistent to assume that a large financial institution such as Lehman Brothers will be required to de-leverage until it reaches an acceptable level of default probability; this implies $\lambda = 0$ in our model, and sets an upper bound for the size of the equity infusion. Implicitly, in a crisis, a financial institution raises capital immediately to cushion against losses and raises. As per our model, under the assumption that $\lambda = 0$ (de-leveraging), Lehman's market value of existing equity in March is $20.754 billion and the market value of existing assets is $115 billion, prior to the equity infusion.

Third, we note that the portfolio of Lehman's existing risky assets and the new capital that is invested in riskless assets will therefore have a lower level of risk. Since the variance of riskless assets is zero and the correlation between the riskless assets and the firm's other risky assets is zero, volatility of assets will decrease in proportion to the amount of capital raised.

Fig. 10 shows the results of the equity infusion strategy. As expected, the figure shows an inverse relationship between the level of equity infusion and default probability. As the size of the equity infusion increases (measured along the horizontal axes) the default probability drops. The solid line in the figure represents the equity necessary in order to reduce the default probability to any chosen level assuming that equity volatility is not affected by the equity infusion and is constant (i.e., the new capital is invested in risky assets with the same level of risk as the firm). The dotted line represents the case where we assume that the equity funds raised are held as cash and consider equity volatility to be a linear combination of the volatility of existing equity volatility and risk free cash (the case discussed in the previous paragraph). The dotted line shows that the equity infusion necessary to reach a benchmark level of default probability, say below 5%, is lower if we assume that the equity infusion will be accompanied by a drop in equity volatility. The figure also shows a default probability under the rollover strategy ($\lambda = 1$), in the absence of equity infusion, of over 81.73% (the point is shown as a square dot). Requiring that Lehman begin de-leveraging and raise equity immediately reduces the probability of default. The magnitude of decrease in default is important. We find that, given market conditions as of March 2008, raising $5 billion in equity would still leave Lehman with an unacceptably high default probability of over 10%, as the solid line suggests. Indeed, our model indicates that

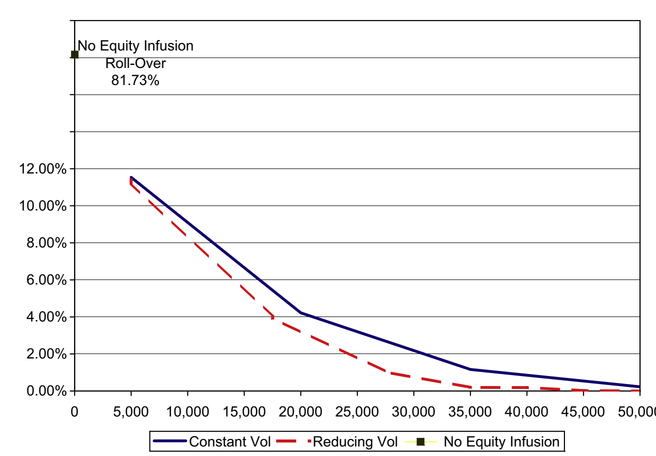


**Fig. 10.** Equity Infusion and Default Probability. Fig. 10 shows the model results under the de-leveraging assumption for hypothetical equity infusions into Lehman Brothers in March 2008. The solid line plots the results for the case when volatility is assumed to be constant and independent of the equity added to the balance sheet. The dashed line plots the results for the case when volatility is assumed to decrease as a function of the equity raised, reflecting the zero volatility of equity proceeds held as cash. The point shown, as a square, on the figure represents the base case default probability under the rollover assumption.

