AN EMPIRICAL-DISTRIBUTION-BASED OPTION PRICING MODEL: A SOLUTION TO THE VOLATILITY SMILE PUZZLE

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Current version: December 4, 2002

We thank Charles Cao for letting us use his data set, and Chyong Ling Chen for research assistance. We also thank Gurdip Bakshi, John Cochrane, Ramon Rabinovitch, Avi Wohl, and participants at a seminar at Rutgers University and at the Twelfth Annual Conference on Financial Economics and Accounting for comments..

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ABSTRACT

The empirical literature documents that, contrary to the implication of the Black-Scholes model, the implied volatilities that are generated by the model vary systematically across moneyness levels (known as the "volatility smile" puzzle). The literature attributes the problem to two unrealistic features of the Black-Scholes model: the assumed stochastic process of the price of the underlying asset and the continuous rebalancing in the absence of transaction costs. In this paper, we construct an alternative valuation procedure to price S&P 500 call options, by using a histogram from past S&P 500 index daily returns. We find that the implied volatilities that are generated by our model do not exhibit substantial relationship to moneyness levels. Consistent with the absence of the smile, payoffs to holding options are also not related to moneyness levels. We also find that these payoffs are more closely related to our implied volatility measures than to the Black Scholes implied volatility measures. Moreover, the implied volatility curves that are generated by our model for our three maturities are much closer to one another than the corresponding curves that are generated by the Black-Scholes model. These findings indicate that our model is more appropriate than the Black-Scholes model to value S&P 500 call options. Furthermore, they also imply that the Black-Scholes model underprices in- and out-of the money call options relative to at-the-money options.

Key words: options, implied volatility, volatility smile, nonparametric model;

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1. INTRODUCTION

Since the introduction of the Black-Scholes model (1973), researchers have studied the empirical performance of the model. Comparing market prices and predicted model prices, early studies find that the model systematically miscalculates (or biases) the impacts of the strike price and the time to maturity on option prices. Starting in the early 1990's, researchers focus on the corresponding biases in the implied volatility. This examination of the implied volatility facilitates better identification, quantification, and thus understanding, of two sources of the bias. The relationship between implied volatilities and strike prices (or moneyness levels) is termed "volatility smile," while the relationship between implied volatilities and times to maturity is termed "volatility term structure." This paper focuses on the former issue.

The volatility smile that is generated by an option pricing model can be attributed to either of the following reasons: (i) the model underprices in- and out-of-the-money options, or (ii) the market overprices in- and out-of-the-money options (i.e., ignoring transaction costs, selling these options should generate abnormal profits.)

One possible reason for mispricing options is that the return distribution of the underlying asset that is assumed by the model does not match the actual distribution. Specifically, the literature has conjectured that the Black-Scholes model may underprice options because the tail probabilities of its assumed return distribution are too small (e.g.

see Duan (1999)). If the incorrect specification causes the model to underprice some options, high implied volatilities are needed to equate the model prices of these options to the corresponding actual prices. Researchers generally address this problem by incorporating either a jump process or stochastic volatility that add mass to the tails of the normal distribution.[1] However, Das and Sundaram (1999) report that the volatility smile decays too fast as maturities increase in models with jumps, and too slowly in models with stochastic volatility. Another possible reason for mispricing options is the assumption of costless continuous rebalancing. Positive transaction costs may induce a volatility smile if they reduce the value of at-the-money options more than the values of in- and out-of-the-money options. This may be the case because at-the-money options have the highest gamma and would need more frequent rebalancing (see Leland (1985), Proposition 2). Constantinides (1998) argues that proportional transaction costs help explain but cannot fully account for the volatility smile of index options.

The current study addresses the first reason by using a historical return distribution instead a return distribution that is based upon a stochastic process. In particular, we use a histogram from past S&P 500 index daily returns to price S&P 500 call options (European style). The use of the histogram has three advantages. First, using a historical return distribution does not assume any unrealistic moments for that distribution. Second, for each option maturity, we use a histogram of returns with the appropriate holding period. Thus, we do not make any unrealistic assumptions regarding the autocorrelation of returns. Finally, using histograms together with no rebalancing

[1] For example, studies that develop theoretical models include Naik and Lee (1990) and Bates (1991 and 1996) for jump-diffusions; Heston (1993), Hull and White (1987), Johnson and Shanno (1987), Scott (1987), and Wiggins (1987) for stochastic volatility; Scott (1997) and Bakshi, Cao and Chen (1997) for

allows us to derive a tractable equilibrium pricing model. The implied volatilities that are generated by using this empirical-distribution-based model eliminate the volatility smile. Consistent with the absence of a smile, we find that profits from selling options are related to our implied volatilities, but not to moneyness levels.

The paper is organized as follows. Section 2 reviews the empirical option pricing literature. Section 3 describes the distribution of the returns on the S&P 500 index and our call option sample. It also demonstrates that the returns on the S&P 500 index are not drawn from a normal distribution. Our model is presented and compared to the Black-Scholes model in Section 4. Section 5 presents our empirical results. The paper is concluded in Section 6.

2. LITERATURE REVIEW

The empirical literature on the Black-Scholes model is voluminous. Following is a brief survey of the issues that are most closely related to the present paper. Starting with Black (1975), the literature documents biases of the Black-Scholes model along two dimensions, moneyness and maturity.[2] Subsequent studies continue to find similar biases regardless of whether they adjust or do not adjust for early exercise premiums (i.e., American style).[3]

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both jumps and stochastic volatility; and Bailey and Stulz (1989) and Duan (1995) for the other stochastic specifications for the volatility.

[2] The first to document biases are Black and Scholes (1972) who find option prices for high (low) variance stocks to be lower (higher) than predicted by the model.

