GRADUATE SCHOOL OF BUSINESS

Global Risk Management: A Quantitative Guide

Credit Risk Management

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Types of Credit Risk

- Bankruptcy
- Rating migration
- Spread change

Types of Credit Risk

- Bankruptcy
  - ~ default
  - US Treasuries interests as risk-free rates

Types of Credit Risk

- Bankruptcy
- Rating migration
  - AAA ~ D
  - represent likelihood of bankruptcy
  - rating change → price changes (stocks & bonds)
  - upgrade → prices rise; downgrade → fall
  - credit analysts (vs. equity analysts)
  - downgrade → price drop → risk

Types of Credit Risk

- Bankruptcy
- Rating migration
- Spread change
  - ratings change infrequently
  - spreads are traded (via bonds)
  - higher spreads → lower bond prices → risk
  - spread risk is market risk
  - higher spreads ultimately lead to downgrade

Bankruptcy

- Probability of default (PD)
- Recovery (or LGD, loss given default)
  - Equity investors
    - Almost nothing
  - Bond investors
    - Recovery
    - Secured – 80~90% (due to drop in value)
    - Senior unsecured – 40%
    - Junior unsecured – 15%
  - Jump to default risk (now popular)

Migration

- One step before bankruptcy
- Rating announcements carry information
- BBB or higher vs. BB or lower
  - buy and sell pressure
  - due to pension fund regulation
- Large literature (details later)

Migration

Global Corporate Transition Matrix (%) (1981-2010)

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC/C	D
AAA	87.91	4.72	2.68	0.68	0.16	0.24	0.14	0.00	0.05	0.00	0.03	0.05	0.00	0.00	0.03	0.00	0.05	0.00
AA+	2.62	76.06	11.67	3.93	0.89	0.66	0.30	0.12	0.12	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.47	1.32	80.64	8.01	2.89	1.41	0.43	0.42	0.14	0.09	0.05	0.04	0.02	0.00	0.00	0.02	0.05	0.02
AA-	0.05	0.13	4.28	76.93	10.02	2.84	0.71	0.27	0.14	0.07	0.04	0.00	0.00	0.04	0.11	0.02	0.00	0.04
A+	0.00	0.11	0.58	4.46	77.42	8.80	2.57	0.71	0.40	0.09	0.09	0.12	0.01	0.09	0.04	0.01	0.00	0.07
A	0.05	0.06	0.28	0.56	5.01	77.73	6.82	2.69	1.15	0.28	0.15	0.15	0.10	0.12	0.03	0.01	0.02	0.09
A-	0.06	0.01	0.11	0.20	0.61	6.78	75.80	7.51	2.36	0.68	0.16	0.15	0.16	0.14	0.04	0.01	0.05	0.08
BBB+	0.00	0.01	0.07	0.09	0.31	1.05	6.93	73.19	8.85	2.01	0.47	0.40	0.17	0.26	0.15	0.02	0.10	0.16
BBB	0.01	0.01	0.06	0.04	0.17	0.48	1.23	7.04	74.22	6.30	1.62	0.83	0.37	0.31	0.17	0.04	0.09	0.23
BBB-	0.01	0.01	0.01	0.07	0.07	0.24	0.40	1.37	8.56	71.12	5.48	2.59	1.03	0.56	0.34	0.22	0.31	0.38
BB+	0.07	0.00	0.00	0.05	0.02	0.15	0.12	0.63	2.29	11.70	62.56	6.43	3.24	1.27	0.83	0.19	0.51	0.56
BB	0.00	0.00	0.06	0.02	0.00	0.10	0.08	0.23	0.74	2.56	8.51	64.26	7.74	2.69	1.37	0.46	0.74	0.80
BB-	0.00	0.00	0.00	0.01	0.01	0.01	0.07	0.13	0.30	0.48	2.06	8.23	63.76	8.43	3.06	0.97	0.91	1.31
B+	0.00	0.01	0.00	0.04	0.00	0.04	0.09	0.06	0.07	0.10	0.34	1.57	6.92	65.02	7.66	2.62	1.96	2.62
B	0.00	0.00	0.02	0.02	0.00	0.09	0.07	0.04	0.11	0.04	0.23	0.39	1.69	8.39	57.67	7.95	5.42	5.90
B-	0.00	0.00	0.00	0.00	0.04	0.07	0.00	0.14	0.07	0.14	0.18	0.21	0.61	3.13	10.22	51.30	10.82	9.15
CCC/C	0.00	0.00	0.00	0.00	0.05	0.00	0.14	0.09	0.09	0.09	0.05	0.23	0.56	1.39	2.91	8.70	43.80	27.43

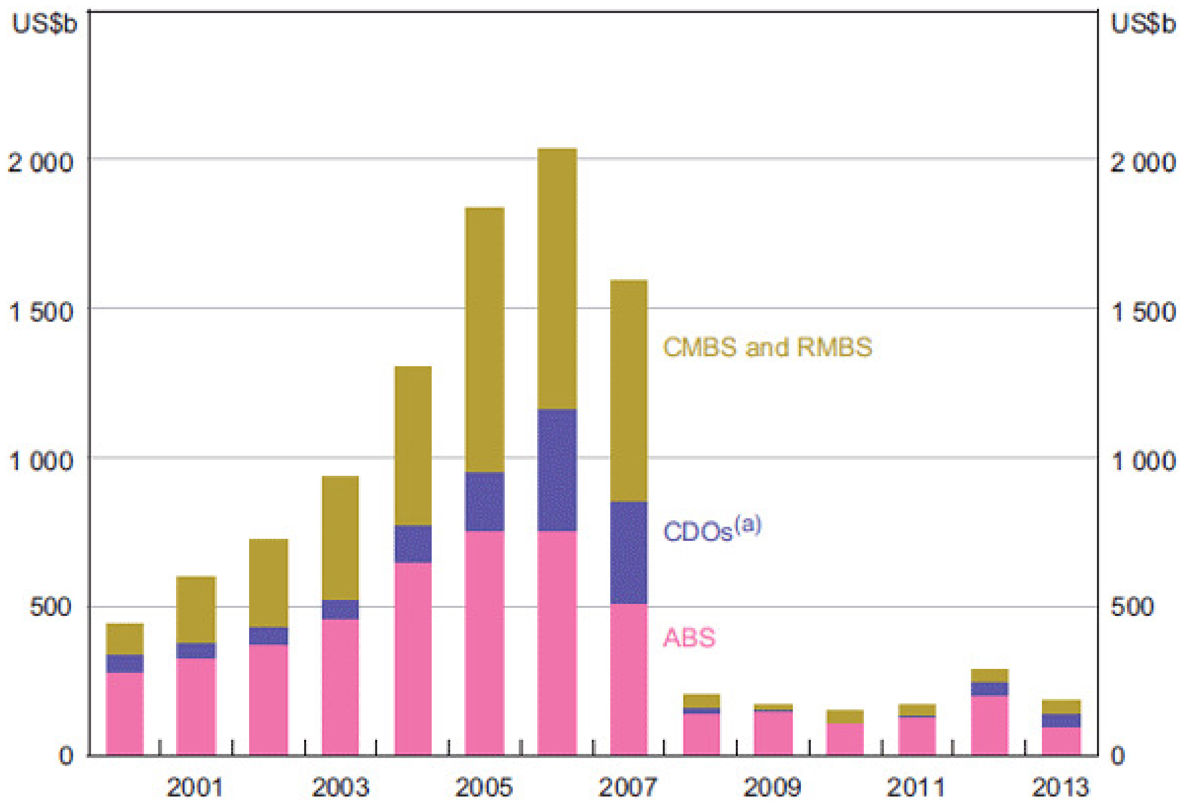
Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's Credit Pro®.

Spreads

- Day to day risk
  - market risk
  - hedgeable
- CVA
  - CDS
  - Correlation risk

The Market

Figure 4: United States – Non-agency Securitisation Issuance



Notes: Year to July 2013; CMBS denotes commercial mortgage-backed securities, RMBS denotes residential mortgage-backed securities, ABS denotes asset-backed securities
(a) Collateralised debt obligations (CDO) issued in US dollars

Source: Securities Industry and Financial Markets Association

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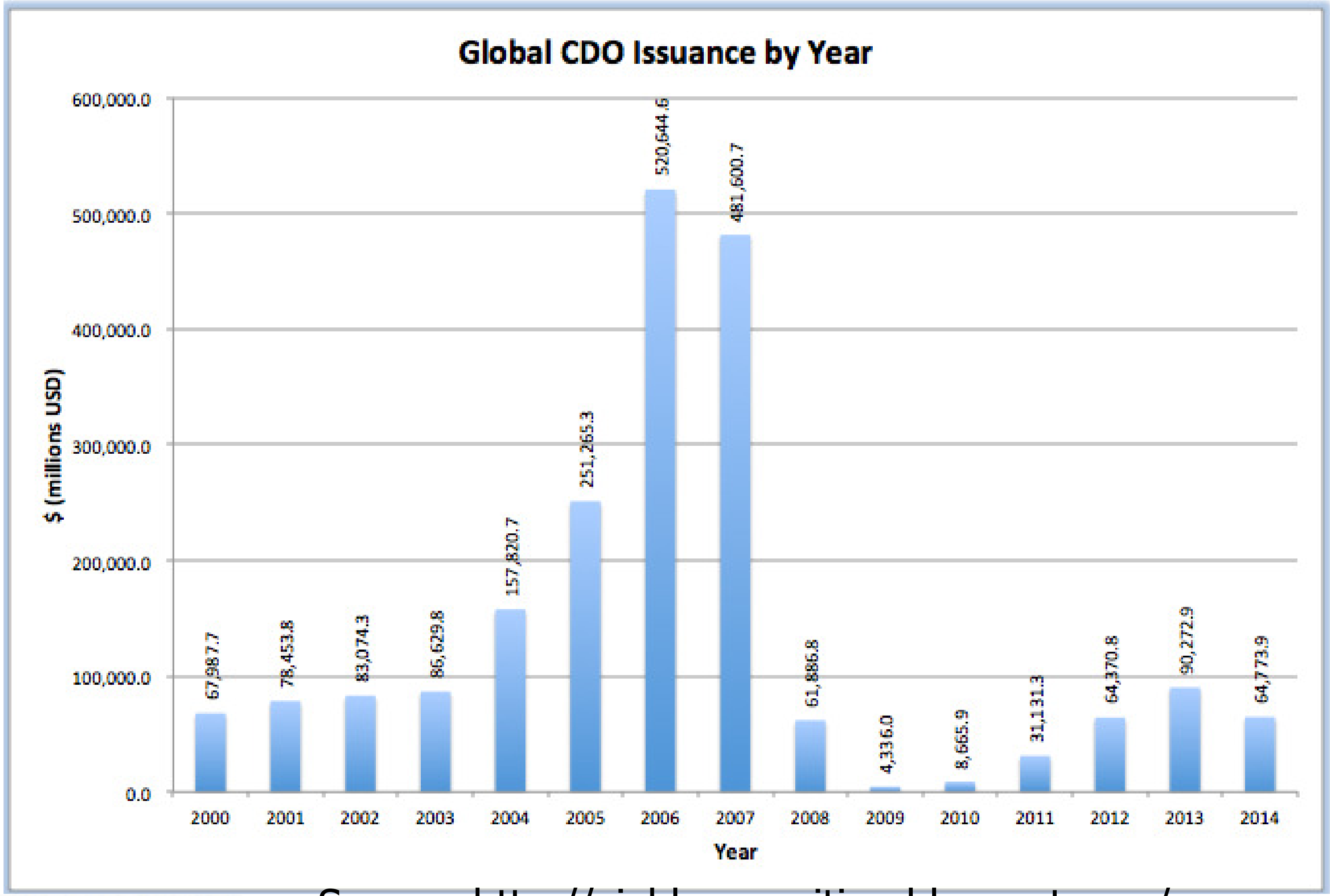
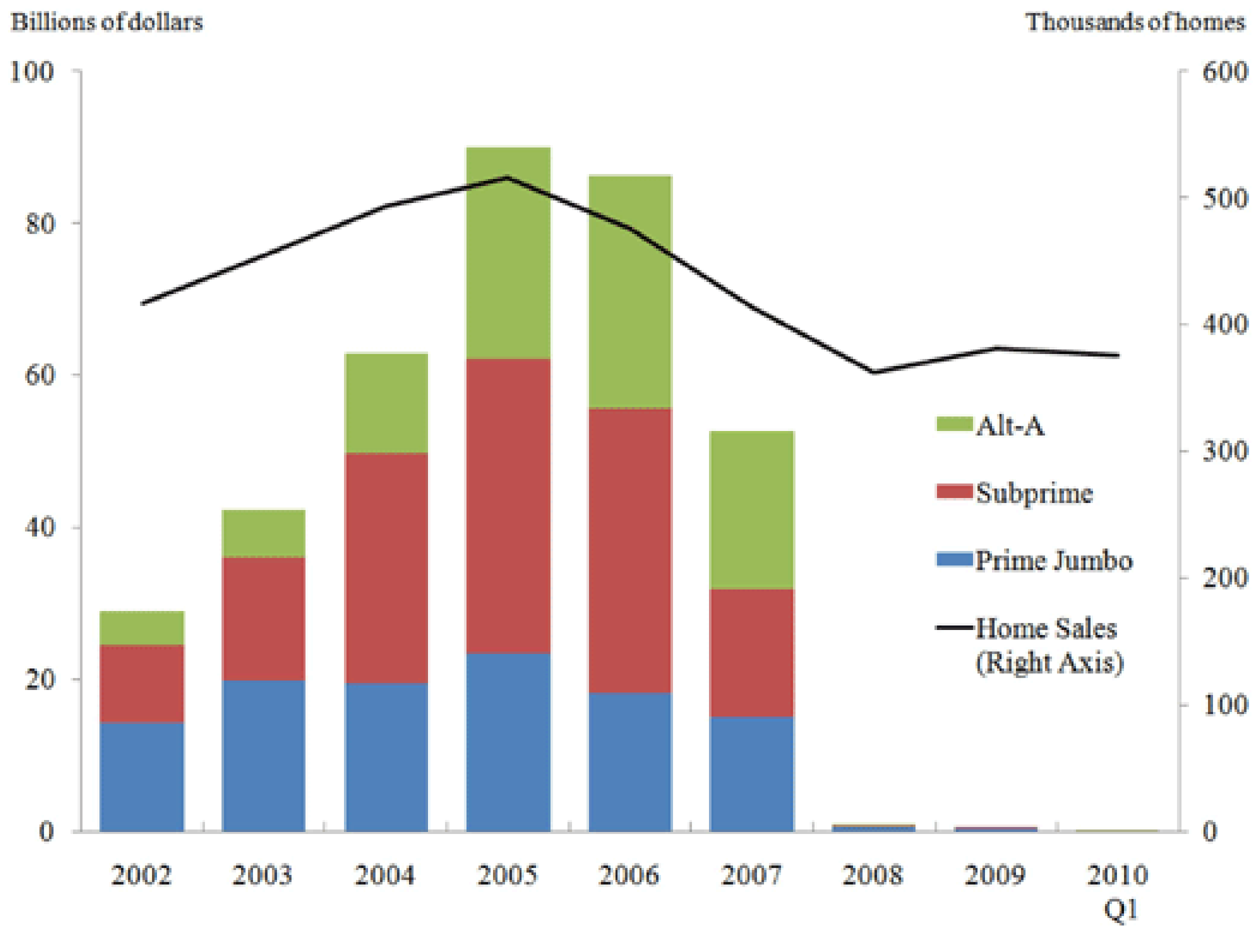
The Market