**Table 6**
Lehman Brothers recovery analysis and implied haircuts.

*Panel A*

| | 2/29/2008 | | | 5/31/2008 | | | 8/31/2008 | | |
|---|---|---|---|---|---|---|---|---|---|
| Seniority Class | Face Value | Recovery | Recovery Rate (%) | Face Value | Recovery | Recovery Rate (%) | Face Value | Recovery | Recovery Rate (%) |
| 1 | 325,616 | 325,616 | 100 | 256,192 | 256,192 | 100 | 221,923 | 221,923 | 100 |
| 2 | 34,014 | 34,014 | 100 | 34,642 | 34,642 | 100 | 25,900 | 25,900 | 100 |
| 3 | 112,128 | 103,345 | 92 | 110,553 | 109,664 | 99 | 77,095 | 77,095 | 100 |
| 4 | 16,157 | 0 | 0 | 17,629 | 0 | 0 | 37,544 | 15,296 | 41 |

*Panel B*

| Date | PD | Implied Haircut | Implied Collateral Value | Actual Collateral Posted | Difference |
|---|---|---|---|---|---|
| 2/29/2008 | 3.13% | $10,200.94 | $335,816.94 | $155,000 | ($180,816.94) |
| 5/31/2008 | 41.45% | $106,180.05 | $362,372.05 | $123,031 | ($239,341.05) |
| 8/31/2008 | 50.44% | $111,933.08 | $333,856.08 | $150,745 | ($183,111.08) |

Panel A: shows the face value, recovery amounts, and recovery rates of Lehman Brothers' debt by Seniority Class at the end of each quarter in 2008. Seniority Class 1 contains short-term secured debt, Seniority Class 2 contains short-term unsecured debt, Seniority Class 3 contains senior long-term debt, and Seniority Class 4 contains junior long-term debt. Face value and recovery amounts are in millions of dollars. Face values are hand collected and grouped from several sources including Lehman Brothers' 10-K and 10-Q filings, the bankruptcy examiner's report [Valukas, 2010] as well as the Financial Crisis Inquiry Report [Financial Crisis Inquiry Commission, 2011]. Recovery amounts are computed using our model.
Panel B: shows the Probability of Default (PD), the implied haircut for Seniority Class 1 (short-term, secured debt), the fair value of the collateral that should have been posted according to our model, the actual collateral posted by Lehman Brothers, and the difference between how much collateral was posted versus how much should have been posted. All amounts are in millions of dollars. Data on actual collateral posted are from Lehman Brothers' 10-Q filings (for February and May 2008) and supplemented from the bankruptcy examiner's report [Valukas, 2010] as well as documents included in the Financial Crisis Inquiry Report [Financial Crisis Inquiry Commission, 2011].

regulators should have required Lehman to raise $15 billion in additional capital to reduce default probability below 5%, the level of default probability in January and February of 2008.

The Fed and other regulators can use our model estimates to validate the capital raising strategy by a bank in financial distress. The model uses market-based inputs (equity value and volatility) and therefore has the additional feature that it is forward-looking and can thus be used by regulators to evaluate financial institutions in dramatically changing market conditions. In this way, our model thus provides a practical tool for managers and regulators in assessing the risk profile and capital adequacy of banks facing financial distress.

### 4.4.3. Recovery analysis and implied haircut

We next use our model to examine the recovery rates and expected losses of the different seniority tranches of Lehman Brothers' debt structure. As discussed in Section 4.1, our seniority analysis uses quarterly data that was hand-collected from different sources to categorize all of Lehman Brothers' debt (public and private) into four tranches. The face values of each of the classes are shown for the first three quarters of 2008 in Panel A of Table 6. Class 1 contains short-term secured agreements including repos, loaned securities, and the controversial Repo105 transactions. Class 2 is comprised of commercial paper and other unsecured short-term debt including lines of credit and short-term portion of long-term debt. Classes 3 and 4 are long-term senior and subordinated debentures, respectively.