[3] Examples that do not take the American premium into consideration include MacBeth and Merville (1979), Emanuel and MacBeth (1982), Rubinstein (1985), Geske et al. (1983), and Scott (1987). Whaley (1982) and Geske and Roll (1984) discuss possible biases if such premiums are not included. Examples that take into consideration of the American premium include Whaley (1986), who adopt the Geske-Roll-

While early studies measure the biases in option prices, more recent studies have measured them in terms of implied volatilities. They consistently document the strike price bias (i.e., the volatility smile) and the time to maturity bias (i.e., volatility term structure) in various contracts. Rubinstein (1994) demonstrates that the implied volatility for S&P 500 index option is a sneer.[4] Shimko (1993) argues that the implied S&P 500 index distribution is negatively skewed and more leptokurtic than a lognormal distribution. Jackwerth and Rubinstein (1996) find that the S&P 500 index futures distribution before the crash of 1987 resembles the lognormal distribution, while the post-crash distribution exhibits leptokurtosis and negative skewness.[5]

The smile puzzle has been largely attributed to two unrealistic assumptions in the Black-Scholes model: the normality of stock returns and costless continuous rebalancing. The Black-Scholes model may underestimate option prices because it underestimates the tail probabilities of the return distribution. Several studies attempt to increase the weight of the tails of the return distribution by introducing jumps or stochastic volatility into the distribution of the underlying asset.[6] Bates (1996), using the Deutsche Mark options from 1984 through 1991, finds that the stochastic volatility model cannot explain the volatility smile. Bakshi, Cao and Chen (1997), using S&P 500 index options from 1988 through 1991, discover that the magnitude of the volatility smile is negatively related to maturity.[7] For maturities less than 60 days, they observe noticeable smiles for the three alternative models: stochastic volatility, stochastic volatility with jumps, and stochastic volatility with stochastic interest rates. Duan (1996) uses a GARCH model to price call

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Whaley model, for American style S&P 500 futures options and Bodurtha and Courtadon (1987), who adopt the approximation algorithm by Mason (1979) and Parkinson (1977), for currency options.
[4] Note that the Black-Scholes sneer found by Rubinstein (1994) is based upon data from one day.
[5] For additional evidence, see Bates (1991 and 1996) and Dumas, Fleming, and Whaley (1998).

options on FTSE 100 index. He demonstrates that the Black-Scholes implied volatilities corresponding to the prices generated by the GARCH model form a sneer similar to the Black-Scholes sneer for most options.

However, Das and Sundaram (1999) indicate that incorporating these features mitigates, but does not eliminate, the smile. They point out that jump-diffusion and stochastic volatility models do not generate skew and extra kurtosis patterns that resemble reality. For example, the extra kurtosis generated by jump diffusion models (stochastic volatility models) declines with the holding horizon faster (more slowly) than in reality. In their Table 1, Das and Sundaram demonstrate that, using jump diffusion models, extra kurtosis for the three-month holding period is less than 8% of the extra kurtosis for the one-week holding period. In their Table 3, Das and Sundaram demonstrate that, using stochastic volatility models, extra kurtosis for the three-month holding period is more than 70% of the extra kurtosis for the one-week holding period. In contrast, the corresponding number during our sample period between January 3, 1950 and April 7, 2000 is 23%.[8]

Eberlein, Keller, and Prause (1998) use a hyperbolic function for the distribution of underlying returns and find that the smile and the time-to-maturity effects are reduced in comparison to the Black-Scholes model, but not completely eliminated. They suggest that options that are not at-the-money face additional risk such as liquidity, and thus are more expensive. Longstaff (1995), Jackwerth and Rubinstein (1996), Dumas et al. (1998), and Peña et al. (1999) report that transaction costs and liquidity contribute to, but do not completely explain, the volatility smile.

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[6] See footnote 1.
[7] See also Eberlein and Prause (1998) and Peña et al. (1999).

In contrast, the literature includes a much smaller number of studies on the relationship between cost of rebalancing and the smile. Assuming proportional transaction costs, Leland (1985, proposition 2) argues that transaction costs should depress the values of near-the-money options more than those of away-from-the-money options.[9, 10] Constantinides (1998) finds that transaction costs cannot fully explain the volatility smile.

In addition to the volatility smile, the literature also documents that the implied volatility depends on option maturity.[11] However, other than the impact of maturity on the volatility smile, this issue is beyond the scope of this paper.

In summary, the literature includes many attempts to explain the volatility smile. Most studies document that the curvature of the smile is negatively related to option maturity. However, to the best of our knowledge, none of the studies successfully and completely explains the phenomenon. In the rest of the paper, we present an empirical-distribution-based model that eliminates the Black-Scholes smile.

3. DATA

Charles Cao has generously provided us with approximated prices of S&P 500 index call option contracts, matched levels of S&P 500 index, and approximated risk free interest

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[8] Detailed calculations are available upon request.

[9] One intuitive explanation for this proposition is that at-the-money options have higher gamma, and thus would need to be dynamically hedged more frequently than in- and out-of-the-money options.

[10] Boyle and Vorst (1992), and Cochrane and Saa –Requejo (2000) derive the upper and lower bounds for option prices if transaction costs are introduced but do not focus on the volatility smile.

[11] For example, see Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Heynen et al. (1994), and Xu and Taylor (1994), Campa and Chang (1995), Jorion (1995), and Amin and Ng (1997).

rates for the period of June 1988 through December 1991.[12] For each day, the approximated option prices are calculated as the average of the last bid and ask quotes. The risk free rate is approximated by the 90-day T-Bill rate.

Although the volatility smile is the focus of this paper, we acknowledge that the implied volatility changes also with contract maturity. To verify the robustness of our tests, we perform our tests over three different maturity samples: 25, 39, and 60 days.[13] These times to maturity are measured, in calendar days, from the trading day to the settlement day of the option. CBOE options expire on the Saturday after the third Friday of each month, but contracts are settled based upon the closing level of the index in the previous trading day (i.e., Friday, unless it is a holiday). The trading day for all three maturities is Tuesday. This day is selected because most holidays fall on Mondays, Thursdays and Fridays. We include only the observations in which contracts are settled based on the Friday closing.[14] This selection process yields a fixed time to maturity for all observations within a maturity sample, eliminating the effect of the volatility term structure. Because the data set includes only one set of observations for a given maturity and strike price for each trading date, we obtain at most one contract for each strike price every month. The number of contracts in our data set varies from month to month. Table 1 presents the number of contracts in each of our three maturity samples and their moneyness distribution.

Daily closing levels for the S&P 500 index for the period of January 3, 1950 to April 7, 2000 are collected and confirmed using Standard and Poor's, CBOE, Yahoo and

[12] The data are used in Bakshi, Cao and Chen (1997).

[13] As we document below, and consistent with Bakshi, Cao and Chen (1997), the volatility smile dissipates with maturity. Our choice of maturities corresponds to the maturities in which they find noticeable smiles.

Bloomberg.[15] Table 2 provides summary statistics for the returns derived from these index levels. These historical return distributions present large skewness and extra kurtosis measures that are significantly different from 0 (the skewness and extra kurtosis values under the Normal Distribution).[16] In addition, also consistent with the findings of previous studies, the extra kurtosis decreases as the holding period lengthens.[17]

4. THE MODEL

The literature includes two distinct series of models for valuing derivative contracts. One series of models (termed equilibrium models) assumes stochastic processes that span the economy and estimate their fixed parameters.[18] Hence, they cannot capture changing market conditions to explain market prices. The other series of models (termed no-arbitrage models) try to completely match market prices.[19] These studies are based on the premise that all market prices are "correct." Hence these models cannot suggest profit opportunities, even if prices are "out of line."