Source: http://viableopposition.blogspot.com/

The Market



Source: Federal Reserve Board

The Market

Markit Credit Indices The market standards for investing, trading and hedging in the credit markets.	Index value	Number of securities	Number of sub-indices	Base date	Year high	Year low
Markit CDX North American Investment Grade	85	125	6	21/10/03	262	82
Markit CDX North American Investment Grade High Volatility	146	30		21/10/03	683	146
Markit CDX North American High Yield	518 / 99.271%	100	3	21/10/03	1894 / 66.65%	498 / 100.052%
Markit CDX North American Crossover	230	35		21/09/05	647	221
Markit CDX Emerging Markets	259 / 110.550%	15		21/01/04	895 / 81.38%	248 / 111.52%
Markit CDX Emerging Markets Diversified	367 / 105.625%	40		04/04/05	837 / 81.15%	295 / 109.15%
Markit iTraxx Europe	73.68	125	4	22/06/04	205	72.526825
Markit iTraxx Europe High Volatility	104.33	30		22/06/04	551	102.617525
Markit iTraxx Non-Financials	72.25	100		22/06/04	324	72.2528
Markit iTraxx Europe Senior Financials	73.07	25		22/06/04	210	72.95054745
Markit iTraxx Europe Subordinated Financials	133.67	25		22/06/04	408	133.1723
Markit iTraxx Europe Crossover	433	50		22/06/04	1150	428.167181
Markit iTraxx Asia ex Japan IG	95.5	50		27/07/04	463.6666667	95.5
Markit iTraxx Asia ex Japan HY	415	20		27/07/04	1375	410
Markit iTraxx Japan	133.5	50		27/07/04	565	130.75
Markit iTraxx Australia	84.5	25		27/07/04	441	83.5
Markit MCDX (municipal CDS)	137	50		06/05/08	291	85
Markit iTraxx LevX Senior	102	40		20/09/06	104	68
Markit iTraxx SovX Western Europe Index	69	15		20/03/09	72.68107848	46
Markit iTraxx SovX CEEMEA Index (theoretical)	248.32	15		20/03/09	639	217.9323429
Markit iTraxx SovX Global Liquid Investment Grade Index (theoretical)	124.73	22		20/03/09	290	97
Markit iTraxx SovX G7 Index (theoretical)	57.26	6		20/03/09	102	33

The Market

Markit Structured Finance Indices The preferred tool for market analysis and risk management in the structured finance markets, providing diversification, transparency and standardised trading.	Index value	Number of securities	Base date	Year high	Year low
The ABX.HE index is the key trading tool for banks and asset managers to hedge asset-backed exposure or take a position in this asset class.					
Markit ABX.HE.PENAAA.06-1	88	20	14/05/08	90.07	82.18
Markit ABX.HE.AAA.06-1	81	20	19/01/06	84.80	59.75
Markit ABX.HE.AA.06-1	33	20	19/01/06	34.88	15.90
Markit ABX.HE.A.06-1	11	20	19/01/06	12.32	7.50
Markit ABX.HE.BBB.06-1	4	20	19/01/06	5.59	3.95
Markit ABX.HE.BBB-.06-1	5	20	19/01/06	5.59	3.90
Markit ABX.HE.PENAAA.06-2	74	20	14/05/08	76.23	53.04
Markit ABX.HE.AAA.06-2	45	20	19/07/06	51.43	28.72
Markit ABX.HE.AA.06-2	11	20	19/07/06	13.00	6.94
Markit ABX.HE.A.06-2	5	20	19/07/06	5.39	3.42
Markit ABX.HE.BBB.06-2	5	20	19/07/06	5.31	2.29
Markit ABX.HE.BBB-.06-2	5	20	19/07/06	5.38	2.34
Markit ABX.HE.PENAAA.07-1	42	20	14/05/08	46.44	28.36
Markit ABX.HE.AAA.07-1	35	20	19/01/07	40.39	23.25
Markit ABX.HE.AA.07-1	4	20	19/01/07	5.50	2.86
Markit ABX.HE.A.07-1	3	20	19/01/07	3.61	2.33
Markit ABX.HE.BBB.07-1	4	20	19/01/07	3.78	2.19
Markit ABX.HE.BBB-.07-1	4	20	19/01/07	3.78	2.19
Markit ABX.HE.PENAAA.07-2	38	20	14/05/08	44.89	24.56
Markit ABX.HE.AAA.07-2	34	20	19/07/07	40.14	23.10
Markit ABX.HE.AA.07-2	5	20	19/07/07	5.68	3.75
Markit ABX.HE.A.07-2	5	20	19/07/07	4.62	2.97
Markit ABX.HE.BBB.07-2	3	20	19/07/07	3.85	2.89
Markit ABX.HE.BBB-.07-2	3	20	19/07/07	3.85	2.88

Source: FDIC

The Market

Created in response to the rapid pace of growth in the CDS of CMBS market, the index provides investors with an efficient, standardised tool to gain exposure to this asset class.					
Markit CMBX.NA.AAA.1	93	25	07/03/06	94.04	75.55
Markit CMBX.NA.AJ.1	76	25	04/01/08	80.05	36.06
Markit CMBX.NA.AA.1	56	25	07/03/06	66.71	19.92
Markit CMBX.NA.A.1	46	25	07/03/06	61.15	16.46
Markit CMBX.NA.BBB.1	30	25	07/03/06	40.21	11.20
Markit CMBX.NA.BBB-.1	25	25	07/03/06	36.27	10.09
Markit CMBX.NA.AAA.2	89	25	25/10/06	90.45	64.92
Markit CMBX.NA.AJ.2	64	25	04/01/08	68.27	28.70
Markit CMBX.NA.AA.2	41	25	25/10/06	51.73	14.75
Markit CMBX.NA.A.2	33	25	25/10/06	45.88	13.75
Markit CMBX.NA.BBB.2	20	25	25/10/06	32.24	8.79
Markit CMBX.NA.BBB-.2	16	25	25/10/06	30.08	8.05
Markit CMBX.NA.BB.2	5	25	25/10/06	12.87	4.21
Markit CMBX.NA.AAA.3	85	25	25/04/07	86.96	59.18
Markit CMBX.NA.AJ.3	55	25	04/01/08	61.81	25.06
Markit CMBX.NA.AA.3	33	25	25/04/07	42.00	12.68
Markit CMBX.NA.A.3	25	25	25/04/07	34.50	10.95
Markit CMBX.NA.BBB.3	16	25	25/04/07	23.13	8.78
Markit CMBX.NA.BBB-.3	14	25	25/04/07	21.85	8.22
Markit CMBX.NA.BB.3	5	25	25/04/07	17.89	5.00
Markit CMBX.NA.AAA.4	83	25	25/10/07	84.95	58.22
Markit CMBX.NA.AJ.4	51	25	04/01/08	60.58	22.67
Markit CMBX.NA.AA.4	31	25	25/10/07	41.00	12.93
Markit CMBX.NA.A.4	24	25	25/10/07	32.49	12.70
Markit CMBX.NA.BBB.4	17	25	25/10/07	24.45	10.64
Markit CMBX.NA.BBB-.4	16	25	25/10/07	22.38	9.79
Markit CMBX.NA.BB.4	5	25	25/10/07	17.32	5.00
Markit CMBX.NA.AAA.5	84	25	22/05/08	86.75	57.40
Markit CMBX.NA.AJ.5	56	25	22/05/08	60.60	22.06
Markit CMBX.NA.AA.5	34	25	22/05/08	40.98	12.84
Markit CMBX.NA.A.5	27	25	22/05/08	32.81	12.62
Markit CMBX.NA.BBB.5	18	25	22/05/08	23.35	10.58
Markit CMBX.NA.BBB-.5	16	25	22/05/08	21.87	9.74
Markit CMBX.NA.BB.5	5	25	22/05/08	17.04	5.00

Model

- For bankruptcy
  - accounting
    - Altman Z
    - Ohlson O
  - finance
    - reduced-form
      - Jarrow-Turnbull 1995, Duffie-Singleton 1997
    - structural
      - Black-Scholes-Merton, Geske 1977, Leland 1990
    - Hybrid
      - Black-Cox 1976, CreditGrades

Model

- For migration
  - Markov chain
    - Jarrow-Lando-Turnbull
    - weak link to default
    - no link to spread
- For spread
  - Black-Scholes
    - no link to default
    - no link to migration

Measures of Credit Risk

- Two building blocks
  – PD (probability of default)
  – LGD (loss given default)
    - = 1 – recovery
    - = exposure at default (note: exposure = notional)
- Measures
  – EL, UL, EAD, JTD, etc.

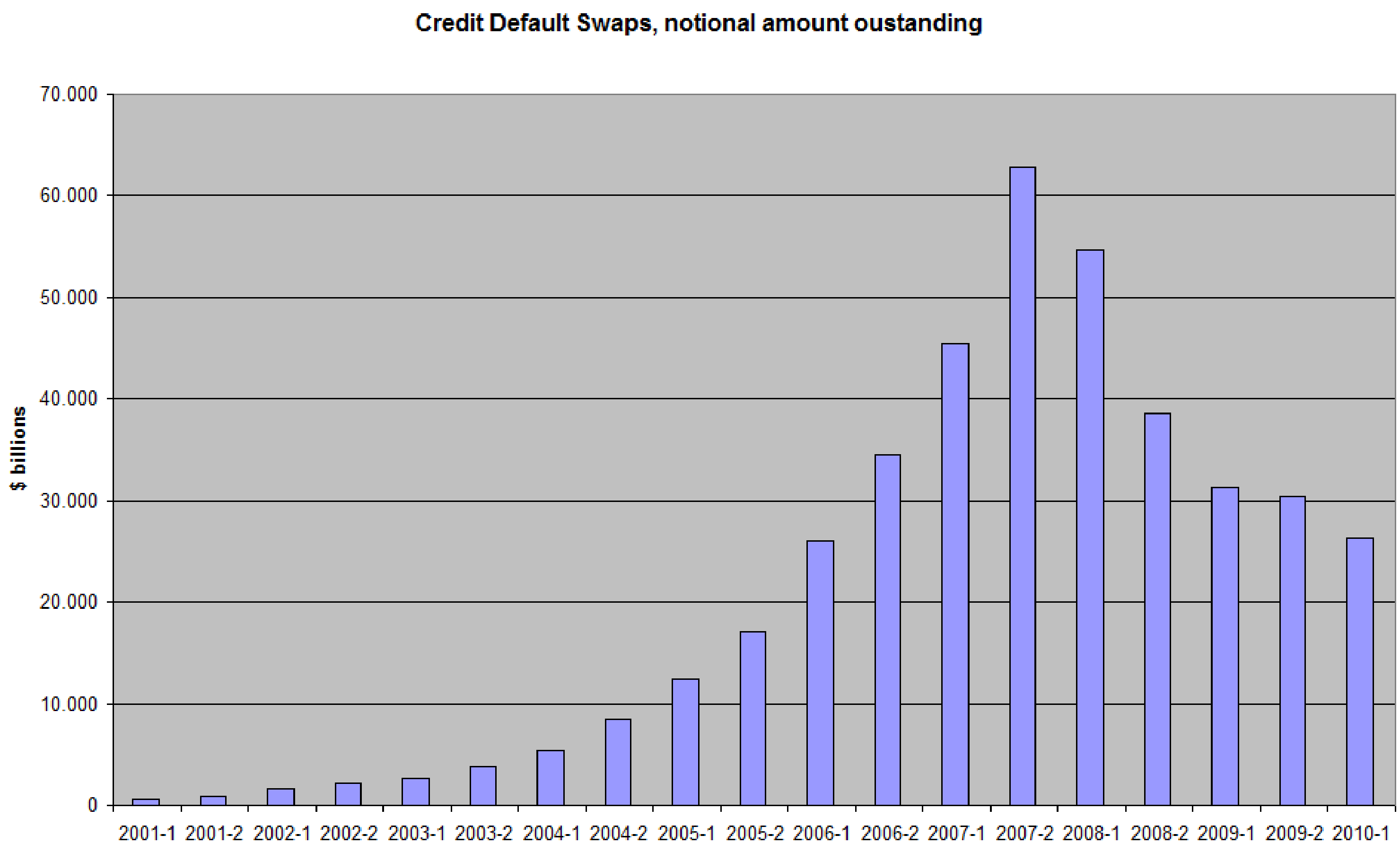
Measures of Credit Risk

- Measures
  - EL/UL (expected loss; unexpected loss)
  - EAD (exposure at default)
  - JTD (jump to default)
  - spread01 (1 bp spread moves)
  - spread duration
  - …, etc.

Instruments to Transfer Credit Risk

- Traditional assets
  - bonds: corporate, municipal, sovereign
  - securitized assets: CMBS, ABS, …, etc.
- Modern derivatives
  - CDS (credit default swap)
  - FTD/NTD (first-to-default/nth-to-default)
  - CDO (collateralized debt obligation)
  - …, etc.

Credit Default Swap (CDS)



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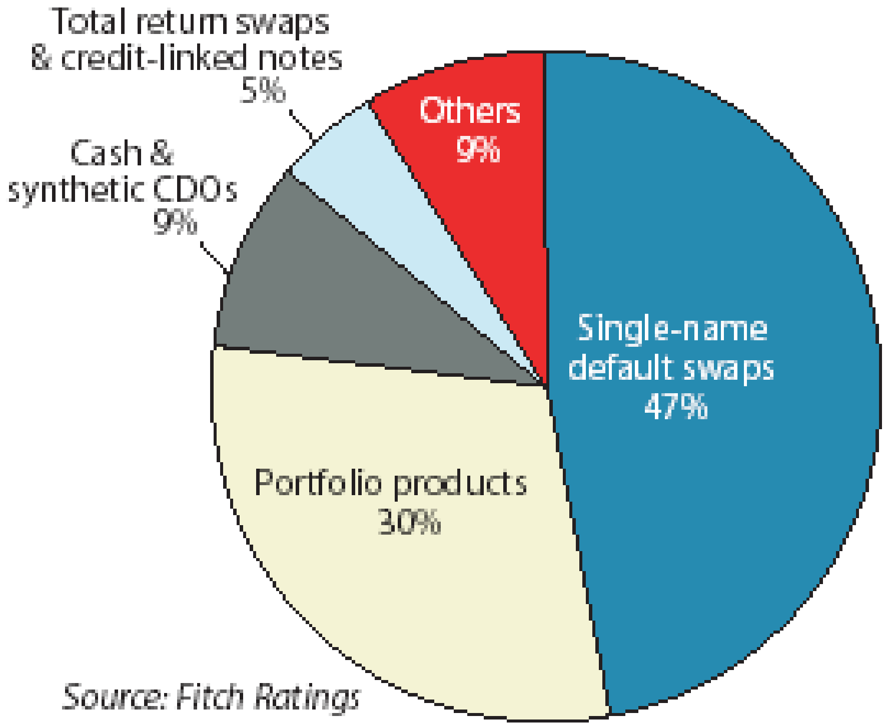
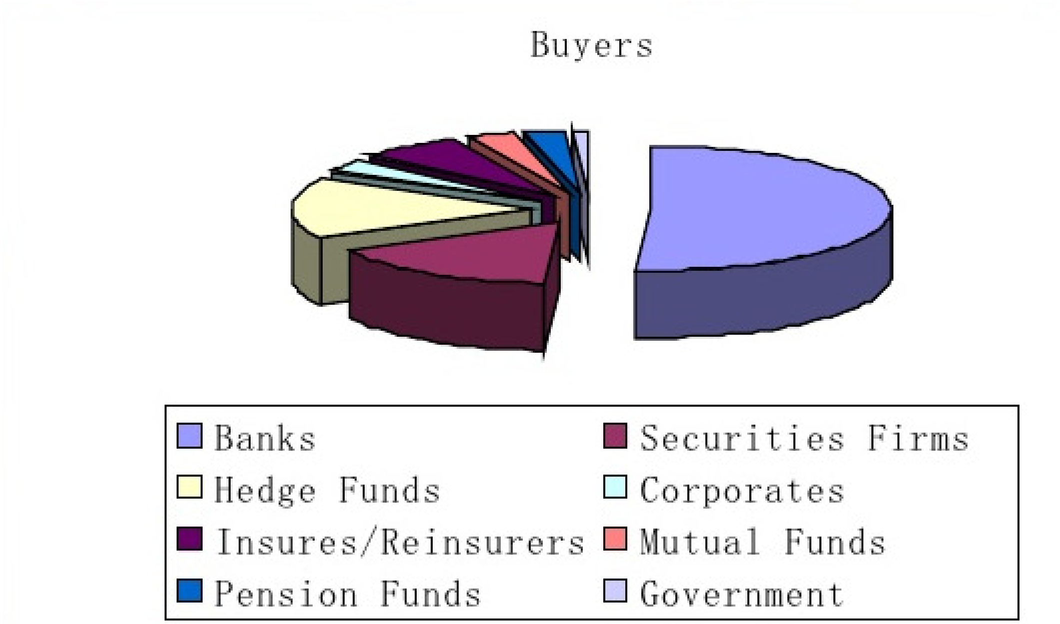
Credit Default Swap (CDS)