We use our structural credit risk model to compute several metrics which provide insight into the mounting distress encountered by Lehman over the course of 2008. First, we compute the 1-year default probability and obtain the endogenous recovery amount for each seniority class. This allows us to compute an expected loss measure and examine the distribution of recovery rates across seniority classes for each period. Lastly – and we believe this to be a major contribution of our model – we focus on the first seniority, which is collateralized debt, and propose an ad hoc measure of an appropriate default-risk-based implied haircut that should have been imposed on Lehman's short-term secured liabilities.

Looking at Panel B of Table 6 we can see that the 1-year default probabilities under the seniority analysis are quite low in early 2008 but shoot up dramatically in the summer.[21] While this may seem contradictory to our default probability analysis along the maturity dimension (Section 4.4.1), note that we are not claiming that the seniority analysis gives the kind of "early warning signal" as our default probability term structure and forward default probabilities did. The real insight comes from examining the respective recovery rates and what we call the default-risk-based implied haircut.

The endogenous recovery is computed as in Eqs. (14)–(16). Looking at Panel A of Table 6, we see that Lehman Brothers' asset value was high enough throughout all of 2008 to ensure, in theory, that short-term secured creditors (Seniority Class 1) would receive 100% recovery. Of course, in practice, there are a multitude of frictions that prevent this from happening. Our model assumes that all of the assets can be sold immediately at the implied market value (which may or may not be below the book value); however, we know that illiquidity of certain asset classes made this close to impossible. In fact, the model predicts that *all* short-term creditors would receive 100% recovery, but we will come back to the implications for the first Seniority Class since they are secured, and we draw some interesting conclusions regarding Lehman's collateralization. First, continuing down the recovery rates in Panel A of Table 6, we see that our model predicts that senior long-term debt holders (Seniority Class 3) would receive close to 100 cents on the dollar for all three quarters, but unfortunately, the same could not be said for the subordinated long-term tranche, which would have been completely wiped out, should Lehman have defaulted anytime in the first half of 2008. This recovery amount does increase to 41 cents on the dollar by the end of Q3, which reflects Lehman's efforts to de-lever by selling off assets and paying down some debt over the summer months. So the recovery analysis indicates a vast difference among recovery percentages across the various classes in Lehman's debt structure. The senior-most creditors should have been in good shape, but the subordinated classes would incur substantial losses.

[21] These default probabilities are actually the same order of magnitude as what would be determined by a Black–Scholes–Merton or KMV model. However, as noted earlier, those models do not have endogenous recovery, nor do they allow for multiple classes of debt and, hence, are not as precise for this kind of analysis.

An interesting application for the default probabilities that are generated by our model is that they can be used, in conjunction with the recovery estimates discussed above, to determine the contemporaneous expected dollar value of non-recoverable losses associated with default. More specifically, we can determine the appropriate risk-neutral implied haircut, which is the amount of additional collateral that Lehman should have pledged in order to cover their potential losses, that should be applied to the outstanding secured liabilities. The implied haircut is calculated as the short-term probability of default multiplied by the expected loss assuming zero recovery.

One might question why we would make such a dire assumption (zero recovery) when the highest Seniority Class is secured and therefore backed by assets. There are several reasons, all of which we believe should be important to regulators and risk managers going forward, considering the lessons learned from Lehman's failure. First, as noted earlier in the paper, Lehman Brothers – as with other broker-dealers – was permitted to re-pledge a portion of the assets that were already pledged to them by counterparties and customers; a practice known as the rehypothecation of assets. And we saw in Table 4, of the almost trillion dollars of collateral that Lehman was allowed to re-pledge, they pledged 80–90% of it. If some of these re-pledged assets were used in repo agreements, it introduces another level of uncertainty in these creditors' recovery. Furthermore, it is also now known that Lehman Brothers was not only pledging the high quality, highly liquid assets that are usually assumed to be involved in repo transactions (i.e., Treasuries, Agencies, etc.), but was also pledging highly risky securitized products, some of which were structured by Lehman themselves and issued by special purpose vehicles that they set up and owned (see Valukas, 2010). For this reason, we perform the thought experiment of seeing what the expected loss would be assuming zero recovery on the short-term secured debt (Seniority Class 1). This Seniority Class is mostly comprised of repos, but we also include loaned securities, because the securities lending business is another important source of collateralized, short-term financing of broker–dealers. An excellent account of this area can be found in Adrian et al. (2012) where the similarities are drawn between this and repo financing. Basically, when a broker–dealer lends securities to a client they require cash collateral on which the client pays interest. This cash collateral is a source of borrowed funds. The difference is that with repos, the security is *sold* and then *repurchased*, whereas with securities lending, it is not outright sold to generate the short-term funds.