In this research, we propose an empirically based pricing model that is more flexible than the equilibrium models and has more predictive power than the no-arbitrage models. We assume that investors use the information from past return-realizations to

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[14] Thus, for example, contracts maturing in April when Good Friday falls on the third Friday of the month are not included in the sample.

[15] The (Ex-dividend) S&P 500 index we use is the index that serves as an underlying asset for the option. For option evaluation, realized returns of this index need not be adjusted for dividends unless the timing of the evaluated option contract is correlated with lumpy dividends. Because we use monthly observations, we think that such correlation is not a problem. Furthermore, in any case, this should not affect the comparison of the volatility smile between our model and the Black-Scholes model.

[16] At 1% level by the *Departure from Normality* test.

[17] See the Literature Review section and footnote 7.

[18] See, for example, Hull and White (1987) for the random volatility model, Merton (1976) for the jump diffusion, and Duan (1995) for GARCH.

estimate future returns on the underlying asset. In particular, we assume that, on each day investors construct histograms from a most recent fixed-length window. However, similar to the no-arbitrage models, investors in our model use option prices to adjust the volatility of the distribution. Formally, the call price is computed as:

Eq. 1 $$C_{t,T,K} = E_t\left[e^{-k_{t,T,K}} \max\{S_T - K, 0\}\right] \quad ,$$

where S_T represents the underlying asset price at the maturity time T, K is the strike price of the option, $k_{t,T,K}$ is the risk-adjusted discount rate, and $E_t[\cdot]$ is the conditional expectation under the real measure. This is consistent with Lemma 3 in Cox, Ingersoll, and Ross (1985). Note that the appropriate risk-adjusted discount rate is a function of the evaluation time (t), maturity time (T), strike price (K), and the current stock and option prices (to be formally presented later).

To facilitate the numerical valuation of Eq. 1 for our model, we normalize the variables as follows:

Eq. 2 $$\begin{cases} C^*_{t,T,K} \equiv \dfrac{C_{t,T,K}}{S_t}, \\ R_{t,T} \equiv \dfrac{S_T}{S_t}, \quad \text{and} \\ K^* \equiv \dfrac{K}{S_t}, \end{cases}$$

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[19] See, for example, Rubinstein (1994) for the implied binomial tree model, Jackwerth and Rubinstein

where S_t represents the current ex-dividend S&P 500 index (SPX) level. Thus, Eq. 1 turns into:

Eq. 3 $$C^*_{t,T,K} = E_t\left[e^{-k_{t,T,K}} \max\{R_{t,T} - K^*, 0\}\right] \quad .$$

In the empirical study, we use a histogram of stock returns to represent the distribution of $R_{t,T}$ in Eq. 3. We will also use the Capital Asset Pricing Model (CAPM) to construct the discount rate, k, for each option.[20] Formally:

Eq. 4 $$k_{t,T,K} = r_t(T-t) + \beta_{t,T,K}(E_t[R_{t,T}] - r_t(T-t)) \quad ,$$

where $E_t[R_{t,T}]$ is the expected return on the S&P 500 index (which serves as both the market portfolio and the underlying asset in our case), for the period [t, T]. The variable r is the risk free rate for which we use the 90-day T-Bill rate as a proxy. The systematic risk for the option, β, is defined as:

Eq. 5 $$\begin{aligned}\beta_{t,T,K} &= \frac{\operatorname{cov}[R_{t,T}, \frac{C_{T,T,K}}{C_{t,T,K}}]}{\operatorname{var}[R_{t,T}]} \\ &= \frac{\operatorname{cov}[R_{t,T}, \frac{1}{C^*_{t,T,K}} \max\{R_{t,T} - K^*, 0\}]}{\operatorname{var}[R_{t,T}]} \quad .\end{aligned}$$

(1996) for a continuous time model, and Ait-Sahalia and Lo (1998) for a state price density model.

We should note that two important adjustments to the option valuation are necessary in practice. First, the options mature on Saturdays, but the settlement is based upon the closing index level from the previous business day. As a result, the expectations specified in Eq. 1 and Eq. 3 are from the current time (t) to last trading day prior to the maturity date (i.e., T minus one day), but the discounting is for the time between the current time (t) and the maturity date (T). Second, because our index return sample is daily, the conditional expectation is based upon the information known a day prior to the trading date (i.e., t minus one day).

We compute option values using histograms that we construct from realizations of S&P 500 returns. We calculate the price at time t of an option that settles at time T using a histogram of S&P 500 index returns for a holding period of $T-t$ minus one day, taken from a 5-year window immediately preceding time t.[21] For example, the 25-calendar-day (and thus 24 calendar day from the trading day to the Friday prior to the settlement day) option price on any date is evaluated using a histogram of 17-trading-day ($17 \approx 24 \times 252/365$) holding period returns. The index levels used to calculate these returns are taken from a window that starts on the 1260-th ($= 5 \times 252$) trading day before the option trading date and ends one day before the trading date. Thus, this histogram contains 1243 17-trading-day return realizations. Note that this distribution is not risk neutral and thus we evaluate the options using Eq. 3. Furthermore, the distribution does not follow a nice functional form, and thus the option value cannot be valued by a closed form formula. Therefore, we evaluate the expectation of Eq. 3 numerically.

[20] This valuation is analogous to Cochrane and Saa–Requejo (2000) who use the Sharpe ratio and costly rebalancing to derive bounds for option prices.
[21] We use three alternative time windows, 2-year, 10-year and 30-year, to check the robustness of our procedure and results.

