Chapter 6

Beyond VaR

6.1 Introduction

In this chapter, three additional risk measures are introduced. The first is designed to correct the biases in volatility. The second is to include another important aspect VaR ignores – capital limitation. Finally, P&L attribution is crucial in assigning responsibilities in fund management.

VaR is far from perfect. Furthermore, VaR cannot detect some very risky situations. For example, there could be a long period over which VaR is satisfactory and yet in a few days in the period the investment suffers a substantial loss which can be so big that wipes out the entire endowment. Or a fund may eventually be profitable but during a turmoil period, substantial withdrawals could terminate the fund. Hence, in addition to VaR, fund management companies often look at drawdowns.

VaR is usually criticized because of its two bad assumptions – normality and volatility. The former is the assumption taken by the parametric VaR for the distribution of returns. This problem is usually circumvented by adopting EVT (see Chapter 4). The latter is more fundamental in that VaR assumes volatility represents risk. However, many believe that volatility cannot properly represent risk, at least not symmetrically. Risk results from fear of uncertainty. VaR uses volatility to proxy uncertainty. When market is bearish, volatility can probably be regarded as risk (this is mainly why VIX is called the fear index). However, when the market is bullish, volatility does not create any fear to investors and hence should not be regarded as risk. As a result, Frank A. Sortino and Lee N. Price develop a measure in which only downside moves are considered risk and then use it to propose the Sortino index.

Lastly, in fund management, VaR which is a risk-return measure is not suitable in evaluating the performance of fund managers. In addition to risk and return, there is a need to evaluate a portfolio manager by his professional skills (this could be his experience, insight, or use of proper models). The measure used here is P&L (profit and loss) attribution. P&L attribution can distinguish a successful manager is not successful because of his good luck and an unsuccessful manager is not unsuccessful because of his skills.

6.2 Sortino Ratio[1][2]

As said earlier, VaR is far from perfect. Although various improvements are proposed to add upon to VaR, still, many are unsatisfied because the underlying measure of risk (i.e. volatility) is flawed. It is widely acceptable that volatility is only risk when the market is going down. It is not necessarily risk when the market is going up. As a result, there has been distinction made for "good volatility" and "bad volatility"[3]. In other words, only one-sided volatility can be viewed as risk. The Sortino ratio is a ratio takes that into consideration and only measure the downside risk.

6.2.1 Various Performance Indices

Before we check out the Sortino ratio, we should first review various famous and popular performance indices.

Sharpe Index

Also known as the Sharpe ratio, the Sharpe index is perhaps the most widely used index in measuring the performance of a stock or a portfolio.

$$I_{\text{Sharpe}} = \frac{\mu_i - r}{\sigma_i}$$

where i can be a single stock or a portfolio, and μ and σ are usually replaced by sample mean and standard deviation as follows:

[1] Sortino, F.A.; Price, L.N. (1994). "Performance measurement in a downside risk framework". Journal of Investing. 3: 50–8.

[2] `http://www.redrockcapital.com/Sortino__A__Sharper__Ratio_Red_Rock_Capital.pdf`

[3] For example, see `http://public.econ.duke.edu/~boller/Papers/jfqa_19.pdf`

$$\begin{aligned} \mu_i &= \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \\ \sigma_i &= \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t}-\mu_i)^2} \end{aligned} \tag{6.1}$$

Jensen Index

In a diversified portfolio, it is the systematic risk (β), not the total risk (σ) that is relevant.

$$I_{\text{Jensen}} = \frac{\mu_i - r}{\beta_i}$$

where β is the coefficient of the following regression:

$$r_{i,t} = \alpha_i + \beta_i r_{M,t} + e_{i,t}$$

and $r_{M,t}$ is the return of the market portfolio.