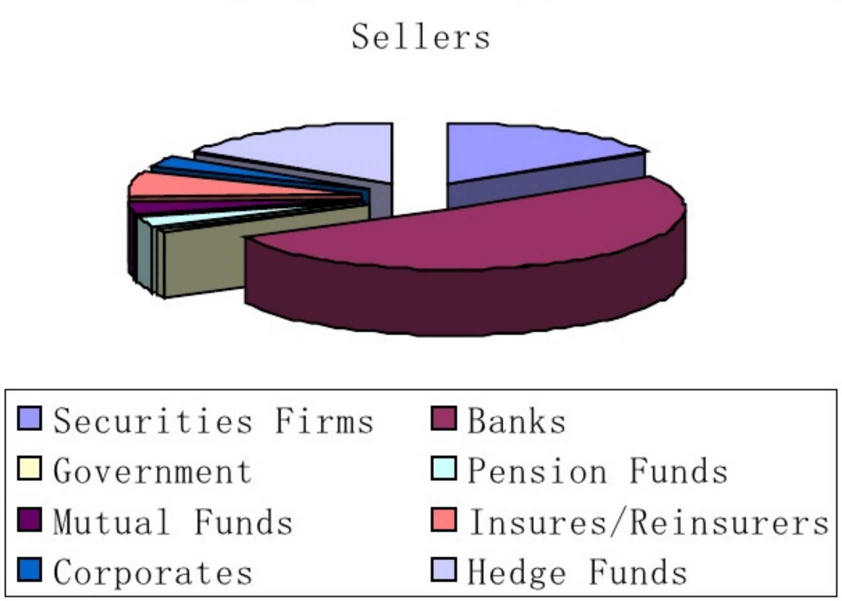
Credit Default Swap (CDS)

- source BBA

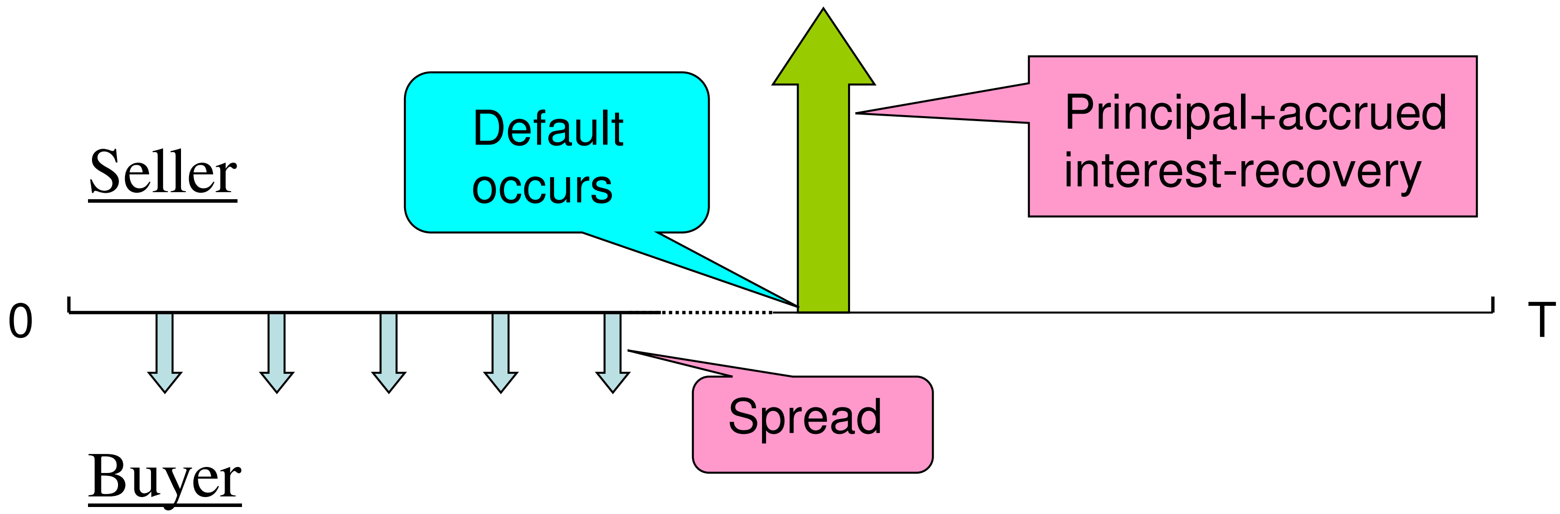


Credit Default Swap (CDS)

- source BBA

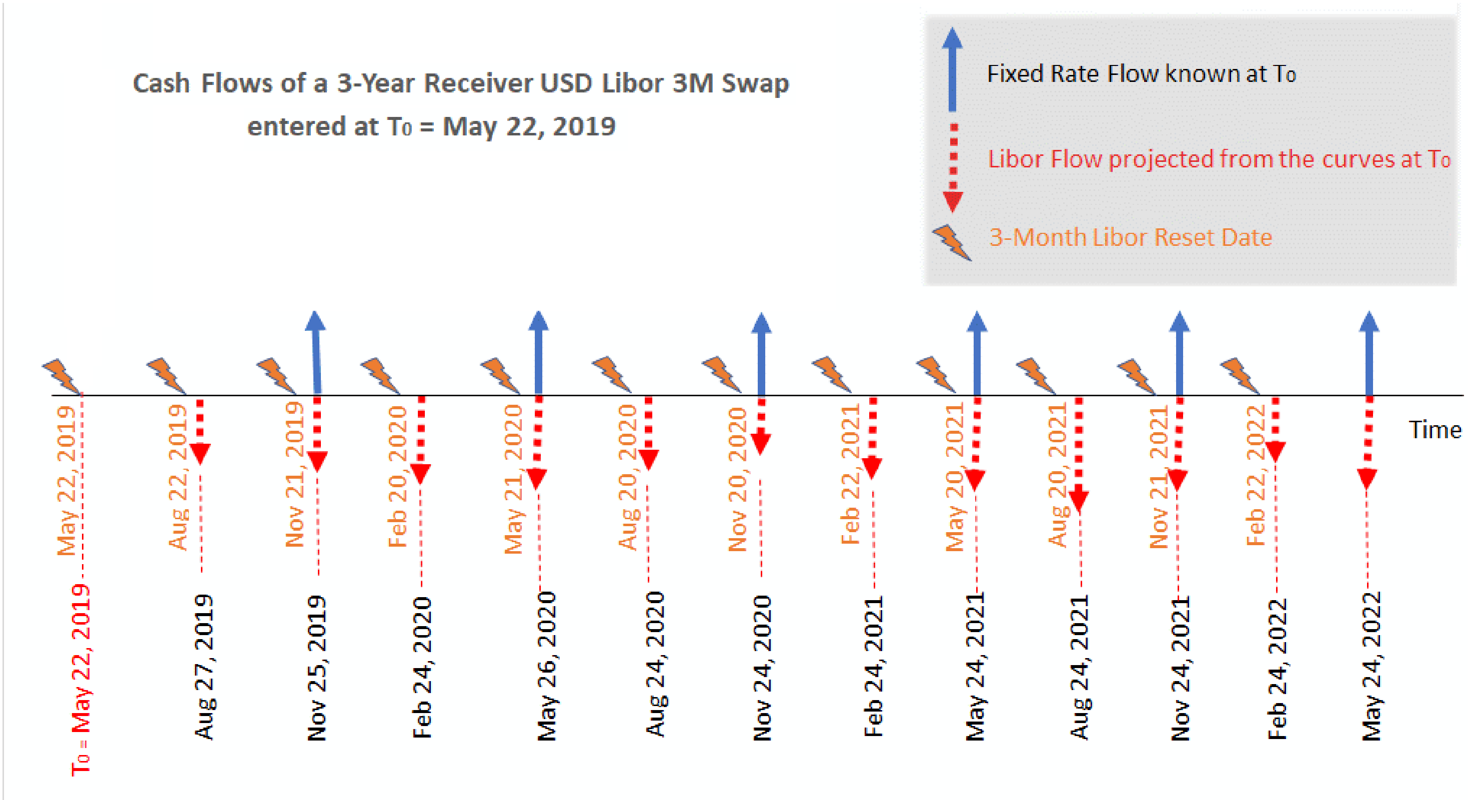


Credit Default Swap (CDS)

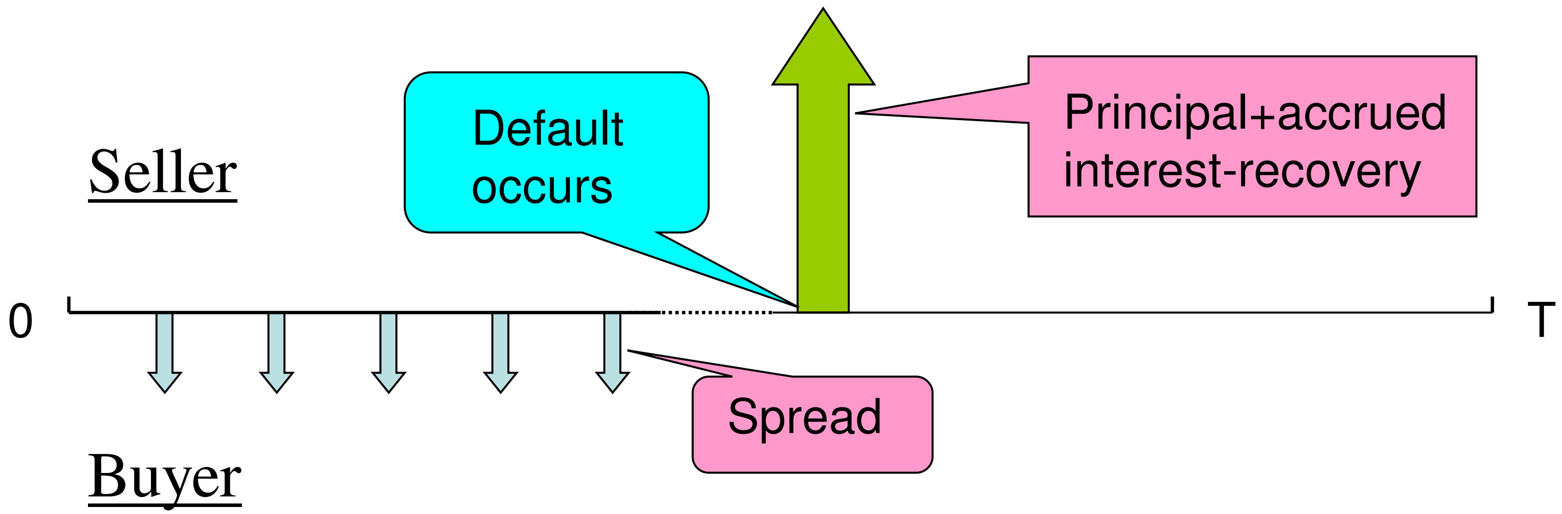


Credit Default Swap (CDS)

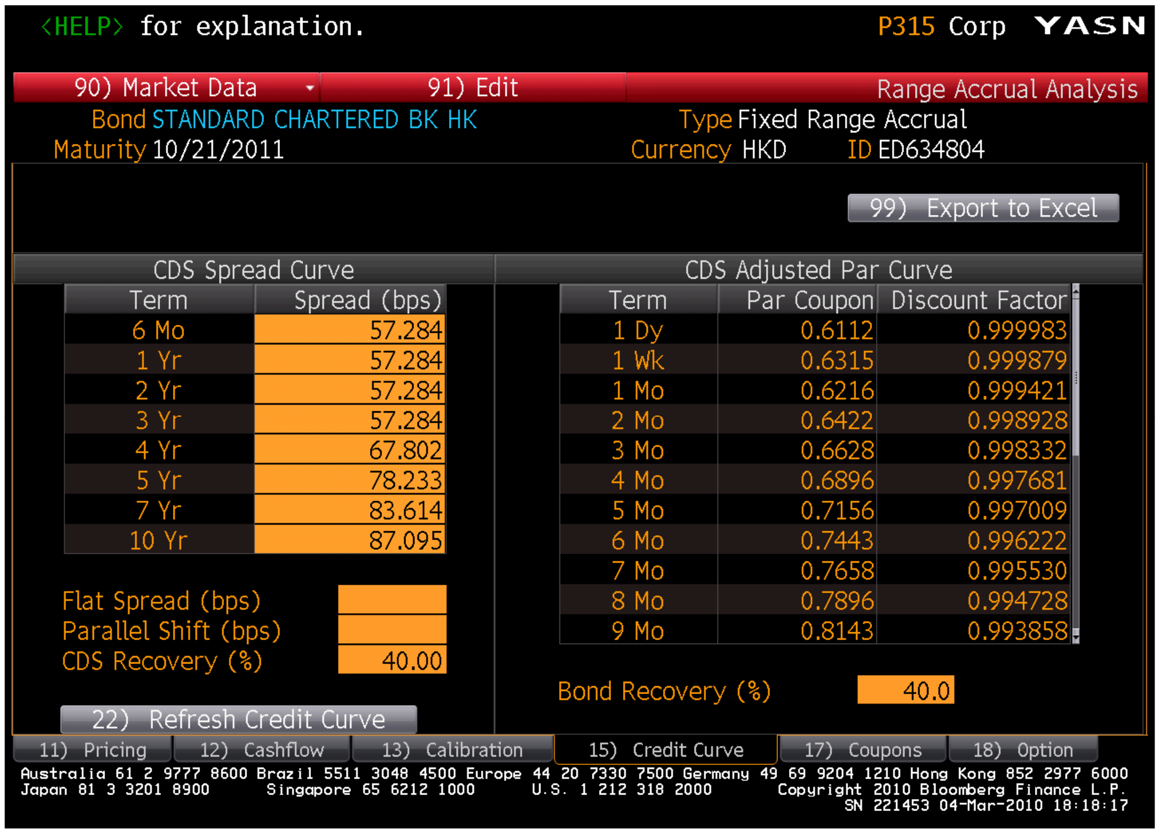
- A swap example:



Credit Default Swap (CDS)

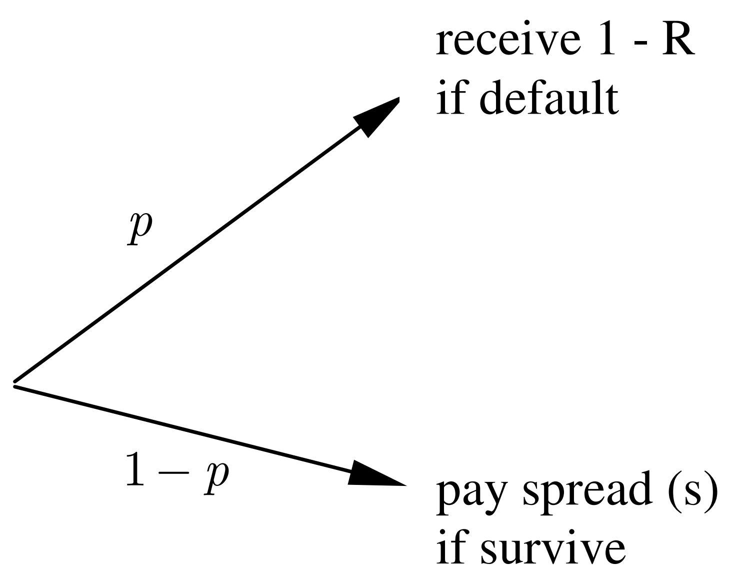


Credit Default Swap (CDS)