In Panel B of Table 6 we see that in February of 2008, our model predicts that Lehman's default probability was 3.13%. Given that, at that time, the firm had \$325.6B in short-term secured liabilities, the expected loss and the associated fair value of the haircut associated with these liabilities should have been \$10.2B and therefore, in equilibrium, the appropriate amount of collateral should have been \$335.8B. Considering that Lehman only posted \$155B of its own collateral, our analysis indicates a deficiency of \$180.8B. In May of 2008, the default probability rose to 41.45%, and the firm had \$256.2B in short-term secured liabilities. At this time, our model indicates that haircuts should have gone up to \$106.2B, which means that Lehman Brothers should have posted over \$360B of collateral. Compared with the \$123B of its own collateral that actually was posted, the model shows a deficiency of \$239.3B. A similar story is associated with August of 2008 where the face value of the short-term secured liabilities was at almost \$222B and the computed probability of default was 50.44%, which implies a haircut of \$111.9B and a fair value collateral amount of almost \$334B. Again, considering that Lehman had only posted \$150.75B of its own collateral, there was a deficiency of over \$183B. From this, we can draw the conclusion that if counterparties and regulators were to use default-risk-based criteria for repo haircuts, then Lehman Brothers was in a position such that their creditors would have demanded substantial collateral in the summer of 2008 to cover the increase in expected losses. In fact, our findings help support the cases that Citi and JP Morgan made when they demanded that Lehman post more collateral in early September 2008 and the concern that JP Morgan expressed as early as July 2008 (see Financial Crisis Inquiry Commission (2011) regarding Lehman's tri-party repo book).

The results further indicate that, over the summer months, Lehman's short-term secured creditors were in great jeopardy, since their securities did not have sufficient credit support despite an expected recovery rate of 100%. Undercollateralization seems to have played a significant role in Lehman's demise. A one-time collateral call of over \$100B is probably unrealistic, but if regulators, clearing banks, and counterparties were using our model with updated market information, it would have suggested that Lehman gradually build up their collateral pool so as to coincide with the increasing risk. This may not have saved the firm (see our earlier analysis with regard to raising capital), but it would have at least provided enough protection for the short-term secured creditors who should have stood to lose the least.

### *4.4.4. Leverage*

Financial institution leverage is defined as the ratio of total assets to equity. It is well-known in financial economics that higher leverage directly increases the risk borne by shareholders. As can be seen in Eq. (7), the asset volatility is multiplied by the leverage ratio when computing the equity volatility. Holding debt constant, when asset values rise, leverage should fall. However, in a recent paper, Adrian and Shin (2010) show empirically that financial institutions' leverage appears to increase when asset values rise. This provides evidence that financial institutions actively manage their leverage, by taking on more debt when their balance sheets grow and reducing debt when assets fall in value. This has important consequences, especially in bad market environments. For instance, in order to reduce debt when asset values are falling, it is likely that they will sell assets putting additional downward pressure on asset prices (the "fire sale" externality). This introduces a previously unexplored dynamic between leverage and market liquidity.[22]

The problem is that typically leverage is measured and reported in book value terms. While financial institutions are supposed to "mark-to-market", the leverage ratio found in financial statements still lags behind the real-time market assessment of a financial institution's risk. Furthermore, it does not capture the nonlinearities between liquidity, asset values, leverage, and volatility. Fortunately, our structural credit risk model is able to address these shortcomings. As a result, we are able to provide new insight into how these dynamic interactions manifested themselves as Lehman Brothers became increasingly distressed over the course of 2008.