Trenor Index

Jack Trenor is the mentor of Fischer Black who invented the Black-Scholes formula. Trenor index is also called Trenor'a alpha.

$$\begin{aligned} I_{\text{Trenor}} &= r_i - \mu_i \\ &= r_i - (r + \beta_i(\mu_M - r)) \end{aligned} \tag{6.2}$$

Again, usually theoretical mean μ and volatility σ are replaced by sample mean and standard deviation. Similarly, the market portfolio is usually replaced by a popular stock index (e.g. S&P 500).[4]

6.2.2 Sortino Ratio

$$I_{\text{Sortino}} = \frac{\mu_i^* - r}{\sigma_i^*}$$

[4]Note that theoretically the market portfolio should be the tangent portfolio on the efficient frontier.

where

$$\begin{aligned}\mu_i^* &= \frac{1}{T^*}\sum\nolimits_{t=1}^{T^*} r_{i,t}^* \\ \sigma_i^* &= \sqrt{\sum\nolimits_{t=1}^{T^*} (r_{i,t}^* - \mu_i^*)^2}\end{aligned} \tag{6.3}$$

and $r_{i,t}^*$ is the return below the risk-free rate (or any benchmark). Hence, μ_i^* and σ_i^* can be viewed as mean and standard deviation of those returns (and T^* is the number of days) that fall below a certain target return.

6.2.3 Omega Ratio[5]

Omega Ratio is first proposed by Con Keating and William F. Shadwick.[6] Omega ratio is closely related to Sortino ratio in the spirit of downside market and lower partial moments of the distribution. Omega ratio captures all of the higher moments of the returns distribution. The performance measure is applied to a range of hedge fund style or strategy indices. Originally, the Omega ratio is defined as follows:

$$\Omega(K) = \frac{\int_K^b [1 - F(r)]dr}{\int_a^K F(r)dr}$$

where K is a threshold and $F(r)$ is the cumulative density function of r (return) over $[a, b]$. [7] Intuitively speaking, Omega ratio is:

$$\begin{aligned}\Omega(K) &= \frac{\sum(\text{winning - benchmarking})}{\sum(\text{benchmarking - losing})} \\ &= \frac{\frac{1}{T^*}\sum_{t=1}^{T}(r_t - K)1_{r_t > K}}{\frac{1}{T^{**}}\sum_{t=1}^{T}(K - r_t)1_{r_t < K}}\end{aligned} \tag{6.4}$$

where 1. is the indicator function which takes a value of 1 if the statement in the subscript is true and 0 if it is false, and T^* and T^{**} are the numbers of those returns in $1_{r_t > K}$ and $1_{r_t < K}$ respectively.

Note that equation 6.4 is theoretically identical to the following:

[5] Materials here are drawn from `https://quantdare.com/omega-ratio-the-ultimate-risk-reward-ratio/`

[6] Keating and Shadwick, 2002, "A Universal Performance Measure," the Finance Development Centre Limited.

[7] Hence, it is common to set $a = -\infty$ and $b = \infty$ under continuous returns.

$$\begin{aligned}\Omega(K) &= \frac{\mathbb{E}[\max\{r-K,0\}]}{\mathbb{E}[\max\{K-r,0\}]} \\ &= \frac{C}{P} \\ &= \frac{\mu-K}{\mathbb{E}[\max\{K-r,0\}]}+1\end{aligned}$$

which is similar to the Sortino ratio if we rewrite the Sortino ratio in its theoretical form as:

$$S = \frac{\mu-K}{\sqrt{\mathbb{E}\left[\max\{K-x,0\}^2\right]}}$$

6.3 Maximum Drawdown

Investopedia:

A maximum drawdown is the maximum observed loss from a peak to a trough of a portfolio, before a new peak is attained. Maximum drawdown is an indicator of downside risk over a specified time period. It can be used both as a stand-alone measure or as an input into other metrics such as "Return over Maximum Drawdown" and the Calmar Ratio. Maximum Drawdown can be expressed in dollar or percentage terms.

As the term suggests, the maximum drawdown is a downward deviation from a defined threshold which is commonly set at the current capital. In other words, a drawdown is how much loss an investment suffers and the maximum drawdown is the maximum loss in a past period (e.g. past 10 years). The purpose of the maximum drawdown is to help avoid the blind spot of an investment's performance. A profitable investment could suffer a maximum drawdown that exceeds the initial capital and in such a case, although the investment is ultimately profitable, it may never be able to reach it because the fund will go bankrupt before it reaches the end of the investment horizon.