Credit Default Swap (CDS)

- Back-of-the-envelope formula

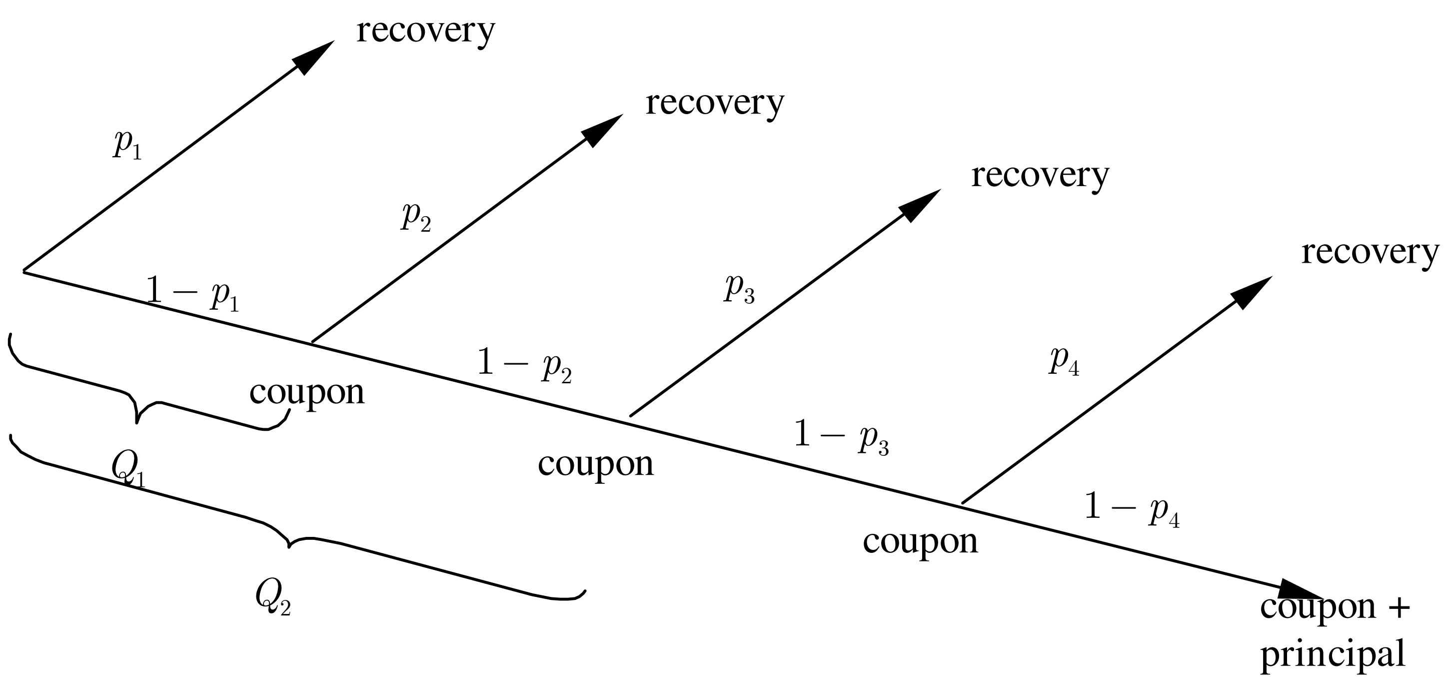


$$p(1 - R) = (1 - p)s \approx s$$

$$p = \frac{s}{1 - R}$$

Credit Default Swap (CDS)

- p_i: default prob; Q_i: survival prob

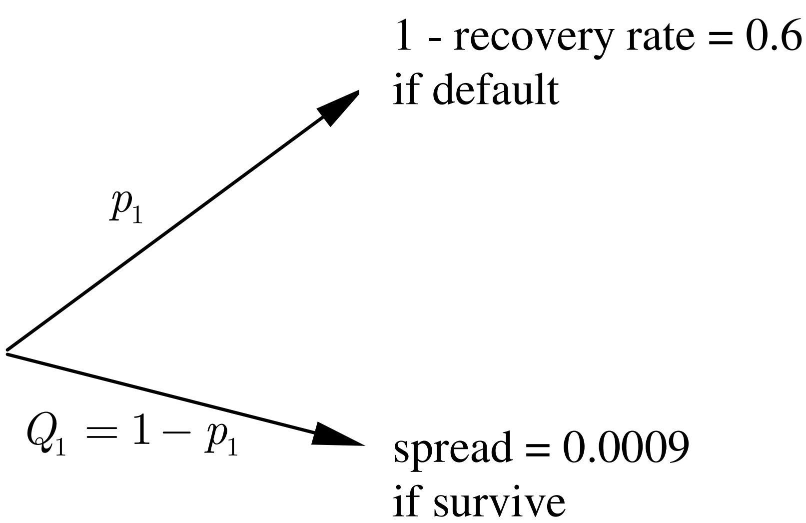


Credit Default Swap (CDS)

- Quotes (Disney 12/23/2005)

term	sprd
1	9
2	13
3	20
5	33
7	47
10	61

Credit Default Swap (CDS)



$$\frac{0.6 \times (1 - Q_1)}{1.05} = \frac{0.0009 \times Q_1}{1.05}$$

$$Q_1 = 0.9985 = e^{-\lambda_1}$$

$$\lambda_1 = 14.99 \text{ basis points}$$

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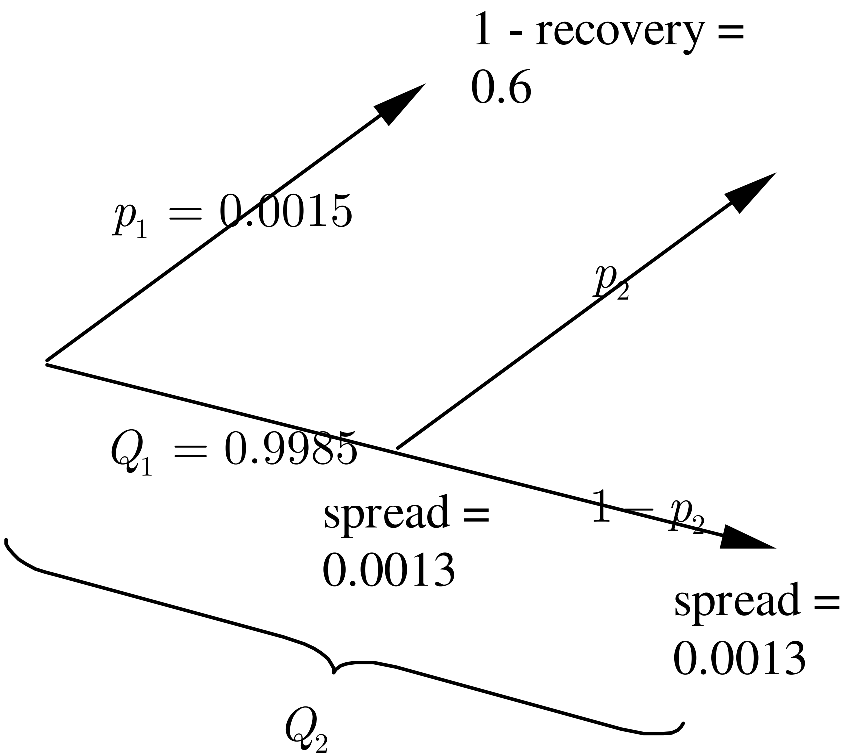
Credit Default Swap (CDS)



$$\text{prem leg} =$$

$$\frac{0.0013 \times 0.9985}{1.05} + \frac{0.0013 \times Q_2}{1.05^2} =$$

$$\frac{0.0015 \times 0.6}{1.05} + \frac{0.9985 \times p_2 \times 0.6}{1.05^2} =$$

def leg

$$\text{sub in } p_2 = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2}{0.9985}$$

$$\text{hence, } Q_2 = 0.9956 = Q_1 e^{-\lambda_2}$$

$$\text{hence, } \lambda_1 = 28.65 \text{ basis points}$$

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Credit Default Swap (CDS)

CDS	Bootstrapping				
	Market	Risk-free	Fwd.	Surv.Pr.	Def.Pr.
Term	Spread	P(t)	lambda(t)	Q(t)	-dQ(t)
1	0.0009	0.9512	0.0015	0.9985	0.0015
2	0.0013	0.9048	0.0029	0.9956	0.0029
3	0.002	0.8607	0.0059	0.9898	0.0058
4		0.7788	0.0092	0.9808	0.0091
5	0.0033	0.7788	0.0092	0.9718	0.009

smoothing

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Credit Default Swap (CDS)

- Bilateral financial contracts
- Allow the transfer of credit risk from one party to another. Using these products, investors may hedge themselves against credit risk
- Related to some risk or volatility
- Do not require initial investment
- Credit event: bankruptcy, failure to pay, restructuring etc.

Credit Default Swap (CDS)

- Two parties: protection buyer, protection seller
- Do not require either of the parties to actually hold the reference asset
- Two ways of settlement: cash and physical
- An over-the-counter contract that provides insurance against credit risk.

Credit Default Swap (CDS)

- The protection buyer pays a fixed fee or premium, often termed as the “spread” to the seller for a period of time.
- When a credit event occur at some point before the contract's maturity, the protection seller pay compensation to the buyer of protection, thus insulating the buyer from a financial loss.

Credit Default Swap (CDS)

- CDS can be viewed as a put option: if one default event occurs, the bond can be put back to the seller at the principal.
- CDS is similar to an insurance contract, providing buyers with protection against specific risks.

Credit Default Swap (CDS)

- CDS benefits
  - a short positioning vehicle not available in the cash market
  - access to maturity exposures not available in the cash market
  - does not require an initial cash outlay
  - access to credit risk not available in the cash market due to a limited supply of the underlying bonds

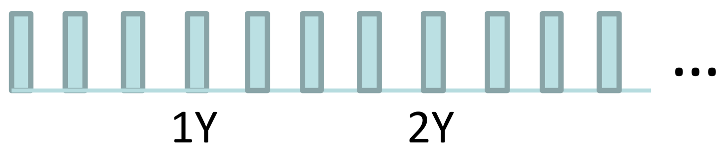
Credit Default Swap (CDS)

– ability to effectively “exit” credit positions in periods of low liquidity
– off-balance sheet instruments which offer flexibility in terms of leverage
– provide important anonymity when shorting an underlying credit

The CDS Big Bang: an example

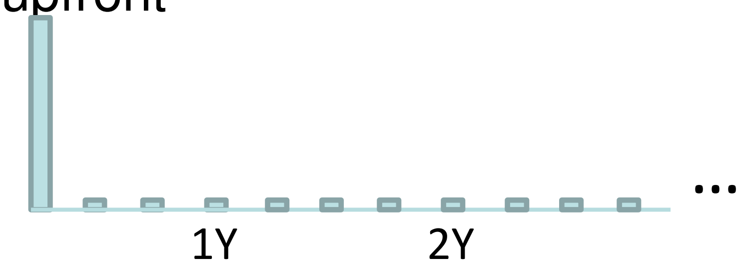
Consider a CDS contract on Bank of America with notional value of 10M dollars. The quoted 5Y CDS spread is 326 basis points. A protection buyer needs to

•Before the Big Bang: pay $326,000 per year

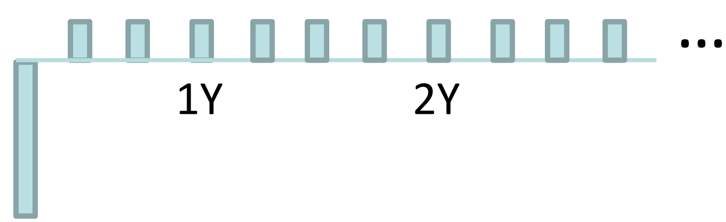


•After the Big Bang:

– fixed 100: pay $990,254 upfront plus $100,000 per year



– fixed 500: receive $756,788 upfront and pay $500,000

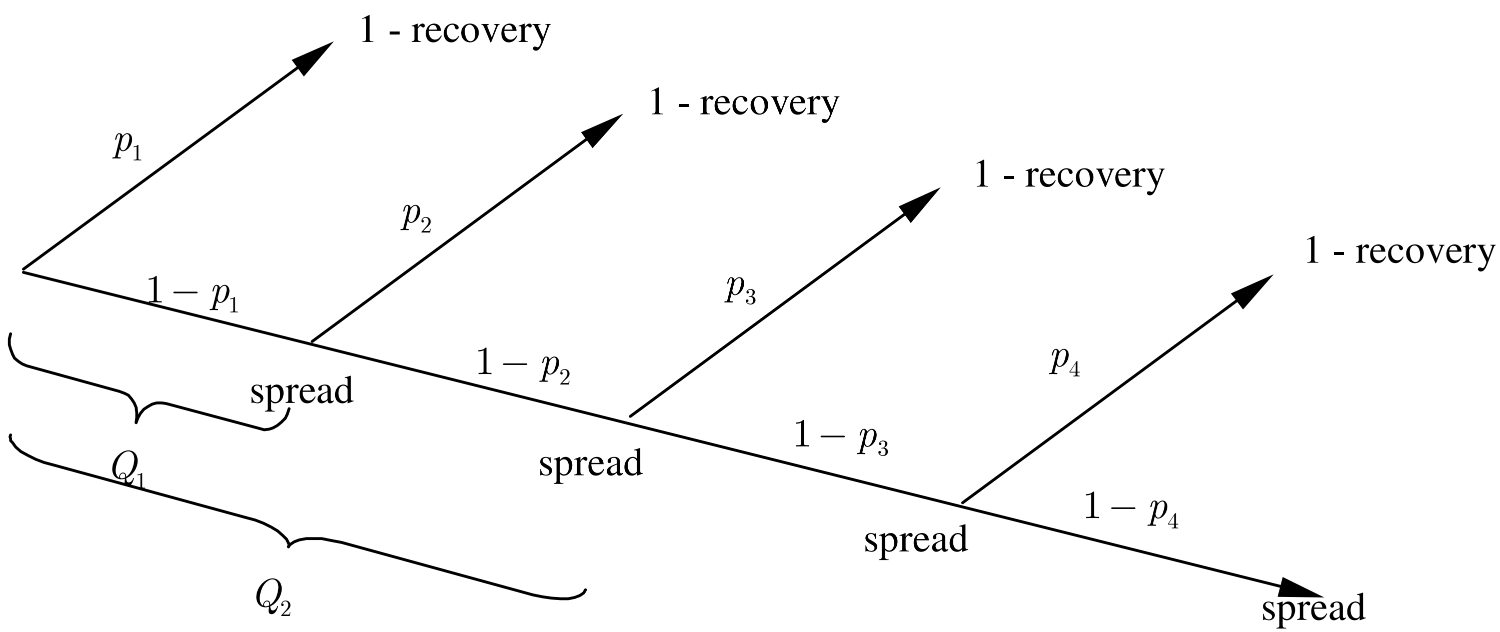


CDS vs. Bond

- CDS is a perfect match/hedge for the corporate floater
  - CDS spread is LIBOR spread
    - Perfectly matched cash flows under default
    - Perfectly matched cash flows under survival
- CDS is a close fit for the corporate fixed
  - CDS spread is close approximation to T yield spread
    - Not so perfectly matched cash flows in either case