We first use the financial statements to compute the book value leverage ratio on a quarterly basis. Then, we use our structural model to compute an implied market value leverage ratio on a monthly basis. The model allows us to do this because, recall, we solve for the implied market value of assets that sets the expected values in Eqs. (5) and (6) equal to the current market value of equity. From the identity in Eq. (3), we can also directly solve for the fair market value of the total debt outstanding. This gives us the implied market value of leverage at time $t$ as

$$Leverage(t) = \frac{A(t)}{E(t)} = \frac{E(t) + D(t)}{E(t)} \tag{18}$$

[22] Brunnermeier and Pedersen (2009) develop a theoretical model for these so-called "liquidity spirals".

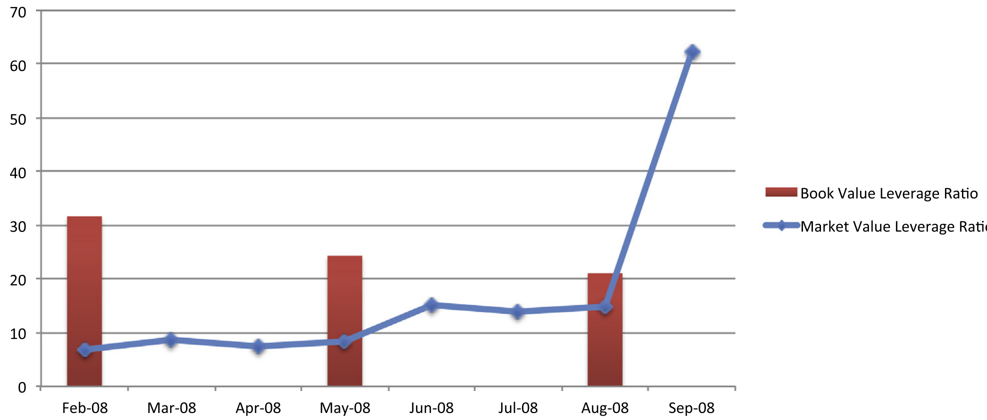


**Fig. 11.** Lehman Brothers leverage. Fig. 11 shows leverage ratios for Lehman Brothers using the standard book value approach (bars) from quarterly reported accounting data and an implied market value of leverage computed monthly using our model (line).

where $A(t)$ is the implied market value of the assets from the model, $E(t)$ is the market value of equity which is observed, and $D(t)$ is the fair market value of debt computed in the model.

The results of the comparison of these two leverage measures for Lehman Brothers are presented in Fig. 11. The red bars show the book value leverage ratio at the end of February, May, and August 2008. We can see evidence that Lehman did appear to de-leverage as their troubles worsened. The leverage ratio went from over 31 in the first quarter of 2008, to 24 in the second quarter of 2008, to 21 in the third quarter of 2008. We know from examining the liabilities in Section 4.1 that they did try to reduce their debt liabilities. It also seems that they tried to shrink their balance sheet by selling off assets. So it appears their behavior was consistent with what was documented in Adrian and Shin (2010) and what we would expect from a financial institution in crisis. Despite the negative externalities of selling assets off as values were falling, they had no choice in order to reduce leverage since their access to equity markets became increasingly restricted.

However, the market value leverage ratio tells a different story; one that should have raised a red flag for regulators in the summer of 2008. The line plots the market value leverage ratio implied by our model. We see that in the first half of 2008, the market value leverage ratio was below 10. In June 2008 the market value leverage ratio increased more than 75%. Between August and September the market value leverage ratio more than quadrupled.