Note that option prices should be based upon the projected future volatility levels rather than the historical estimates. We assume that investors believe that the distribution of index return over the time to maturity follows the histogram of the last five years except for the standard deviation. Thus, for each contract, we infer this projected volatility by calibrating the model to the market price. We solve for an adjusted volatility of the distribution, $\hat{v}_{t,T,K}$, such that the resulting histogram yields the observed market price of the option. The adjusted volatility is used to calculate adjusted returns, $\hat{R}_{t,T,K,i}$:

Eq. 6 $$\hat{R}_{t,T,K,i} = \frac{\hat{v}_{t,T,K}}{v_{t,T}}(R_{t,T,i} - \overline{R}_{t,T}) + \overline{R}_{t,T} \quad , \quad i = 1, \ldots, N \ ,$$

where $\overline{R}_{t,T}$ and $v_{t,T}$ are, correspondingly, the mean and standard deviation of the original distribution $R_{t,T}$. Note that this re-scaling from R to $\hat{R}$ changes the standard deviation from $v_{t,T}$ to $\hat{v}_{t,T,K}$, but does not change the mean, skewness, or kurtosis. The preservation of these moments is a constraint that we impose on our model so that the calibration of our model matches that of the Black-Scholes model. This is also a conservative approach because we could have reached better calibration (flatter smile) had we included all moments in the calibration. If our model is correct, the adjusted volatilities should not vary systematically across moneyness levels for a given maturity.[22] As a result, we can use this test to examine the validity of our (and in fact any) model.

The expected option payoff is calculated as the average payoff where all the realizations in the histogram are given equal weights. Thus, Eq. 3 is numerically calculated as:

$$\text{Eq. 7} \quad C^{*\text{Our}}_{t,T,K} = e^{-k_{t,T,K}} \frac{\Sigma_{i=1}^{N} \max\{R_{t,T,i} - K^*, 0\}}{N} \quad ,$$

where N is the total number of realized returns and $R_{t,T,i}$ is the i-th return in the appropriate histogram.

We solve for $\hat{v}_{t,T,K}$ by numerically finding the solution to the simultaneous equation system that includes Eq. 4, Eq. 5, and Eq. 7, where the variable $R_{t,T,i}$ in these equations is replaced by $\hat{R}_{t,T,K,i}$ as given by Eq. 6. We compare the performance of our model with that of the Black-Scholes model. The Black-Scholes model assumes a log normal diffusion process for the SPX:

$$\text{Eq. 8} \quad \frac{dS_t}{S_t} = \mu dt + \sigma dW_t \quad ,$$

where μ is the expected rate of return on the SPX, σ is the instantaneous standard deviation of the SPX return, and W_t represents the Wiener process whose differential has 0 mean and dt variance. The Black-Scholes call option formula on the SPX is:

$$\text{Eq. 9} \quad C^{\text{BS}}_{t,T,K} = S_t N(h_{t,T,K}) - e^{-r_t(T-t)} K N(h_{t,T,K} - \sigma\sqrt{T-t}) \quad ,$$

[22] As described earlier, this hypothesis is valid only when all prices are "correct."

where

$$h_{t,T,K} = \frac{\ln(S_t / K) + r_t (T - t) + \sigma^2 (T - t) / 2}{\sigma \sqrt{T - t}} \quad .$$

To facilitate the comparison between the Black-Scholes model and our model, we perform the similar normalization:

Eq. 10 $C^{*\mathrm{BS}}_{t,T,K} = N(h_{t,T,K}) - e^{-r_t (T-t)} K^* N(h_{t,T,K} - \sigma \sqrt{T - t})$,

where

$$h_{t,T,K} = \frac{\ln(1 / K^*) + r_t (T - t) + \sigma^2 (T - t) / 2}{\sigma \sqrt{T - t}} \quad .$$

We solve for the implied volatility of the Black-Scholes model, denoted as $\hat{\sigma}$, by substituting the market price of the call option into the pricing equation.

In the next section we evaluate the performances of the Black-Scholes model and our model. We compare the payoffs to selling a naked option contract with the implied volatilities generated by the Black-Scholes model and our model. On any trading day, the present values of the cash flows that are generated by selling naked option contracts (referred to as payoffs) should be positively related to their implied volatilities, but should not be otherwise systematically related to moneyness levels. We conclude that the implied volatilities generated by our model match the payoffs better than the implied volatilities that are generated by the Black-Scholes model. Our findings indicate that our model is more appropriate to model S&P 500 call option prices.

5. EMPIRICAL RESULTS

We first compute the implied volatilities for S&P 500 call options that are generated by our model and by the Black-Scholes model in our three fixed-maturity samples (25, 39 and 60 days). We remove observations in which the implied volatilities do not converge. To mitigate the impact of observations with extreme moneyness levels on our results, we screen out the observations with moneyness more than 10% out-of-the-money or 30% in-the-money.[23] Excluding observations with extreme moneyness values should yield more reliable regression estimates when the estimated functional form is an approximation of the true function. The resulting sub-sample includes over 95% of the observations in the original sample. We present the annualized implied volatilities in Figures 1a through1c.

To examine the existence of a smile in S&P 500 index options, we run the following regressions:

$$\text{Eq. 11} \quad \begin{cases} a. \quad \hat{\sigma} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e \\ b. \quad \dfrac{\hat{v}}{\sqrt{T-t}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e \end{cases}$$

[23] The choice of near-the-money final sample is also consistent with previous studies. See, for example, Bakshi, Cao and Chen (1997).

where $\hat{\sigma}$ and $\frac{\hat{v}}{\sqrt{T-t}}$ are the annualized implied volatilities derived from the Black-Scholes model and our model, respectively. To simplify the exposition of Eq. 11 and the following regression equations, we suppress from all variables the complex subscripts that indicate the trade date t, the maturity date T, and the strike price K. The variable M, our moneyness variable, is defined as:

Eq. 12 $M \equiv 10(S-K)/K$.

Thus, the value of M in our final sample ranges between –1 and 3, where it is positive for in-the-money and negative for out-of-the-money options. We include the third and fourth powers of the moneyness measure in our regressions so not to restrict ourselves to the quadratic shape of the smile. We present the estimates of Eq. 11 for our three maturity samples in Table 3 and present the fitted values in Figures 2a and 2b. Figure 2a confirms the existence of a volatility smile generated by the Black-Scholes model as documented in previous studies. It also confirms the observations in previous studies that the volatility smile dissipates as the time to maturity lengthens. Figure 2b presents the corresponding fitted implied volatilities for our model, which are almost flat. Note also that the fitted curves that are generated by our model for the three maturities (presented in Figure 2b) are much closer to one another than the corresponding curves that are generated by the Black-Scholes model (presented in Figure 2a). While the volatility term structure is not our focus in this paper, the absence of dissipation under our model may indicate that our model may also help resolve the volatility term structure.

Recall that an option pricing model may generate an implied volatility smile either because the model underprices in- and out-of-the-money options, or because the market overprices these options. If our model is correct, then our flat fitted implied volatilities indicate that the volatility smile under the Black-Scholes model is due to the first motivation. To further verify that the second motivation is not an important determinant of the volatility smile that is induced by the Black-Scholes model, we calculate the payoffs generated by selling naked options and examine the relationship between the payoffs and moneyness.