CDS vs. Bond

- p_i: default prob; Q_i: survival prob



Structural Models

- Black-Scholes-Merton model

Structural Models

- Black-Scholes-Merton model
  - the Black-Scholes model (1973) is for option
  - Black and Scholes recognize:
    - equity (E) = call (C); stock (S) = asset (A)

$$S \geq K \quad C = S - K \qquad\qquad A \geq K \quad E = A - K$$

$$S < K \quad C = 0 \qquad\qquad A < K \quad E = 0$$

  - as a result, debt = covered call

$$A \geq K \quad D = K$$

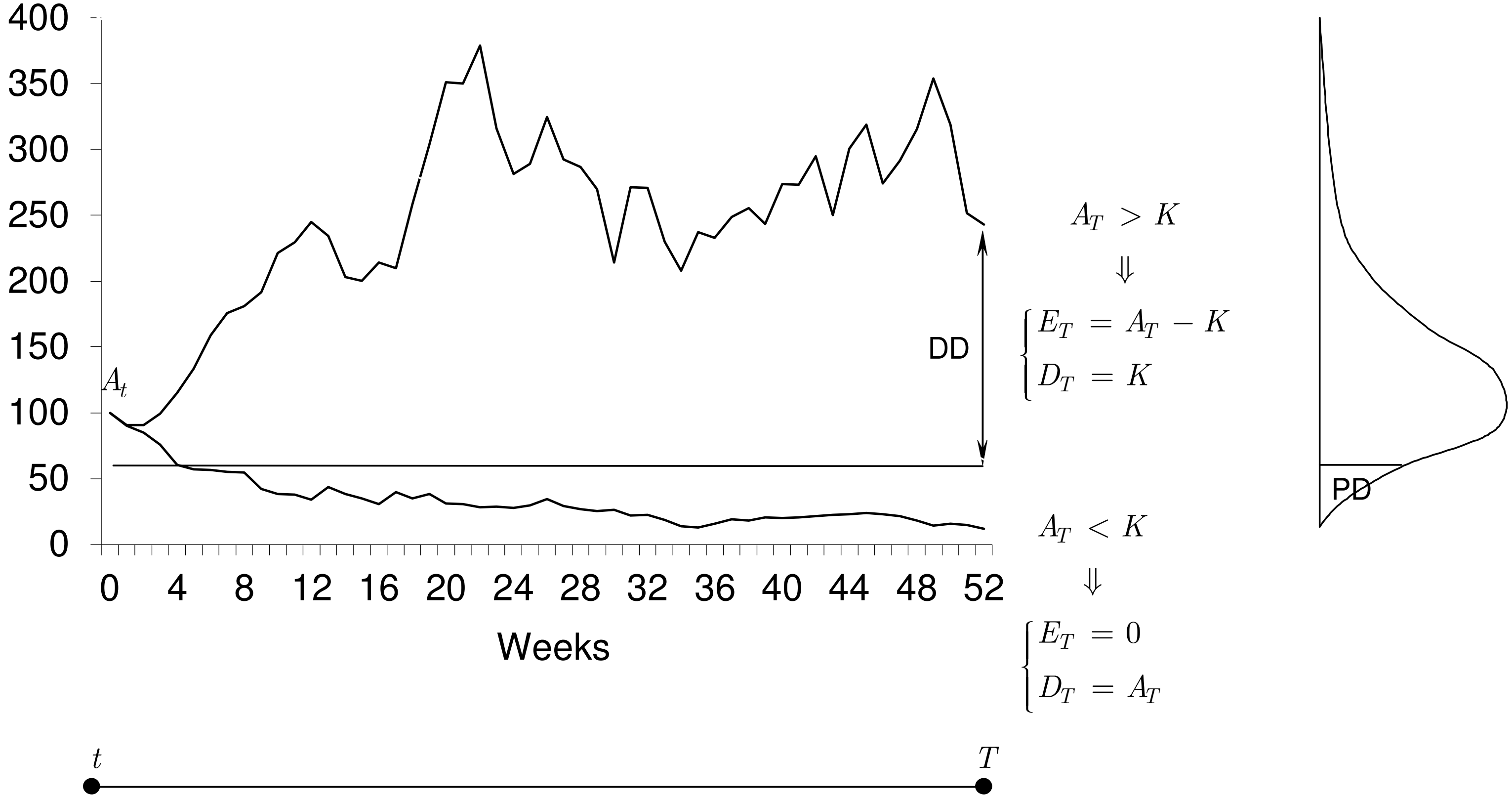
$$A < K \quad D = A$$

Structural Models

- Merton (1974) took it and did a wide variety of analyses of credit risk
- Stephen Kealhofer, Michael McQuown and Oldrich Vasicek founded KMV in 1988
  - later acquired by Moody's in 2002
  - use the Merton model to provide quantitative ratings – first in the world!!!
- Merton model = KMV model

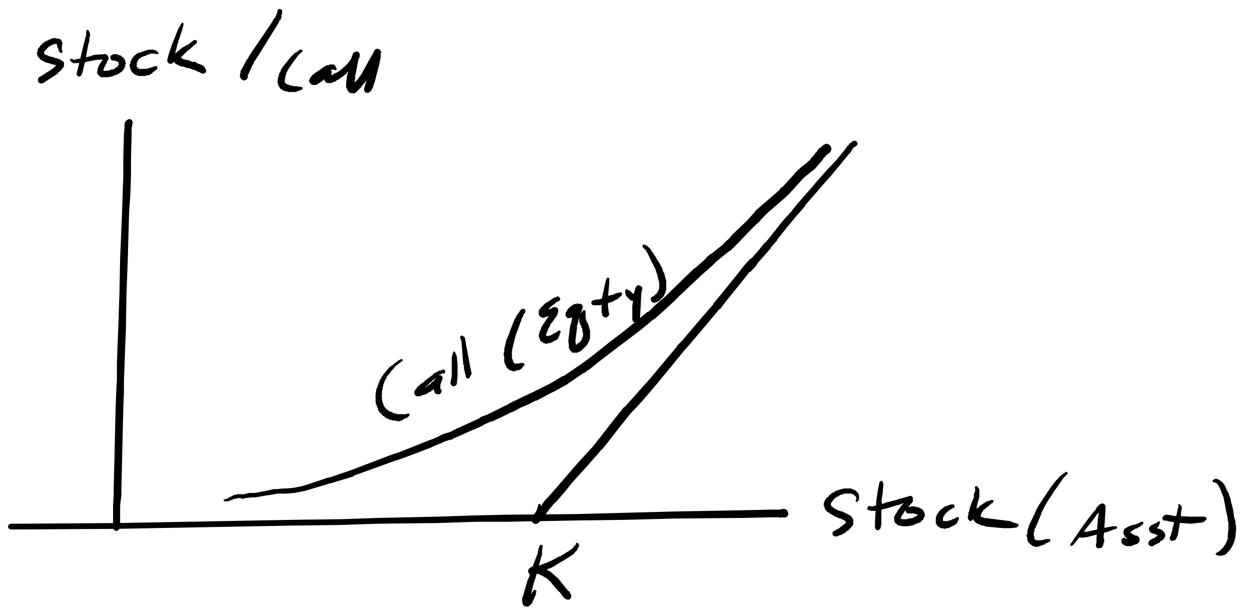
Structural Models

- KMV (Black-Scholes-Merton) method



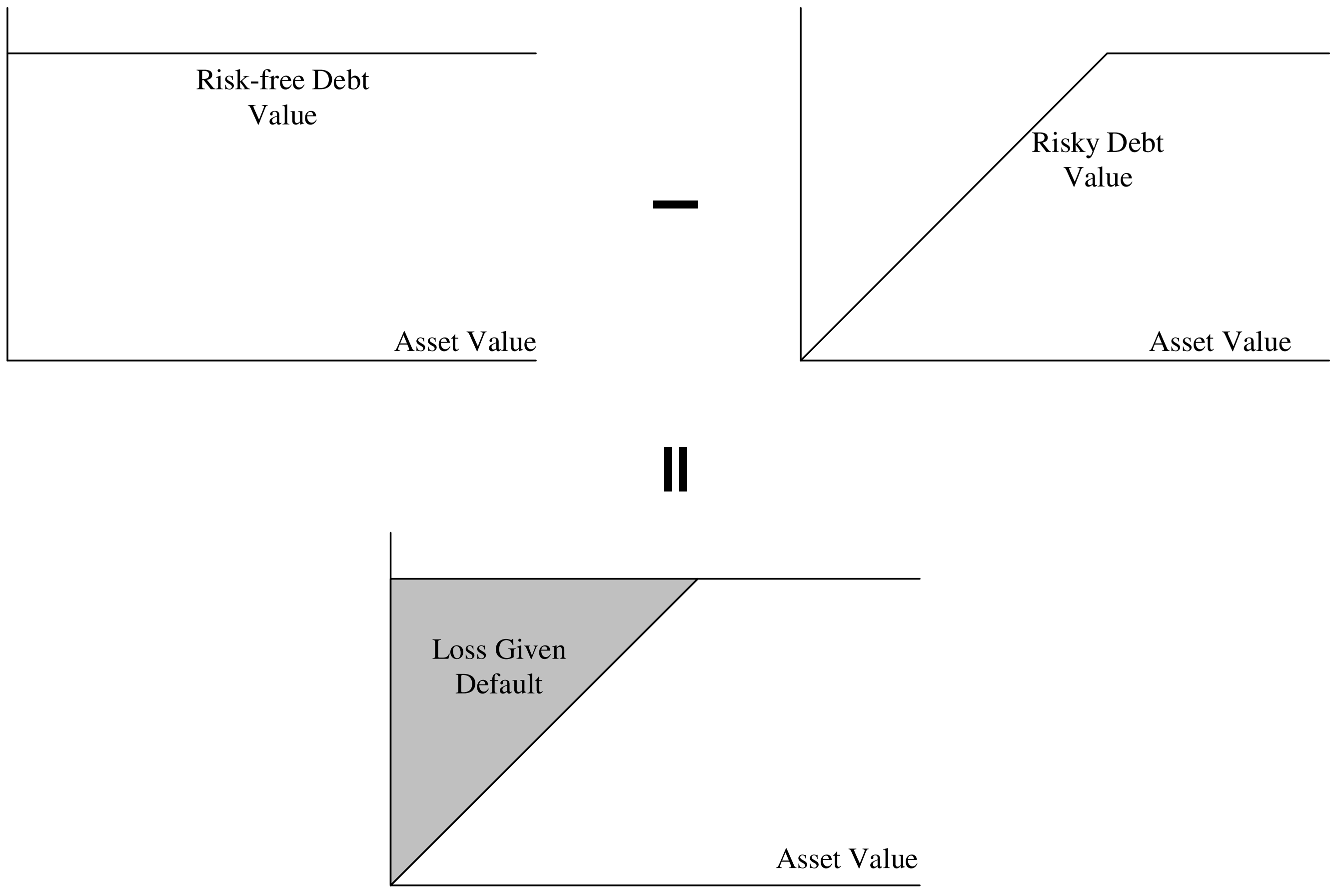
Structural Models

- Equity vs. Asset



Structural Models

- KMV



Structural Models

- Equity is a call option (Black-Scholes)

$$E(t) = A(t)N(d_1) - e^{-r(T-t)}KN(d_2)$$

$$\sigma_E = [A(t) / E(t)]\sigma_A N(d_1)$$

- where

$$d_2 = \frac{\ln A(t) - \ln K + (r - ½\sigma_A^2)(T - t)}{\sigma_A \sqrt{T - t}}$$

$$d_1 = d_2 + \sigma_A \sqrt{T - t}$$

Structural Models

- KMV
  - K 1-year debt (=STD+0.5×LTD); σ_A asset vol; σ_E equity vol; r risk-free rate; T-t time horizon; E equity value; A asset value; N(.) normal probability;
- Note

$$D(t) = A(t) - E(t) = \underbrace{A(t)[1 - N(d_1)]}_{\text{Recovery value}} + \underbrace{e^{-r(T-t)} K N(d_2)}_{\text{Survival value}}$$

- PD

$$Q_1 = N(d_2)$$

$$p = 1 - N(d_2)$$

Structural Models

- KMV
  - DD = d_2

$$d_2 = \frac{\mathbb{E}[\ln A(T)] - \ln K}{\sqrt{\mathbb{V}[\ln A(T)]}}$$

  - Expected recovery

$$\begin{aligned} R &= \mathbb{E}[A(T) \mid A < K] \\ &= A(t)(1 - N(d_1)) \end{aligned}$$

Structural Models

- Multiple debts
  - Geske
    - discrete time
    - flexible capital structure
  - Leland
    - continuous time
    - steady state capital structure

Structural Models

- Geske (2-pd)
  - Equity value (market cap)

$$E_0 = A_0 M(h_{1+}, h_{2+}; \rho) - e^{-r(T_2 - t)} K_2 M(h_{1-}, h_{2-}; \rho) - e^{-r(T_1 - t)} K_1 N(h_{1-})$$

  - K1 debt value

$$D_{0,1} = A_0 N(-y_{1+}) + e^{-r(T_1 - t)} K_1 N(y_{1-})$$

  - K2 debt value

$$\begin{aligned} D_{0,2} &= A_0 - D_{0,1} - E_0 \\ &= A_0 (N(y_{1+}) - M(h_{1+}, h_{2+}; \rho)) - e^{-r(T_1 - t)} K_1 (N(y_{1-}) - N(h_{1-})) \\ &\quad + e^{-r(T_2 - t)} K_2 M(h_{1-}, h_{2-}; \rho) \end{aligned}$$

Structural Models

- Geske
  - Total liabilities

$$D_{0,1} + D_{0,2} = \underbrace{A_0[1 - M(h_{1+}, h_{2+}; \rho)]}_{\text{Recovery}}$$

$$+\underbrace{e^{-r(T_1 - t)} K_1 N(h_{1-})}_{\text{1st Yr Survival}} + \underbrace{e^{-r(T_2 - t)} K_2 M(h_{1-}, h_{2-}; \rho)}_{\text{2nd Yr Survival}}$$

  - Survival probabilities (and PDs)

$$Q_1 = N(h_{1-})$$

$$Q_2 = M(h_{1-}, h_{2-}; \rho)$$

$$p_1 = 1 - Q_1$$

$$p_2 = \frac{Q_1 - Q_2}{Q_1}$$

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Structural Models

- Geske
  - default barrier is A <> sum of all debts: (K_1 + D_{12})

At time T_1				
Before K_1 is paid off			After K_1 is paid off (with equity)	
A_1 or $A(T_1)$	K_1 $D_{12}(A_1)$ $E_1^{(b)}$		A_1 or $A(T_1)$	$D_{12}(A_1')$ $E_1^{(a)} = E_1^{(b)} + K_1$

Structural Models

- Geske

Balance Sheet
as of year 0

assets	400	Maturity $t=1$ debt	90
		Maturity $t=2$ debt	80
		Equity	130
total	400	Total	400

note: both debts have face values of $100

Table 14.2: Balance Sheet at Year 0

Balance Sheet
as of year 1 before payment of first debt

assets	450	Maturity $t=1$ debt	100
		Maturity $t=2$ debt	90
		Equity	260
total	450	Total	450

Balance Sheet
at year 1 after payment of first debt

assets	450	maturity $t=2$ debt	90
		old equity	260
		new equity	100
total	450	Total	450

note: issue new equity to pay for the first debt

Table 14.3: Balance Sheet at Year 1

Balance Sheet
as of year 1 before payment of first debt

assets	186.01	one-year debt	100
		two-year debt	86
		Equity	0.01
total	186.01	Total	186.01

Balance Sheet
as of year 1 after payment of first debt

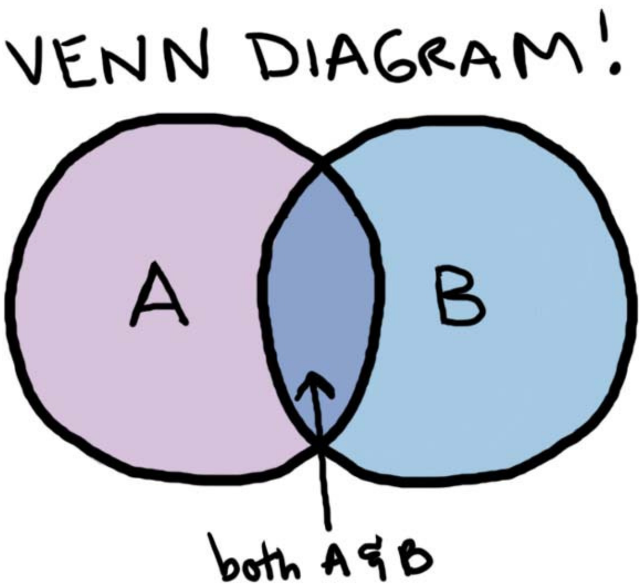
assets	186.01	two-year debt	86
		old equity	0.01
		new equity	100
total	186.01	Total	186.01

note: issue new equity to pay for the first debt

Table 14.4: Balance Sheet at Year 1

First to Default (FTD)