Sterling Ratio

Wikipedia:

The original definition was most likely suggested by Deane Sterling Jones (a company no longer in existence). The Sterling ratio (SR) is a measure of the risk-adjusted return of an investment portfolio.

$$R_S = \frac{R}{|D - 10\%|}$$

where R is compounded ROR, D is the average annual drawdown.

If the drawdown is put in as a negative number, then subtract the 10%, and then multiply the whole thing by a negative to result in a positive ratio. If the drawdown is put in as a positive number, then add 10% and the result is the same positive ratio.

To clarify the reason he (Deane Sterling Jones) used 10% in the denominator was to compare any investment with a return stream to a risk-free investment (T-bills). He invented the ratio in 1981 when t-bills were yielding 10%. Since bills did not experience drawdowns (and a ratio of 1.0 at that time), he felt that any investment with a ratio greater than 1.0 had a better risk/reward tradeoff. The average drawdown was always averaged and entered as a positive number and then 10% was added to that value.

$$R_S = \frac{r_P - r}{L}$$

where r_P is annual portfolio return and L is average largest drawdown.

Calmar Ratio

Investopedia:

The Calmar ratio is a gauge of the performance of investment funds such as hedge funds and commodity trading advisors (CTAs). It is a function of the fund's average compounded annual rate of return versus its maximum drawdown. The higher the Calmar ratio, the better it performed on a risk-adjusted basis during the given time frame, which is mostly commonly set at 36 months.

The Calmar ratio was developed and introduced in 1991 by Terry W. Young, a California-based fund manager. He argued that the ratio offered a more up-to-date reading of a fund's performance than the Sterling or Sharpe ratios, other commonly used gauges, because it was calculated monthly while they were done annually. The monthly update also made the Calmar ratio smoother than what Young called the "almost too sensitive" Sterling ratio.

The Calmar ratio is, in fact, a modified version of the Sterling ratio. Its name is an acronym for California Managed Account Reports. Young also referred to the Calmar ratio as the drawdown ratio.

The ratio is defined as:

$$I_{\text{Calmar}} = \frac{r_P - r}{D^*}$$

where D^* is the maxmimum drawdown.

6.4 P&L Attribution

The P&L attribution is to explain where the profits and losses come from. Due to the fact that luck plays a critical role in trading, managers need to make sure that their traders make money not due to luck but due to skills (or talents). As a result, P&L attribution has become essential in trading and fund management business.

6.4.1 Taylor's Series Expansion and Explanatory Risk Factors

Greeks are key to explain relative importance of each risk factor. We define the basic Greeks as follows:

$\Delta = \frac{\partial C}{\partial S}$ = partial derivative of target (e.g. call) with respective to the underlying (e.g. stock)

$V = \frac{\partial C}{\partial \sigma}$ = partial derivative of target (e.g. call) with respective to the volatility

$\mathrm{P} = \frac{\partial C}{\partial r}$ = partial derivative of target (e.g. call) with respective to the the interest rate

$\Theta = \frac{\partial C}{\partial t}$ = partial derivative of target (e.g. call) with respective to time (known as time decay)

$\Gamma = \frac{\partial^2 C}{\partial S^2}$ = partial derivative of target (e.g. call) twice with respective to the underlying

Cross Greeks can be defined, as a few examples, as follows:

$$\Delta_V = V_\Delta = \frac{\partial^2 C}{\partial S \partial \sigma} \quad \text{(Vanna)}$$

$$\Gamma_V = \frac{\partial^2 C}{\partial \sigma \partial \sigma} \quad \text{(Volga)}$$

$$\Delta_{\mathrm{P}} = \mathrm{P}_\Delta = \frac{\partial^2 C}{\partial S \partial r}$$

$$\mathrm{P}_V = V_{\mathrm{P}} = \frac{\partial^2 C}{\partial r \partial \sigma}$$

where the first two are popular in FX.