The payoff of the short naked call strategy is defined as:

Eq. 13
$$\begin{aligned}\Pi_{t,T,K} &= C_{t,T,K} - e^{-k_{t,T,K}} C_{T,T,K} \\ &= C_{t,T,K} - e^{-k_{t,T,K}} \max\{S_T - K, 0\}\end{aligned}$$

where $k_{t,T,K}$ is the risk-adjusted discount rate for the call contract. Note that this risk-adjusted discount rate is model-dependent and calculated numerically in our model according to Eq. 4 and Eq. 5 (where $\hat{R}_{t,T}$ replaces $R_{t,T}$). In the Black-Scholes model, the call beta in Eq. 5 is defined as follows:

Eq. 14
$$\begin{aligned}\beta^{\mathrm{BS}}_{t,T,K} &= \frac{\partial C^{\mathrm{BS}}_{t,T,K} / C^{\mathrm{BS}}_{t,T,K}}{\partial S_t / S_t} \\ &= \frac{S_t}{C^{\mathrm{BS}}_{t,T,K}} N(h_{t,T,K}).\end{aligned}$$

We calculate the betas in Eq. 14 separately for each observation using the implied volatilities generated by the Black-Scholes model (implying that $C^{\mathrm{BS}}_{t,T,K} = C_{t,T,K}$). Thus, the betas calculated in Eq. 14 are consistent with the betas calculated in Eq. 5 because both use implied or adjusted volatilities. We run the following regressions:

$$\text{Eq. 15} \quad \begin{cases} a. \quad \Pi^{\text{Our}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \\ b. \quad \Pi^{\text{BS}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \\ c. \quad \Pi^{\text{RF}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \end{cases}$$

where the superscript of the dependent variable indicates the source of the discount rate: our model, Black-Scholes model, and the risk free rate. We use the three discount rates in order to verify that our results are not driven by the calculation of the risk-adjusted discount rates or by the difference between the discount rates that are generated by our model and the Black-Scholes model. The results are summarized in Panels A through C of Table 4. Note that the corresponding coefficients in the panels that present the estimated coefficients using our and the Black-Scholes discount rates are very similar to one another. Furthermore, these coefficients are generally similar to the corresponding estimated coefficients from the regressions that use the risk free rate to discount cashflows. Thus, any difference between the results of our model and the Black-Scholes model should not be attributed to the use of different discount rates.[24] None of the 30 estimated coefficients (for the constant term and the four moneyness powers for three maturities) using our and the Black-Scholes discount rates is significantly different from zero at the 10% significance level.[25] We conclude that trading profits are not significantly related to moneyness levels. This is consistent with the absence of a

[24] In addition, consistent with Merton, Scholes, and Gladstein (1978) and Coval and Shumway (2001), our discount rates are decreasing in our moneyness measure.

volatility smile in our model (and thus with the view that the Black-Scholes volatility smile is generated because the Black-Scholes model underprices in- and out-of-the-money options relative to at-the-money options).

Although the payoffs from selling naked options are not related to moneyness levels, they should be positively related to the implied volatilities that are generated by the correct model (i.e., the $\hat{v}_{t,T,K}$ if our model is correct and the $\hat{\sigma}$ if the Black-Scholes model is correct). Thus, we next run regressions of profits with respect to implied volatilities that are generated by our model and the Black-Scholes model. The comparison of implied volatilities and payoffs raises two concerns. First, implied volatilities are determined *ex-ante* (i.e., based on the expectations of traders on the trading date), while payoffs are *ex-post* results (i.e., based also on the realizations of returns on the maturity date of the sample period). Thus, realized profits should be affected by the expected profit (if any) on the trading date (which should be reflected in the implied volatilities), as well as by the unexpected return on the index between the trading and the maturity dates. Second, the derivative of profits with respect to the implied volatility measure is not independent of the moneyness level. The first concern that the *ex-post* realizations need not accurately reflect the *ex-ante* expectations is difficult to analyze because *ex-ante* expectations are not observable. We address the first concern by including in the regression the annualized realized index return between the trade date and the maturity date, denoted as ρ. We use this realized return to control for the difference between the *ex-post* realizations of the index returns and the *ex-ante* expectations (i.e., the *ex-post* surprises). We address the second concern by replacing the

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[25] Also, none of the 6 F-Statistics is significant at the 10% level (the two for 39 days are very close to 10%).

implied volatilities with a measure, *D*, which represents for each contract the dollar difference that corresponds to the difference between the volatilities implied by the two models.[26] Thus, for each maturity we estimate the following five regressions:

$$\text{Eq. 16}\quad \begin{cases} a. \quad \Pi^{\text{Our}} = a + b_1 \dfrac{\hat{v}}{\sqrt{T-t}} + b_2 \rho + e \quad , \\ b. \quad \Pi^{\text{BS}} = a + b_1 \hat{\sigma} + b_2 \rho + e \quad , \\ c. \quad \Pi^{\text{Our}} = a + b_1 \hat{\sigma} + b_2 \dfrac{\hat{v}}{\sqrt{T-t}} + b_3 \rho + e \quad , \\ d. \quad \Pi^{\text{BS}} = a + b_1 \hat{\sigma} + b_2 \dfrac{\hat{v}}{\sqrt{T-t}} + b_3 \rho + e \quad , \\ e. \quad \Pi^{\text{Our}} = a + b_1 \rho + b_2 D + e \quad . \end{cases}$$

Results for all three maturities are reported in Table 5. Eq. 16a uses our discount rate to obtain the payoffs. These payoffs are explained by our adjusted volatility and the realized rate of return on the index, controlling for the first concern. Eq. 16b uses the Black-Scholes implied volatility to explain payoffs that are obtained using the Black-Scholes discount rate. We interpret the positive (and significantly different from zero) coefficients on our implied volatility measure and the negative (and largely insignificantly different from zero) coefficients on the Black-Scholes implied volatility measure as indicating the validity of our model. However, the similar R-square measures indicate that the realized returns explain most of the variation in the payoffs.