- Joint default



B \ A	0	1	
0	80%	0	80%
1	10%	10%	20%
	90%	10%	100%

$$p(A \cap B) = \begin{cases} p(A \mid B)p(B) & \text{or} \\ p(B \mid A)p(A) & \end{cases}$$

$$\begin{aligned} p(A \cap B) &= p(B \mid A)p(A) \\ &= p(A) \\ &= 10\% \end{aligned}$$

First to Default (FTD)

$$\begin{aligned} p(A \cap B) &= p(B \mid A)p(A) \\ &= p(A) \\ &= 10\% \end{aligned}$$

$$p(B \mid A) = \frac{p(A \cap B)}{p(A)} = \frac{10\%}{10\%} = 100\%$$ B completely depends on A

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)} = \frac{10\%}{20\%} = 50\%$$ A only 50% depends on B

Default correlation = 0.6667 (highest possible)

First to Default (FTD)

B \ A	0	1	
0	70%	10%	80%
1	20%	0%	20%
	90%	10%	100%

$$
\begin{aligned}
p(A \cap B) &= p(B \mid A)p(A) \\
&= p(A) \\
&= 0\%
\end{aligned}
$$

$$p(B \mid A) = \frac{p(A \cap B)}{p(A)} = \frac{0\%}{10\%} = 0\%$$ B opposite of A

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)} = \frac{0\%}{20\%} = 0\%$$ A opposite of B

Default correlation = -0.1667 (lowest possible)

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First to Default (FTD)

- Default correlation reaches 1 as $p_A = p_B$
- Default correlation reaches –1 as $p_A + p_B = 1$
- Multi-party becomes more complex

First to Default (FTD)

- In the event of perfect dependency, i.e. p(B|A)=1, the basket valuation is:

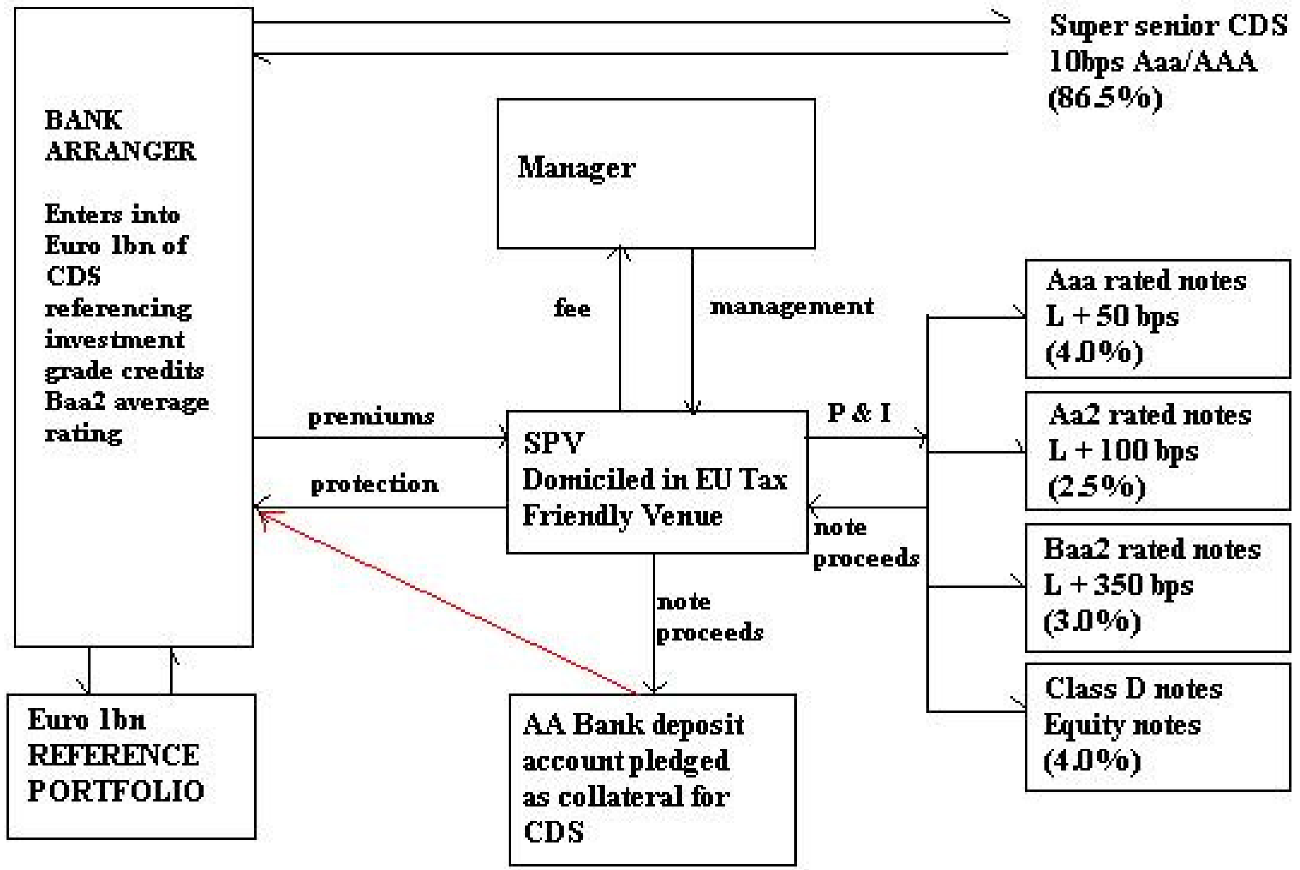
$$\begin{aligned} V &= \frac{1}{1+r}[p(A) + p(B) - p(B \mid A)p(A)] \\ &= \frac{1}{1+r} p(B) \end{aligned}$$

First to Default (FTD)

- In the event of perfectly negative dependency, i.e. p(B|A)=0, the basket valuation becomes:

$$
\begin{aligned}
V &= \frac{1}{1+r}[p(A) + p(B) - p(B \mid A)p(A)] \\
&= \frac{1}{1+r}[p(A) + p(B)]
\end{aligned}
$$

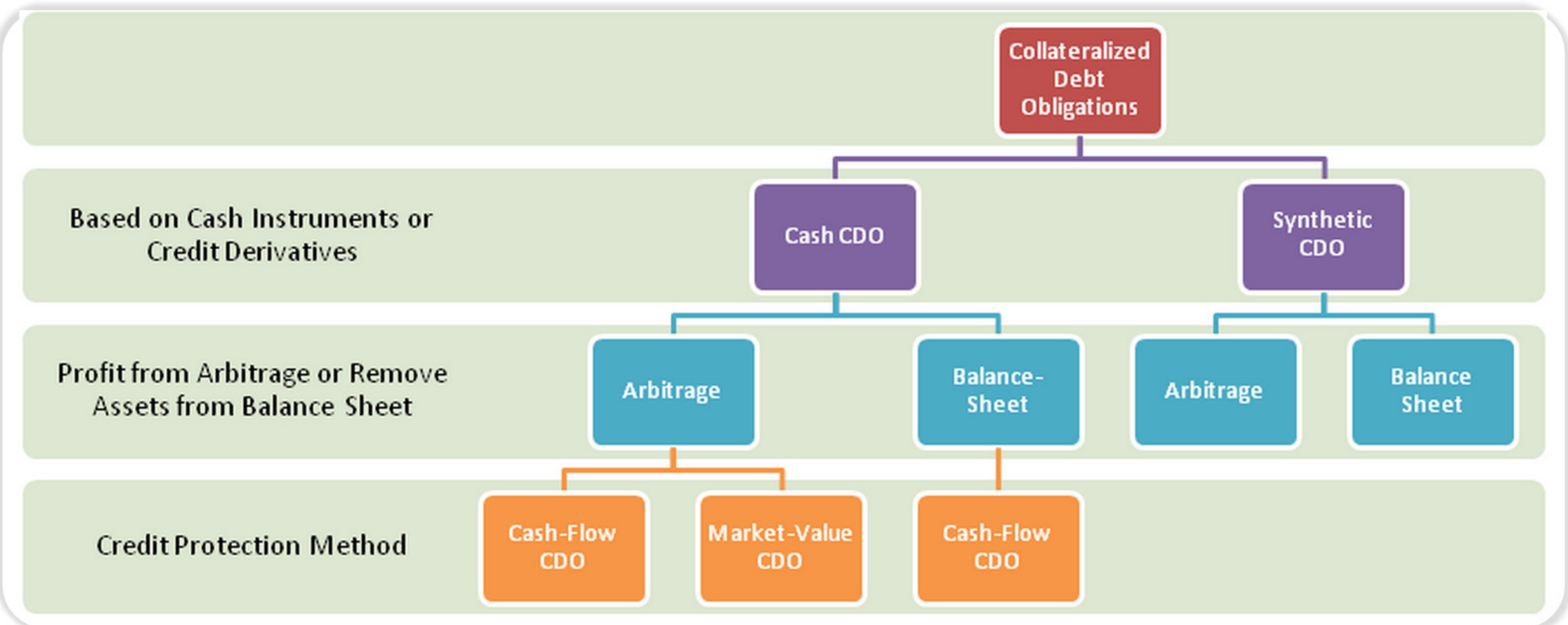
Collateral Debt Obligation (CDO)



Collateral Debt Obligation (CDO)

- Types
  - cash CDO (real bonds)
  - synthetic CDO (CDS)
- by action
  - cash-flow CDO (boxed)
  - market-value CDO (non-boxed)
- by sponsor
  - arbitrage CDO (active)
  - balance-sheet CDO (passive)

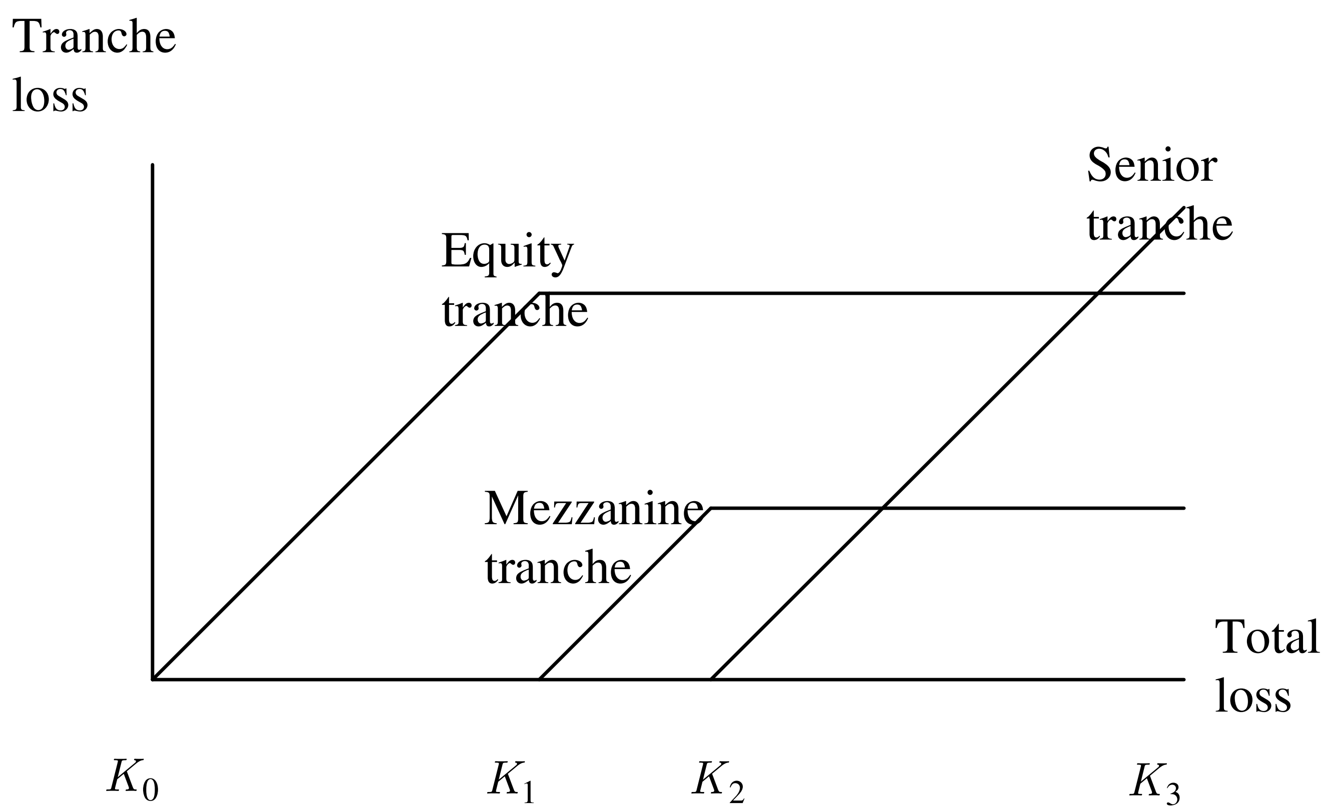
Collateral Debt Obligation (CDO)



– http://thismatter.com/money/bonds/types/cdo.htm

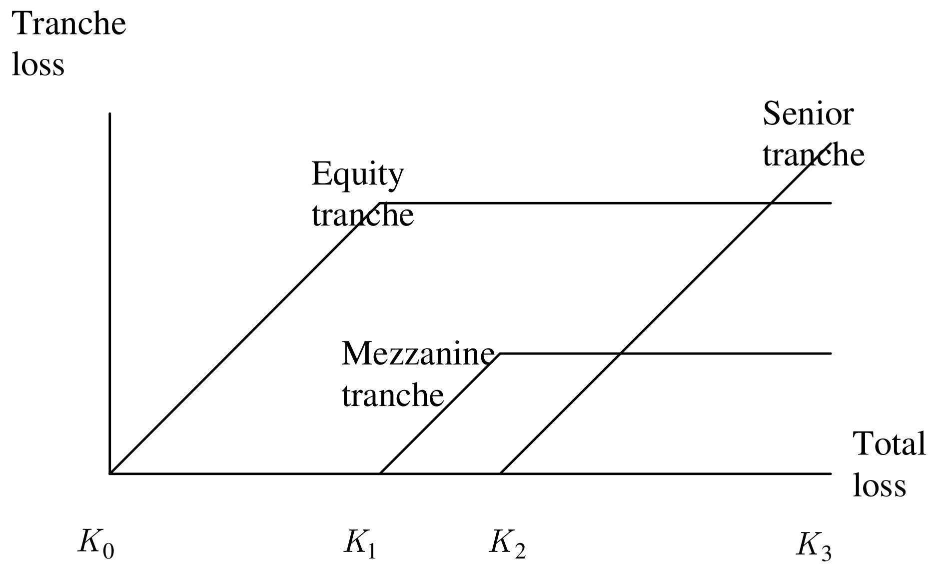
Collateral Debt Obligation (CDO)

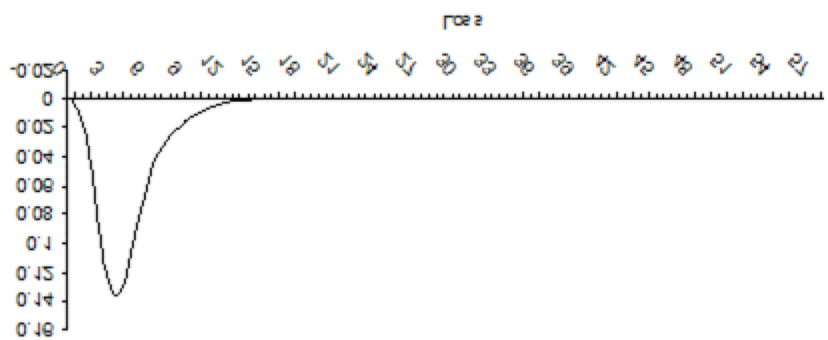
- Waterfall



Collateral Debt Obligation (CDO)