Take a single product as an example (e.g. call option). The explanatory factors are price, volatility, and time – which translate to Delta, Gamma, Vega, and Theta. If the BS model is the correct model, then we know that:

$$\begin{aligned} dC &= \left(\mu S C_S + \sigma^2 S^2 C_{SS} + C_t\right) dt + \sigma S C_S dW \\ &= \underbrace{\left(\mu S \Delta + \sigma^2 S^2 \Gamma + \Theta\right)}_{\text{explanatory factors}} dt + \underbrace{\sigma S \Delta dW}_{\text{unexplained}} \end{aligned} \tag{6.5}$$

and there is no Vega. If the volatility and the risk-free rate are both random, then we must first define the volatility and interest rate processes. Usually we write both of them as mean-reverting square root processes:

$$\begin{aligned} dS &= \mu S dt + \sqrt{V} S dW_1 \\ dV &= \alpha(\beta - V)dt + \sqrt{V}\gamma dW_2 \\ dr &= a(b - r)dt + \sqrt{r} g dW_3 \end{aligned} \tag{6.6}$$

where $dW_i dW_j = \rho_{ij} dt$. Then the P&L attribution becomes:

$$\begin{aligned} dC &= C_S dS + C_V dV + C_r dr + C_{SS}(dS)^2 + C_{VV}(dV)^2 + C_{rr}(dr)^2 \\ &\quad + C_{SV}(dS)(dV) + C_{Sr}(dS)(dr) + C_{Vr}(dV)(dr) + C_t \\ &= \begin{bmatrix} \mu S C_S + \alpha(\beta - V)C_V + a(b-r)C_r + C_t + \\ V S^2 C_{SS} + \gamma^2 V C_{VV} + g^2 r C_{rr} + V S \gamma \rho_{12} C_{SV} + r S g \rho_{13} C_{Sr} + \sqrt{rV}\gamma g \rho_{23} C_{Vr} \end{bmatrix} dt \\ &\quad + \sqrt{V} S C_S dW_1 + \sigma \sqrt{V} C_V dW_2 + g\sqrt{r} \mathrm{P} dW_3 \\ &= \underbrace{\begin{bmatrix} \mu S \Delta + \alpha(\beta - V)V + a(b-r)\mathrm{P} + \Theta + \\ V S^2 \Gamma + \gamma^2 V V_V + g^2 r \mathrm{P}_{\mathrm{P}} + V S \gamma \rho_{12} \Delta_V + r S g \rho_{13} \Delta_{\mathrm{P}} + \sqrt{rV}\gamma g \rho_{23} V_{\mathrm{P}} \end{bmatrix}}_{\text{explanatory factors}} dt \\ &\quad + \underbrace{\sqrt{V} S \Delta dW_1 + \gamma\sqrt{V} V dW_2 + g\sqrt{r}\mathrm{P} dW_3}_{\text{unexplained}} \end{aligned} \tag{6.7}$$

As we can see, the model can expand to as many random factors as we wish. As shown in the exhibit earlier, the random factors can include FX, multiple key interest rates, credit, prepayment, and any other risk factors. At the end, the equation can be very long and the number of parameters can become quite unimaginable. Some simplifications must be necessary.

The first simplification is as easy as just ignoring higher order (and cross) Greeks. We can assume that higher order Greeks such as Vega-Vega and Delta-

Vega effects are small and ignore them. The second simplification is to adopt the VaR methodology and choose several key benchmark indices.

6.4.2 Pictorial P&L Attribution

Figure 6.1 is taken from http://www.pnlexplained.com/ on 12/15/2009 which describes very well the concept of P&L attribution.