Because our implied volatility and the Black-Scholes implied volatility are positively correlated, we use both to explain the payoffs in Eq. 16c and Eq. 16d. In both

[26] *D* is defined as the difference between our model's price using our own implied volatility (which equals the observed market price) and our model's price using the Black-Scholes implied volatility. An

specifications, the coefficients on our implied volatilities are positive while those on the Black-Scoles implied volatilities are negative (all are significantly different from zero). These results confirm our conclusion that the payoffs are more positively related to our implied volatilities than to the Black-Scholes implied volatilities. In Eq. 16e, we address the second concern by replacing the implied volatilities with the variable that represents the dollar difference between the two implied volatilities. Consistent with the better explanatory power of our model, the coefficients of this variable are positive and significantly different from zero for all three maturities. Finally, we test the robustness of our results to sample changes. We repeat the analyses with alternative return periods for the histogram construction: 2, 10 and 30 years. We also screen our sample to include the top half contracts when ordered by their trading date volume. Our tests indicate that our results are very robust to all these changes.

6. CONCLUSION

Our model contributes to the literature by generating implied volatilities that eliminate the volatility smile for the S&P 500 call options. We confirm the evidence found in previous studies of a volatility smile in the implied volatilities that are generated by the Black-Scholes model from the prices of S&P 500 call option contracts. Furthermore, also consistent with the previous literature, the curvature of the Black-Scholes implied volatility curves are negatively related to contract horizon. Previous studies unsuccessfully attempt to eliminate the smile by modifying one of two unrealistic

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alternative measure uses the Black-Scholes model to generate prices. Our results are robust to using this alternative measure.

assumptions of the Black-Scholes model: the normality of the return distribution and costless continuous rebalancing. We propose a model that simultaneously assumes an empirical-based return distribution and no rebalancing. Our model improves over previous studies in a number of ways. First, our empirical distribution does not suffer from the criticisms provided by Das and Sundaram (1999). Second, our implementation algorithm does not require complete markets or continuous trading. Thus, it allows for dependence between the discount rate and moneyness. Third, our model uses a more flexible return distribution. In contrast to the fixed normal distribution in the Black-Scholes model, the assumed distribution in our model (which is estimated from past returns) varies with the option horizon and trade date.

The implied volatility curves that are generated by our model for our three maturities (25-, 39-, and 60-day) are much closer to one another than the corresponding curves that are generated by the Black-Scholes model. Consistent with the absence of a smile, we also find that profits from selling options are not related to moneyness levels. Furthermore, our regressions indicate that these profits are related to our implied volatilities but not to the Black-Scholes' implied volatilities. These regressions control for the impact of *ex-post* realized returns, as well as take into account the potential difference in the sensitivity of premiums to volatilities across moneyness levels. Finally, our results are robust to a number of alternative model specifications including various time horizons for the histogram and sample choices.

These results are consistent with the view that our model is more appropriate than the Black-Scholes model to value S&P 500 call options. Furthermore, they also imply that the Black-Scholes model underprices in- and out-of the money call options relative to at-the-money options. Thus, in order to equate the option price to the actual price, the Black Scholes model generates relatively high implied volatilities for in- and out-of the money options relative to at-the money options.

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Table 1

Moneyness of the Call Options

The number of positive-volume observations of the CBOE S&P 500 in-the-money and out-of-the-money call options. The trading day for all option contracts is not earlier than June 1988 and not later than December 1991. The moneyness is defined as $(S-K)/K$.

Maturity Moneyness	25 days	39 days	60 days
> 10% out-of-the money	0	0	4
5%~10% out-of-the money	15	45	61
0~5% out-of-the money	130	150	132
0~5% in-the money	130	133	113
5~10% in-the money	105	97	59
10~20% in-the money	68	59	45
20~30% in-the money	22	16	15
30~40% in-the money	5	5	1
> 40% in-the money	1	0	1
Total	476	505	431

Table 2

Distribution Statistics of S&P 500 Returns (annualized) for Various Holding Horizons from 1/3/1950 to 4/7/2000

	25-day	39-day	60-day
Mean	0.1039	0.1034	0.1028
Standard Deviation	0.1412	0.1415	0.1413
Skewness	-0.4913	-0.4391	-0.4596
Extra Kurtosis	3.3950	2.4725	2.2356

Both skewness and extra kurtosis are significantly different from 0 (normality) at less than 1%.

Table 3
Smile Regressions

Regression Result of:

$$\text{Eq. 11:} \begin{cases} a. & \hat{\sigma} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e \\ b. & \dfrac{\hat{v}}{\sqrt{T-t}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e \end{cases},$$

where $\hat{\sigma}$ and $\hat{v}$ are the implied volatilities derived from the Black-Scholes model and our model respectively and M is defined as $10(S-K)/K$. Regression coefficients are listed on the left column and t statistics are listed on the right column. The trading day for all call option contracts is not earlier than June 1988 and not later than December 1991. The t statistic is reported to the right of each corresponding coefficient.

BS model						
	25-day		39-day		60-day	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Const	0.1690	72.2222	0.1736	75.5905	0.1766	62.3660
M	0.0557	11.7542	0.0503	10.7836	0.0491	7.6439
M2	0.0832	7.6436	0.0403	5.0472	0.0307	4.2286
M3	-0.0306	-3.1469	-0.0259	-3.0057	-0.0226	-2.5151
M4	0.0033	1.4228	0.0057	2.4406	0.0046	1.7241
R2(adjusted)	0.8679		0.6575		0.4670	
Our model						
	25-day		39-day		60-day	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Const	0.1917	68.8509	0.1967	72.6575	0.2018	62.9921
M	-0.0077	-1.3576	-0.0041	-0.7399	-0.0042	-0.5844
M2	0.0405	3.1299	0.0108	1.1468	0.0078	0.9515
M3	-0.0216	-1.8683	-0.0240	-2.3600	-0.0227	-2.2342
M4	0.0038	1.3724	0.0078	2.8275	0.0075	2.4710
R2(adjusted)	0.0778		0.0482		0.0616	
No. of Obs.	456		479		407	

Table 4
Regression Results on the Trading Profits from Selling Naked Calls:

$$\text{Eq. 15}\quad \begin{cases} a. \quad \Pi^{\text{Our}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \\ b. \quad \Pi^{\text{BS}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \\ c. \quad \Pi^{\text{RF}} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e, \end{cases}$$

where Π^{Our}, Π^{BS}, and Π^{RF} are the present values of the profits where the superscripts denote the sources of the discount rates: our model, the Black-Scholes model, and the risk free rate respectively and *M* is defined as $10(S-K)/K$. The t statistic is reported to the right of each corresponding coefficient.