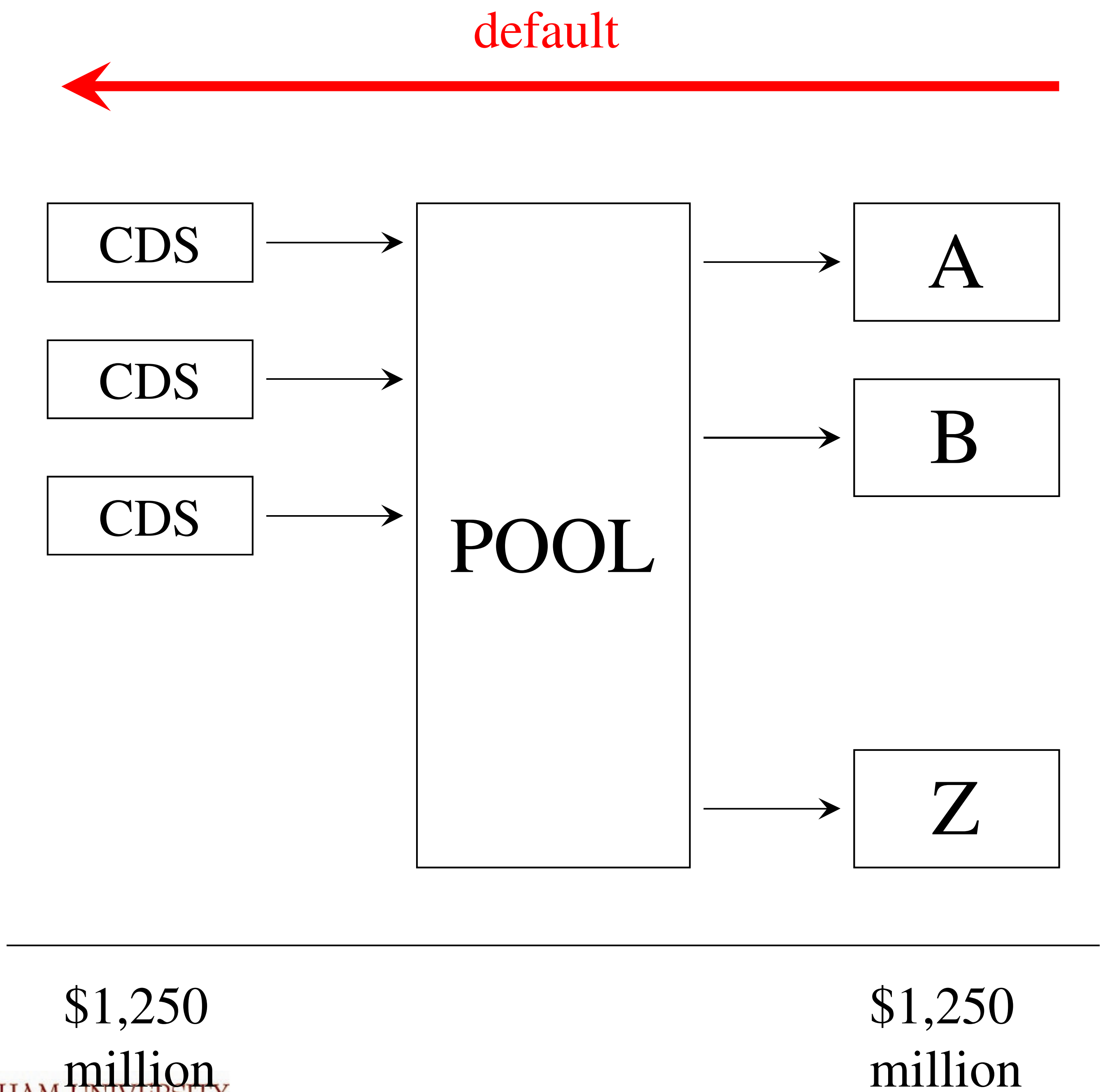
- Valuation
  - loss distribution

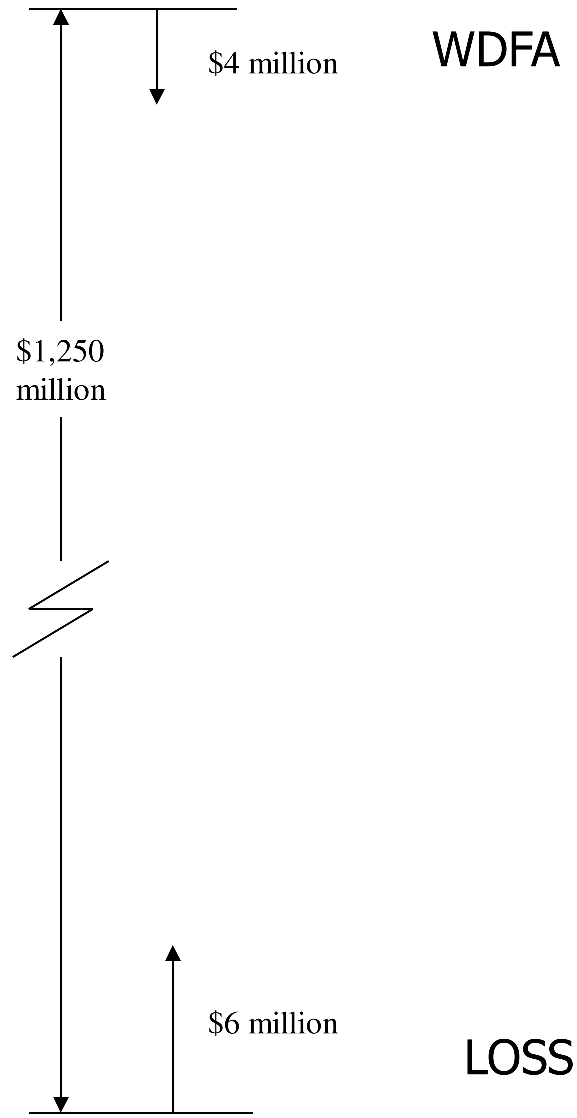




Synthetic CDO

- Risky bond + CDS = risk-free bond
  - hence, risky bond = risk-free bond - CDS
  - i.e. risky bond = long risk-free bond and short CDS (provide protection)
  - e.g. $100 mil risky bonds = $100 mil Treasury and $100 mil CDS (which has no value)
  - Treasury is collateral
    - if no collateral, then no treasury


default
CDS
CDS
CDS
POOL
A
B
Z
$1,250 million
$1,250 million


$4 million
WDFA
$1,250
million
$6 million
LOSS

Collateral Debt Obligation (CDO)

- CDX
  - a CDX CDO is a CDO with 125 credit default swaps (8% each) with US$10 million notional
  - 0-3%, 3-7%, 7-10%, 10-15%, and 15-30%.
  - very liquid (more liquid than single name CDS)

Collateral Debt Obligation (CDO)

- Copula (how to correlate defaults)
  - Gaussian copula (solve the dependency problem)
  - Key equations

$$x_i = \sqrt{\rho}\hat{W}_M + \sqrt{1-\rho}\hat{W}_i$$

$$\begin{aligned}
\hat{p}_{i|f} &= \widehat{\Pr}\left(x_i < K_i \mid W_M = f\right) = \widehat{\Pr}\left(\sqrt{\rho}f + \sqrt{1-\rho}W_i < K_i\right) \\
&= \widehat{\Pr}\left(W_i < \tfrac{\sqrt{\rho}f - K_i}{\sqrt{1-\rho}}\right) \\
&= N\left(\tfrac{K_i - \sqrt{\rho}f}{\sqrt{1-\rho}}\right) \\
&= N\left(\tfrac{N^{-1}(\hat{p}_i) - \sqrt{\rho}f}{\sqrt{1-\rho}}\right)
\end{aligned}$$

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Collateral Debt Obligation (CDO)

- Vasicek (approx.) Model
  - Conditional binomial model
  - Equations

$$\begin{aligned}\Pr(\ L = i/m\ ) &= \binom{m}{i} \Pr(A_1 < K_1, \cdots, A_i < K_i, A_{i+1} > K_{i+1}, \cdots, A_m > K_m) \\ &= \binom{m}{i} \int_{-\infty}^{\infty} \Pr(A_1 < K_1, \cdots, A_i < K_i, A_{i+1} > K_{i+1}, \cdots, A_m \\ &\quad > K_m | W_M = f) dF(W_M < f)\end{aligned} \tag{15.16}$$

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Collateral Debt Obligation (CDO)

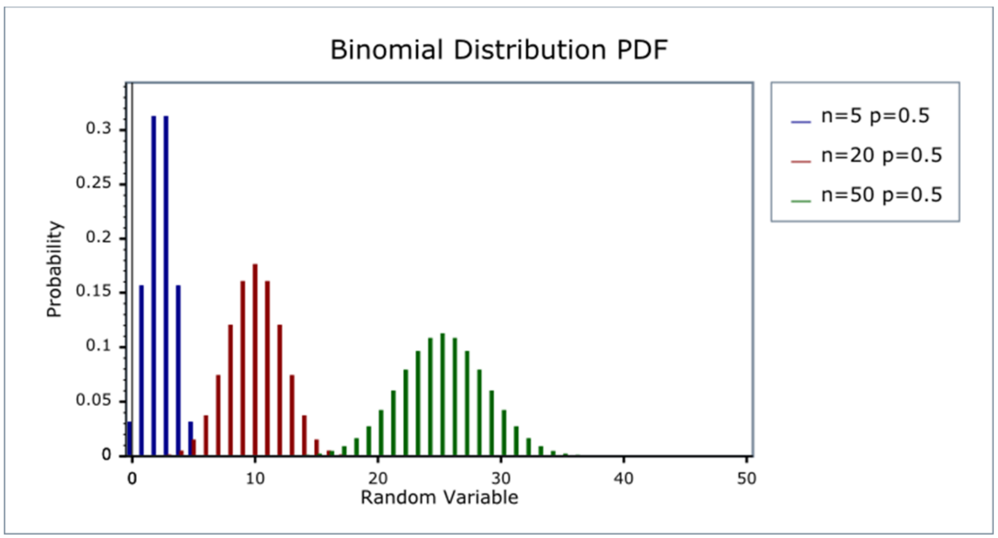
- Vasicek (approx.) Model

$$\Pr(L = i/m) = \binom{m}{i} \int_{-\infty}^{\infty} \Pr(A_1 < K_1 | W_M = f) \cdots \Pr(A_i < K_i | W_M = f)$$
$$\Pr(A_{i+1} > K_{i+1} | W_M = f) \cdots \Pr(A_n > K_n | W_M = f) dF(W_M < f)$$
$$= \binom{m}{i} \int_{-\infty}^{\infty} \prod_{j=1}^{i} N\left(\frac{N^{-1}(p_j) - \sqrt{\rho} f}{\sqrt{1-\rho}}\right) \prod_{j=i+1}^{m} N\left(-\frac{N^{-1}(p_j) - \sqrt{\rho} f}{\sqrt{1-\rho}}\right)$$
$$dF(W_M < f) \tag{15.17}$$

$$\Pr(L = i/m) = \binom{m}{i} \int_{-\infty}^{\infty} N\left(\frac{N^{-1}(p) - \sqrt{\rho} f}{\sqrt{1-\rho}}\right)^{i} N\left(-\frac{N^{-1}(p) - \sqrt{\rho} f}{\sqrt{1-\rho}}\right)^{m-i} dF(W_M < f) \tag{15.18}$$

Collateral Debt Obligation (CDO)

- Moody's binomial model
  - BET (binomial expansion technique)
  - N names, K independent names (smaller K, higher correlation; K=N, total independence)



Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where
n = the number of trials (or the number being sampled)
x = the number of successes desired
p = probability of getting a success in one trial
$q = 1 - p$ = the probability of getting a failure in one trial

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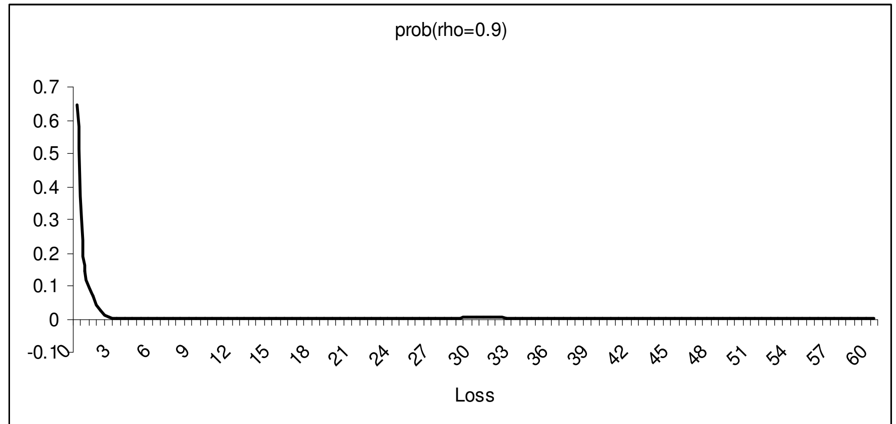
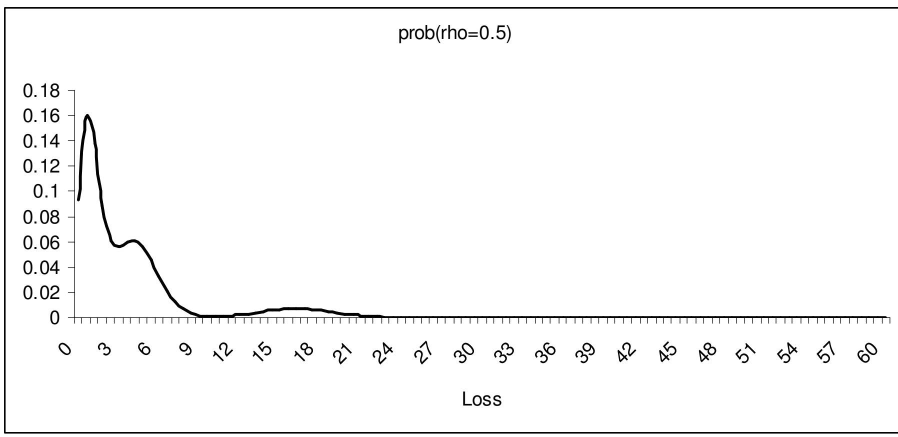
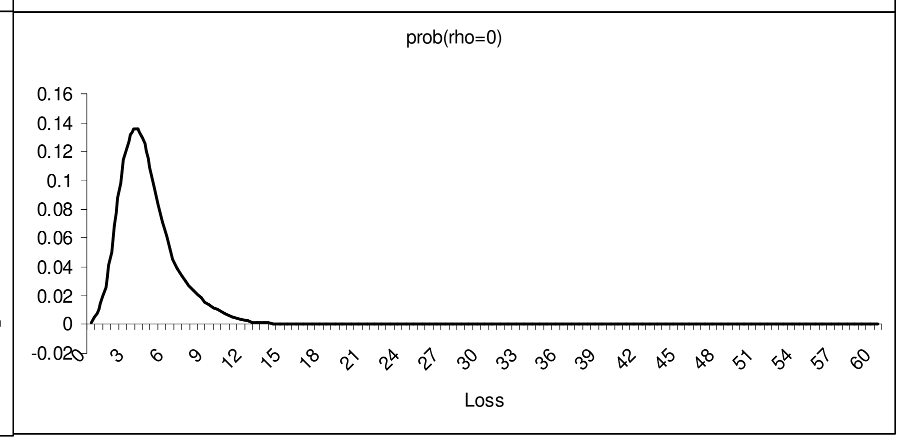
Collateral Debt Obligation (CDO)

- Loss distribution from coupla
  - Fourier inversion
  - Recursive algorithm







Collateral Debt Obligation (CDO)