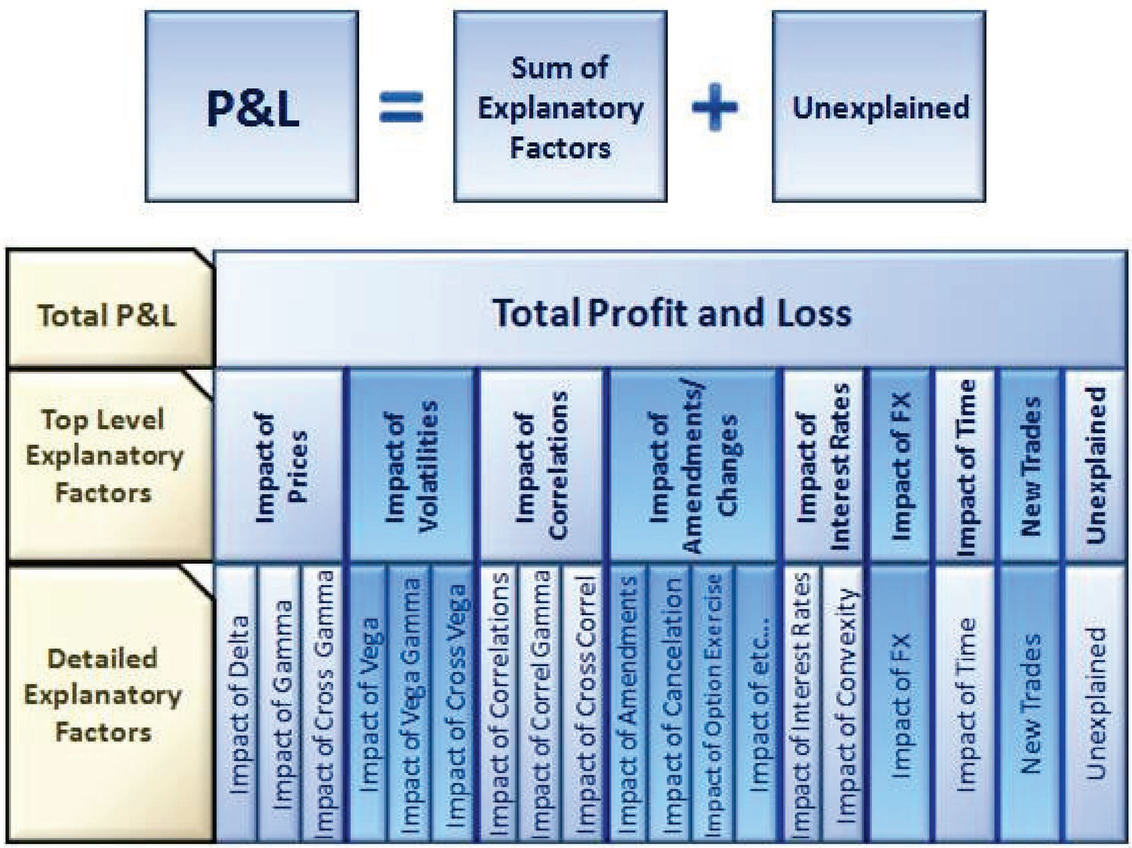

Figure 6.1: P&L Decomposition

At the bottom layer, we see various first order Greeks such as Delta and Vega, and various higher order Greeks such as Gamma and cross Greeks (cross Gamma and Vega-Gamma).

6.4.3 How Does It Work Empirically

The theory explained in Section 6.4.1 often does not apply in reality simply because only very few models exist for the large universe of securities. Usually, these models are for derivatives and hence "cash" products like stocks, FX, commodities, etc. will not be able to leverage upon Taylor's series expansion because they are not derivatives. However, we do recognize that these cash products are significantly influenced by macro economic conditions such as GDP, inflation, exports and imports, etc. In other words, we can view these cash products as derivatives of more fundamental economic conditions. However, unfortunately, it is hard to develop an analytical model of these cash products as a result of those macro economic factors.

As a result, regression is used to specify the relationship of these cash products and the macro economic factors. One can specify any functional form such as follows:

$$r_i = b_0 + \sum\nolimits_{k=1}^{K} b_k x_k + e_i$$

where for example x_1 can be GDP and x_2 can be GDP^2 and x_3 can be GDP^3. Interaction terms such as GDP$\times$Inflation can also be included.

Then the partial derivatives are simply the regression coefficients and the decomposition depicted in Section 6.4.2 can be implemented. Certainly, regression results are not stable. They vary from period to period and also are sensitive to how the sample is chosen. Interested reader can use the regression model (x_k's are polynomials of the stock price) to price a call option and compare it against the Black-Scholes price.