While mark-to-market accounting and book value leverage ratios might be sufficient in normal times, they can be misleading in times of crisis, especially for distressed financial institutions. When asset values are high, the relationship between asset and equity values is approximately linear. Therefore, increases in asset value result in increases in equity of the same magnitude. The proportion of debt in the capital structure shrinks and leverage decreases. Financial institutions respond proactively by taking on more debt to keep leverage ratios close to a target level. When asset values are high, they can do the reverse with relative ease: the balance sheet shrinks, leverage increases, and the financial institution reduces its debt. The linear relationship does not hold when conditions become bad, even though book value measures assume they do. The structural model captures the highly convex relationship between asset value and equity value; and it is precisely in the bad times – when asset values are low – that this convexity has a bigger impact. When asset values are low relative to the amount of debt outstanding, a further decrease in asset prices does not elicit a change in equity values of the same magnitude. Equity, as the residual claim, will already be valued closer to zero than assets; but due to limited liability, they cannot go down much further. Therefore, even though asset values fall more than equity values, their percentage change and rate is less than equity. The result is that leverage increases. These are also the states where liquidity dries up in the credit markets; so actively managing leverage is no longer an option for the financial institutions. This accurately describes what happened to Lehman Brothers after the second quarter of 2008.

We do not interpret these results as being contradictory to Adrian and Shin (2010). Their analysis does indicate that financial institutions manage their leverage actively, or at least try to. However, during times of crisis and severe distress, market values deviate from book values and tell a very different story. We can see

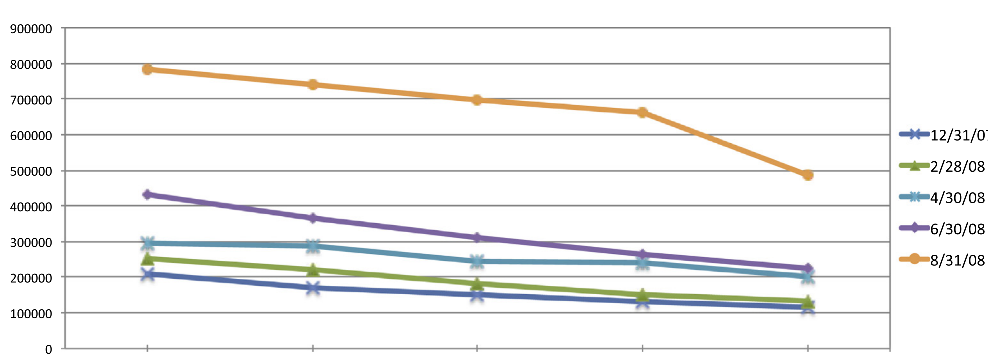


**Fig. 12.** Default barrier for Lehman Brothers. Fig. 12 shows the default barrier for our model with the refinancing parameter $\lambda = 0.5$ for every other month over the year. The scaling factor, $\alpha$, is set equal to 0.02.

from our structural analysis of Lehman Brothers that, when times are bad, drops in asset values increase leverage. Increased leverage further increases volatility which can feed back into asset prices, leading to a vicious downward-spiraling cycle. This negative feedback loop is also exacerbated by reduced liquidity, making it difficult to sell assets, raise capital, or maintain access to necessary credit channels. This reflects the nonlinearities in the relationships among the critical factors – volatility, asset values, leverage, and liquidity – that give rise to the buildup in risk exhibited by Lehman Brothers leading up to their ultimate failure. We believe that these results serve as further justification for the use of market information in supervising and regulating financial institutions and the additional insight that is provided from structural credit risk models.

#### *4.4.5. The default barrier*

Lastly, we return to the endogenous default barrier introduced in Section 3.1. Recall, this is the asset value that sets the equity value exactly equal to the next debt payment. The default barrier is therefore a function of the liability structure *and* the refinancing parameter, $\lambda$. Fig. 12 plots Lehman Brothers' default barrier as computed by our model for every other month from December 2007 to August 2008.