A: discount rate is generated by our beta						
	25-day		39-day		60-day	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Const	0.0589	0.0939	0.0838	0.1105	0.4600	0.5247
M	-2.0639	-1.6243	1.9344	1.2558	-1.5567	-0.7824
M2	-3.2117	-1.1011	1.0780	0.4094	-1.0651	-0.4739
M3	3.9427	1.5142	-4.0747	-1.4321	1.7771	0.6399
M4	-0.8884	-1.4285	1.1654	1.5030	-0.5745	-0.6951
F stat. / F sig.	1.4808	0.2069	1.9503	0.1010	0.7809	0.5381
R2(adjusted)	0.0042		0.0079		-0.0022	
B: discount rate is generated by the Black-Scholes beta						
	25-day		39-day		60-day	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Const	-0.0417	-0.0653	-0.0126	-0.0164	0.2291	0.2548
M	-2.0696	-1.6022	2.0794	1.3323	-1.6173	-0.7927
M2	-3.1273	-1.0546	1.2569	0.4711	-0.9467	-0.4108
M3	3.8975	1.4723	-4.2733	-1.4821	1.8435	0.6473
M4	-0.8812	-1.3938	1.2080	1.5376	-0.6049	-0.7136
F stat. / F sig.	1.4211	0.2259	1.9043	0.1086	0.6524	0.6254
R2(adjusted)	0.0037		0.0075		-0.0034	
C: discount rate is risk free rate						
	25-day		39-day		60-day	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Const	-0.8654	-1.2844	-1.1221	-1.3572	-1.4982	-1.5450
M	-3.2306	-2.3668	0.7460	0.4440	-3.7243	-1.6924
M2	-2.6278	-0.8386	1.5708	0.5469	-0.6856	-0.2758
M3	3.8968	1.3931	-4.1511	-1.3375	2.2946	0.7470
M4	-0.9036	-1.3525	1.1701	1.3836	-0.7346	-0.8035
F stat. / F sig.	2.2681	0.0610	2.2634	0.0614	1.8768	0.1136
R2(adjusted)	0.0110		0.0105		0.0086	
# of obs.	456		479		407	

Table 5
Regression Results of Profits from Selling Naked Call Options:

$$\text{Eq. 16}\begin{cases} a. \quad \Pi^{\text{Our}} = a + b_1 \dfrac{\hat{v}}{\sqrt{T-t}} + b_2 \rho + e \quad , \\ b. \quad \Pi^{\text{BS}} = a + b_1 \hat{\sigma} + b_2 \rho + e \quad , \\ c. \quad \Pi^{\text{Our}} = a + b_1 \hat{\sigma} + b_2 \dfrac{\hat{v}}{\sqrt{T-t}} + b_3 \rho + e \quad , \\ d. \quad \Pi^{\text{BS}} = a + b_1 \hat{\sigma} + b_2 \dfrac{\hat{v}}{\sqrt{T-t}} + b_3 \rho + e \quad , \\ e. \quad \Pi^{\text{Our}} = a + b_1 \rho + b_2 D + e \quad . \end{cases}$$

where Π^{Our} and Π^{BS} are the present values of the profit where the discount rates are generated by our model and by the Black-Scoles model, respectively, $\hat{v}/\sqrt{T-t}$ is the annualised implied volatility using our model, ρ is the realized index returns, $\hat{\sigma}$ is the implied volatility using the Black-Scoles model, and D is the dollar difference that corresponds to the difference between the volatilities implied by the two models. The t statistic is reported to the right of each corresponding coefficient.

Panel A	25 days (# of obs. = 456)									
Specification	16*a*		16*b*		16*c*		16*d*		16*e*	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Intercept	6.8315	6.8488	11.9637	19.4569	6.4858	6.6985	6.5367	6.7574	11.1205	31.7581
BS Imp.Vol			-3.6553	-1.6595	-14.3048	-5.5472	-14.2280	-5.5227		
Our Imp. Vol.	21.5033	4.4213			40.2530	6.9422	40.7821	7.0401		
Ann. Ret.	-8.1618	-39.7449	-8.1931	-39.6595	-8.2928	-41.3980	-8.4663	-42.3042	-8.0830	-39.8864
OurIV-BSIV									1.1549	4.7473
F stat. /F sig.	791.3917	1.5E-148	787.4434	3.6E-148	572.526	1.7E-153	597.7602	7.2E-157	797.8299	3.6E-149
R2 (adj'ed)	0.7765		0.7756		0.7903		0.7974		0.7779	

Panel B	39 days (# of obs. = 479)									
Specification	16*a*		16*b*		16*c*		16*d*		16*e*	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Intercept	15.2677	12.7705	19.5743	20.6804	16.2312	13.0443	16.3684	13.0314	18.4697	35.6139
BS Imp.Vol			-3.9064	-0.9053	-12.0171	-2.6104	-11.0320	-2.3740		
Our Imp. Vol.	18.9985	3.1185			25.9396	3.9223	25.4601	3.8137		
Ann. Ret.	-14.3719	-40.3321	-14.2599	-39.9544	-14.3070	-40.2948	-14.5187	-40.5082	-14.1404	-40.8769
OurIV-BSIV									0.8879	3.2557
F stat. /F sig.	835.6350	1.9E-156	826.3259	1.5E-155	566.1660	2.1E-156	571.4073	3.8E-157	837.5684	1.2E-156
R2 (adj'ed)	0.7774		0.7754		0.7801		0.7817		0.7778	

Panel C	60 days (# of obs. = 407)									
Specification	16*a*		16*b*		16*c*		16*d*		16*e*	
	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.	Coef.	t stat.
Intercept	26.8230	21.6645	32.1208	26.3619	27.7075	21.5674	27.7969	20.3735	29.6271	37.2371
BS Imp.Vol			-5.9959	-1.0199	-14.3760	-2.4035	-23.9798	-3.7751		
Our Imp. Vol.	21.4210	3.2917			29.5394	4.0476	47.4129	6.1173		
Ann. Ret.	-24.4365	-37.7280	-25.7130	-37.4767	-24.2069	-37.1867	-27.0266	-39.0941	-23.5938	-40.5788
OurIV-BSIV									0.8574	3.9155
F stat. /F sig.	815.4973	1.4E-142	812.5546	2.6E-142	552.0191	2.6E-142	603.0131	1.4E-148	826.5235	1.6E-143
R2 (adj'ed)	0.8005		0.7999		0.8028		0.8165		0.8026	

Figure 1a
Implied Volatility Scatter Plot: 25-day
Black-Scholes Model versus Our Model