There are several features about Lehman's default barrier that are worth mentioning. First of all, we note that the figure shows the forward-looking default boundary. That is, the numbers in the figure represent the minimum market value of assets for Lehman Brothers to be solvent t-years in the future. Second, the forward-looking default boundaries all decline over time. This is because $\lambda = 0.5$, and half of the outstanding debt is paid down every year while half is rolled-over with new debt. Third, the default boundary reflects the market value of the illiquid assets on the books (due to netting).

The results shown in Fig. 12 have to be interpreted as follows. Standing at December 2007 (the lowest default barrier), we see that the default level one year out is $210 billion, as shown in the figure. This implies that only if the market value of Lehman Brothers' illiquid assets were to fall below $210 billion at the end of the first year, would the firm be in economic default. As the results show, the probability of this happening is relatively low given the market data.

Going forward in 2008, the numbers are dramatically different (this is done by moving vertically from line to line). In June 2008 the default barrier one year out is above $430 billion. Here the probability of the market value of Lehman's illiquid assets falling below $430 billion is much higher and, as indicated by our previous results, the mounting distress could have been predicted several months earlier. In September 2008, the 1-year forward default boundary is over $780 billion, implying near certain economic default. Given the high volatility, it is theoretically possible for Lehman to meet that target, but it is unlikely, as indicated by the high default probabilities at that time. The corresponding default probability is therefore very high, as seen in Figs. 8 and 9.

## 5. Conclusion

This paper presents new generalized binomial lattice, structural credit risk model which incorporates elements from both the Geske (1977) and Leland and Toft (1996) frameworks. The model is applied in a clinical analysis of the distress and failure of Lehman Brothers. The model uses market inputs, namely equity value and equity volatility, in order to estimate the forward default probabilities that serve as an "early warning signal" for financial institutions in distress. Additional insights from the model include estimating the amount of equity infusion needed to lower default risk to acceptable levels when a financial institution is in distress, and computing default-risk-based haircuts and implied collateral requirements for secured transactions.

Default in our model is defined in terms of an *endogenous* boundary, which is the point at which the firm will no longer be able to raise capital. In order to estimate economic default risk, it is important to begin with a deconstruction of a financial institution's balance sheet along several dimensions. First, it is important to note that a large portion of a financial institution's assets and liabilities are highly liquid – e.g., repo and reverse repo transactions – and these should be netted out to find the level of illiquid *Net Debt*. Second, it is important to take into account maturity and seniority characteristics of the firm's illiquid debt. And lastly, it is important to model the policy the firm pursues in managing its capital structure and how it refinances debt at maturity. By incorporating details of the liability structure, the refinancing of debt, and endogenous default, our model overcomes many of the hurdles that have previously limited the applicability of structural credit risk models to financial institutions.

We use hand-collected data to create a complete and comprehensive depiction of Lehman Brothers' liabilities along both the maturity and seniority dimensions. Our analysis of the maturity structure of debt allows us to construct a full term structure of default probabilities and then use this term structure to compute forward default probabilities. The evolution of the forward default probabilities indicates, as early as March 2008, that the firm would likely lose access to external capital within the next two years. Our analysis of the seniority structure supports Lehman's secured creditors asking for more collateral in the summer of 2008. Furthermore, we show that the firm's attempts to bolster its equity capital in the spring of 2008 were insufficient to mitigate the mounting risk. Our estimates indicate that Lehman needed an equity infusion of $15 billion or more to reduce default probabilities below 5%. Overall, our findings support regulators' suspicions that over-reliance on short-term funding and insufficient collateral compounded the effects of dangerously high leverage and resulted in undercapitalization and excessive risk exposure for Lehman Brothers. Going forward, the analytic tools from our model can be used by regulators and risk managers to diagnose financial distress and prescribe a course of action for addressing the sources of risk in large, complex financial institutions.

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