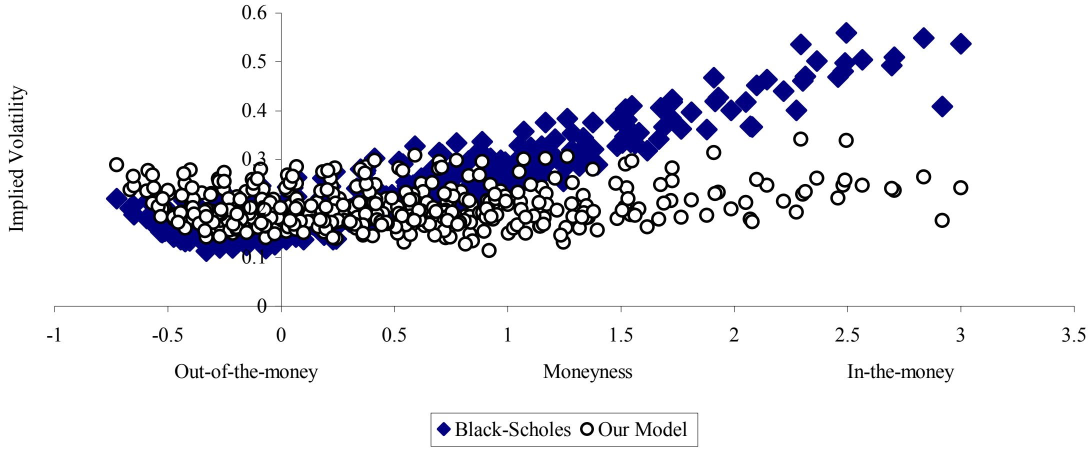

Figure 1b
Implied Volatility Scatter Plot: 39-day
Black-Scholes Model versus Our Model

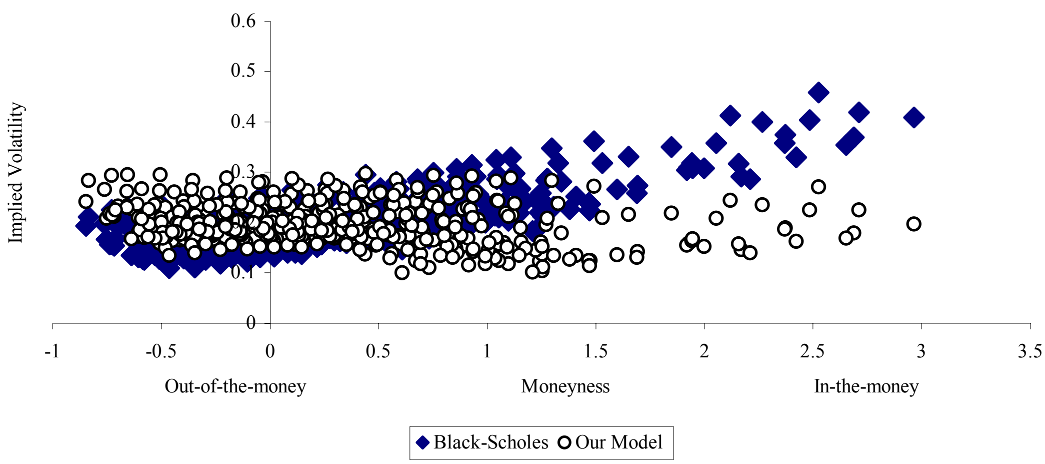

Figure 1c
Implied Volatility Scatter Plots: 60-day
Black-Scholes Model versus Our Model

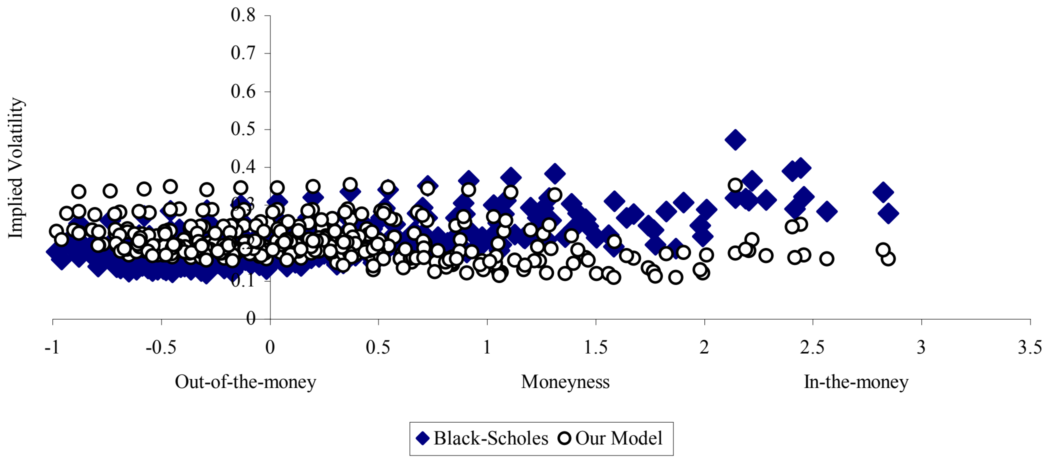

Figure 2a
Fitted Implied Volatility:
Black-Scholes Model

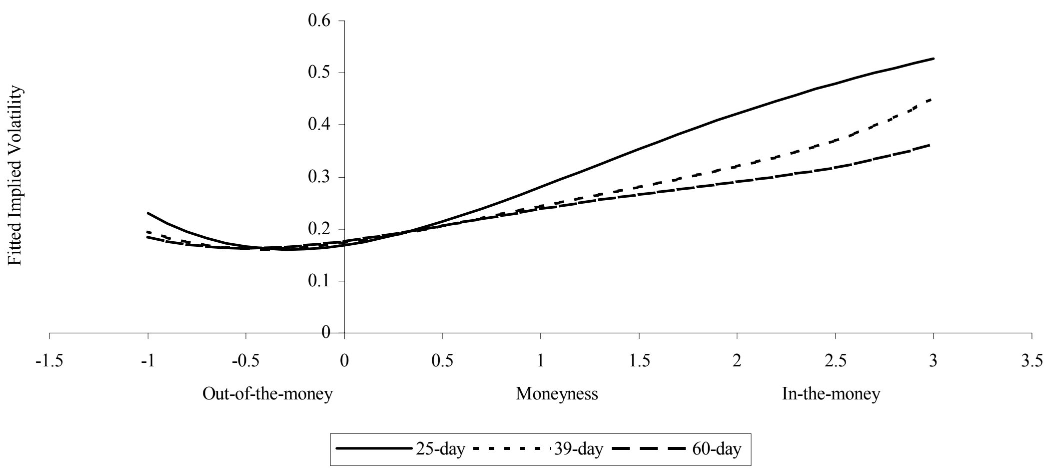

Figure 2b
Fitted Implied Volatility:
Our Model