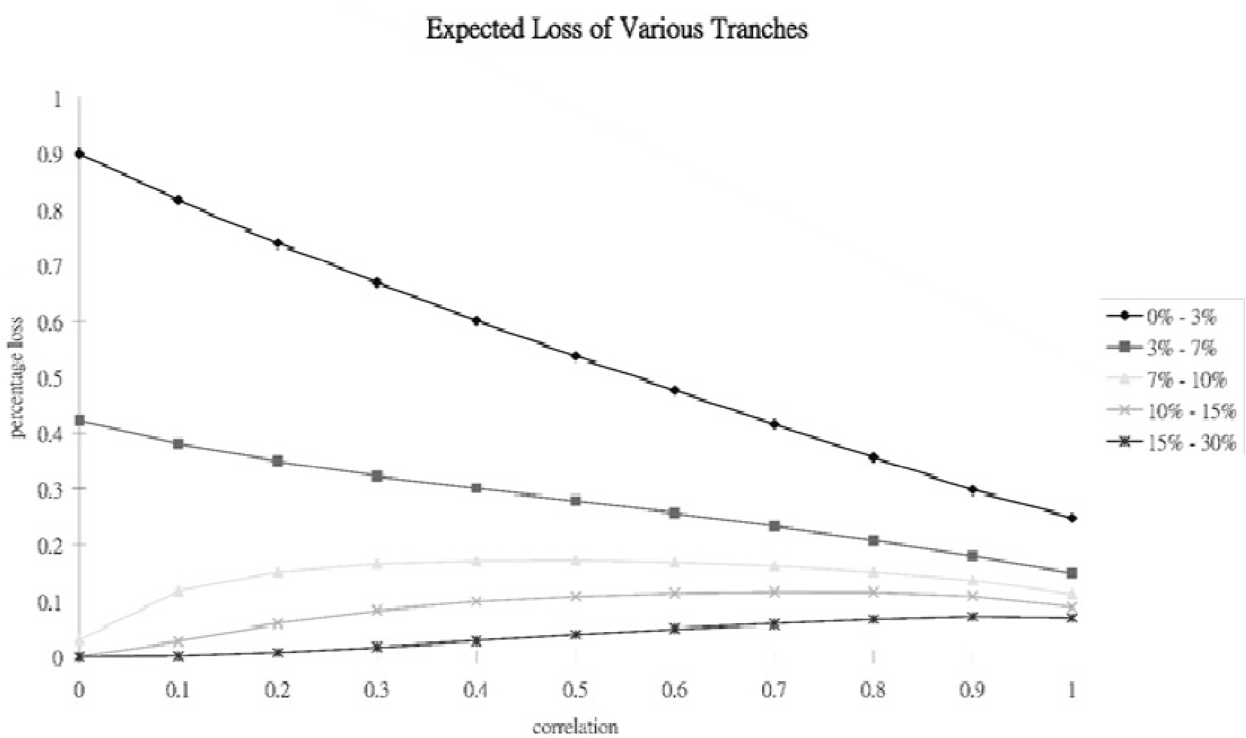
- Model ranks
  - nTD (best, most complete)
  - Coupla (simplified, good enough)
  - Vasicek (one spread for all)
  - Moody's BET (no explicit correlation, subjective)

Collateral Debt Obligation (CDO)

- Problems identified with such a loss distribution
  - thin tranches (100 tranche CDO)
  - CDO^2, CDO^3, ...
  - mezzanine tranches difficult to price

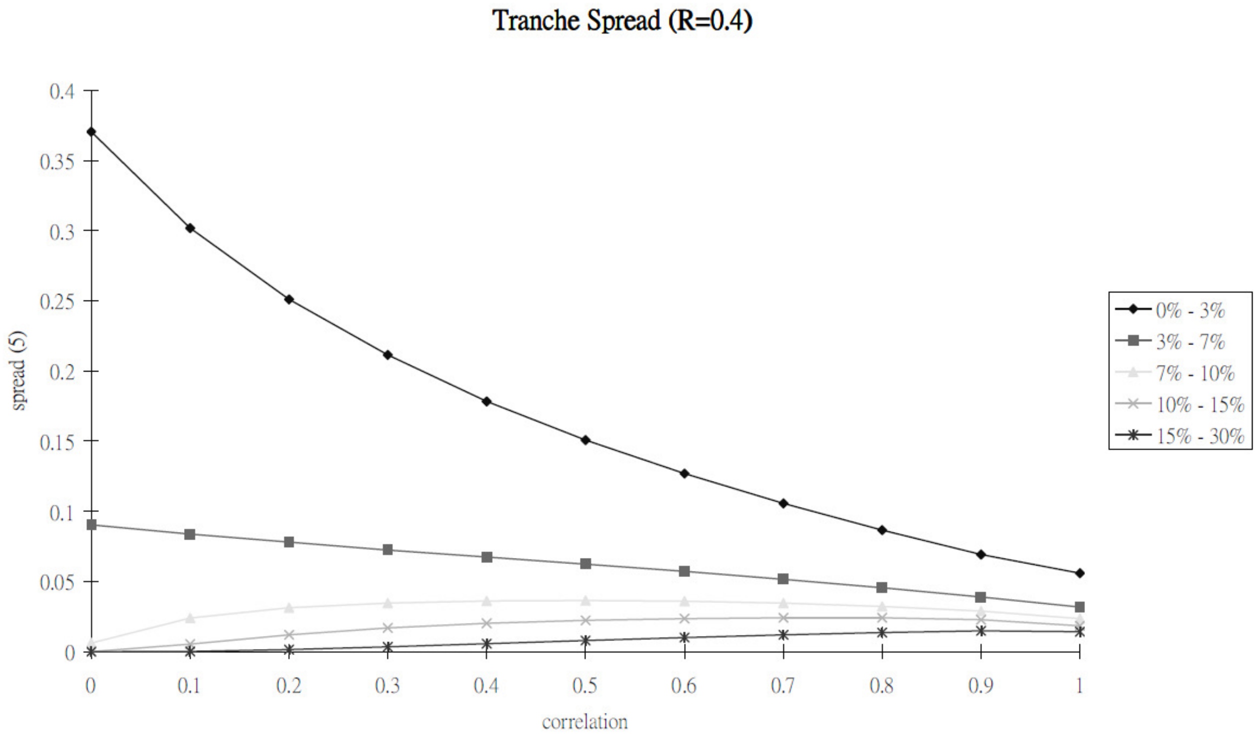
Collateral Debt Obligation (CDO)

- Tranche expected loss



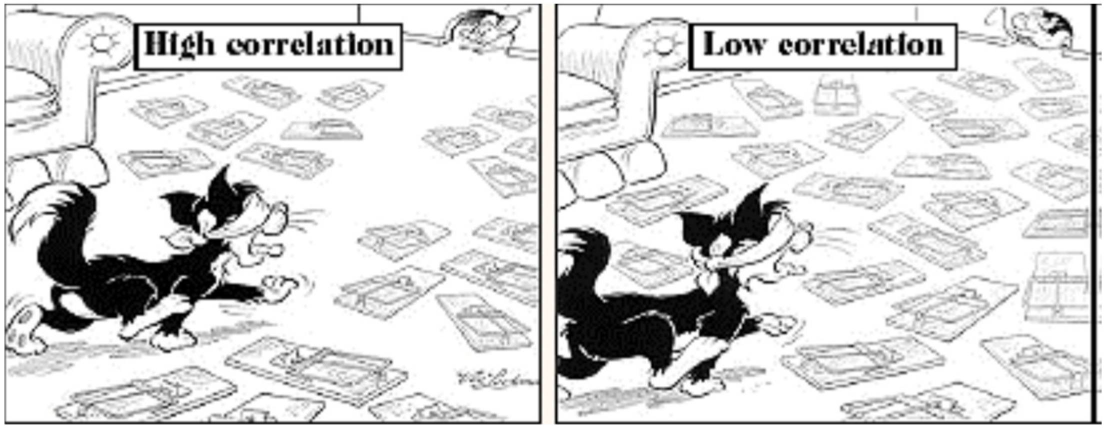
Collateral Debt Obligation (CDO)

- Tranche spread



Collateral Debt Obligation (CDO)

- A Cat analogy
  - Cats have nine lives (JPM)

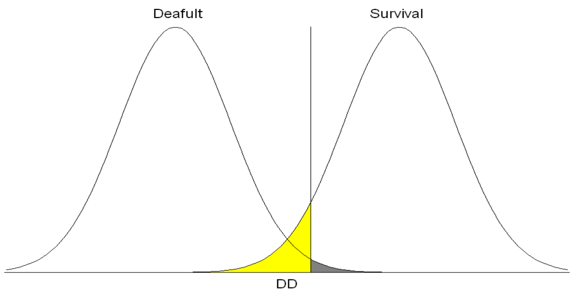


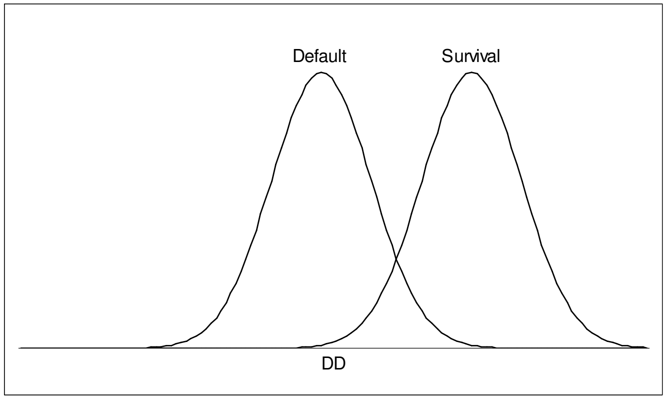
Default Prediction

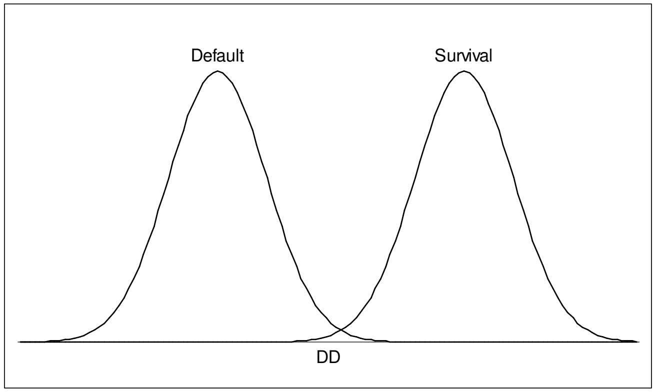
- Early warning signal
- Quantitative rating
- KMV-Moodys
- Altman’s Z
- Olhson’s O

Default Prediction

- Basics
  - default firms (left)
  - survival firms (right)
  - models compute DDs
  - good model (right) vs. bad model (left)







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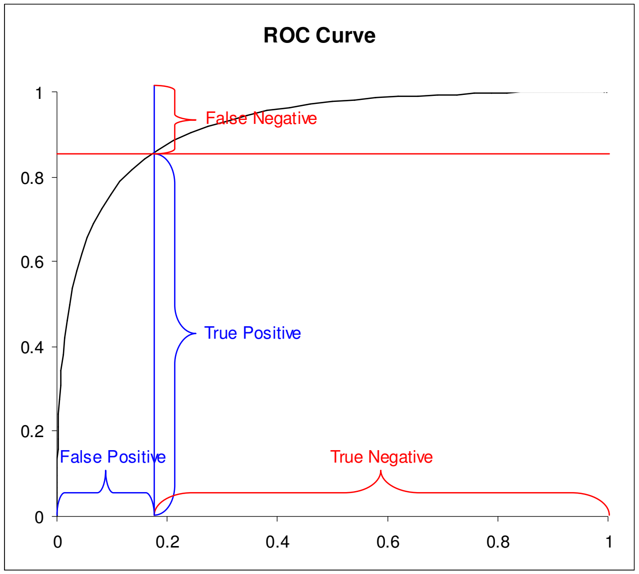
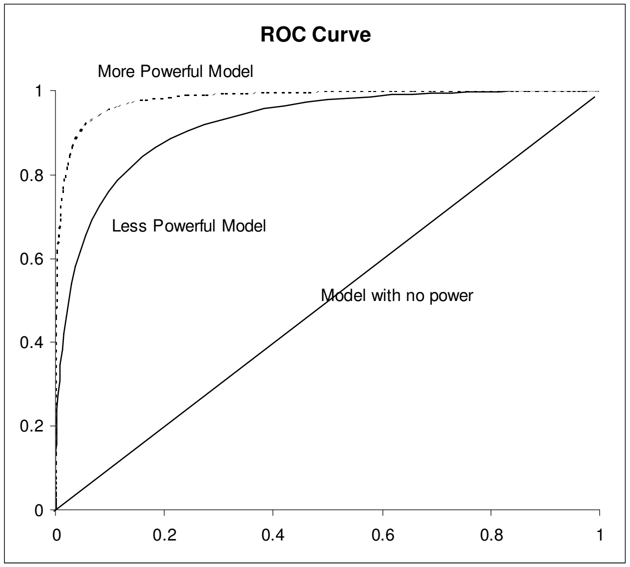
Default Prediction

- ROC (Receiver Operating Characteristic)
  - used in medical research
  - false positive and false negative (vaccine)
  - now used in default model testing





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Default Prediction

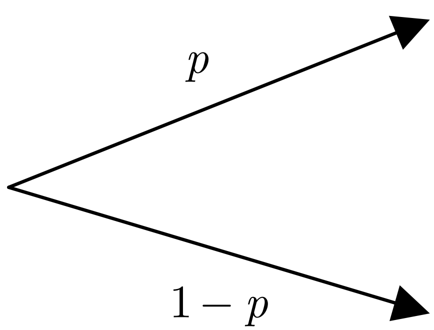
- Steps
  - collect all default firms (not many)
  - collect all survival firms (many)
  - calculate DDs for all firms
  - sort them by DD (horizontal axis)
  - for each DD bucket, compute % default
  - plot ROC
  - compute area

Default Prediction

- Questions
  - two samples are not in proportion
    - one extremely few and one extremely many
  - hard to decide what to include
    - too many survival firms
  - hard to get default data
    - some defaults happened long ago

EL and UL

- Basic intuition (single name)



Default => $1-R$

No default => 0

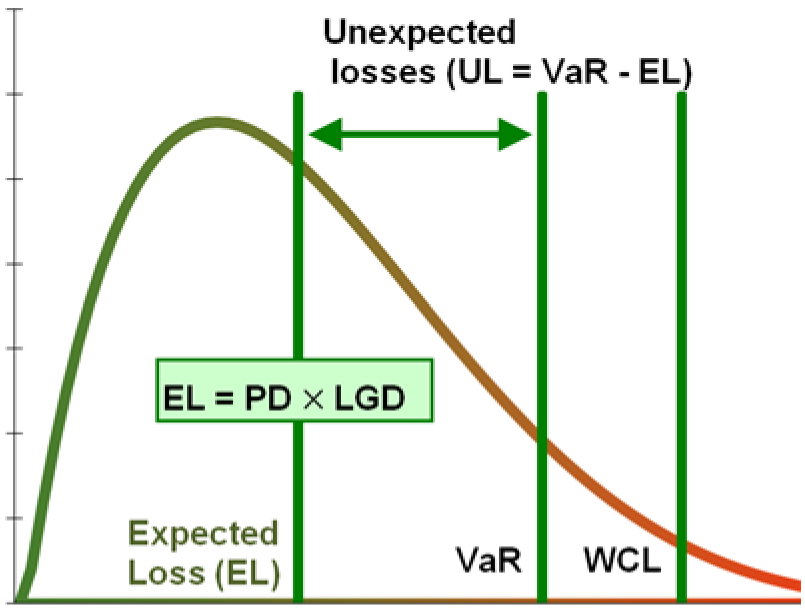
$$\mathrm{EL} = p(1-R)$$

$$\mathrm{UL} = \sqrt{p(1-p)}(1-R)$$

UL highest when p = 0.5

EL and UL

- Portfolios
  - difficult to measure accurately
  - use standard deviation



EL and UL

- CECL (current expected credit losses)
  - fair value accounting
  - June 2016
  - replaces ALLL (allowance for loan and lease losses)
  - no specific methodology (IRB)

Credit VaR

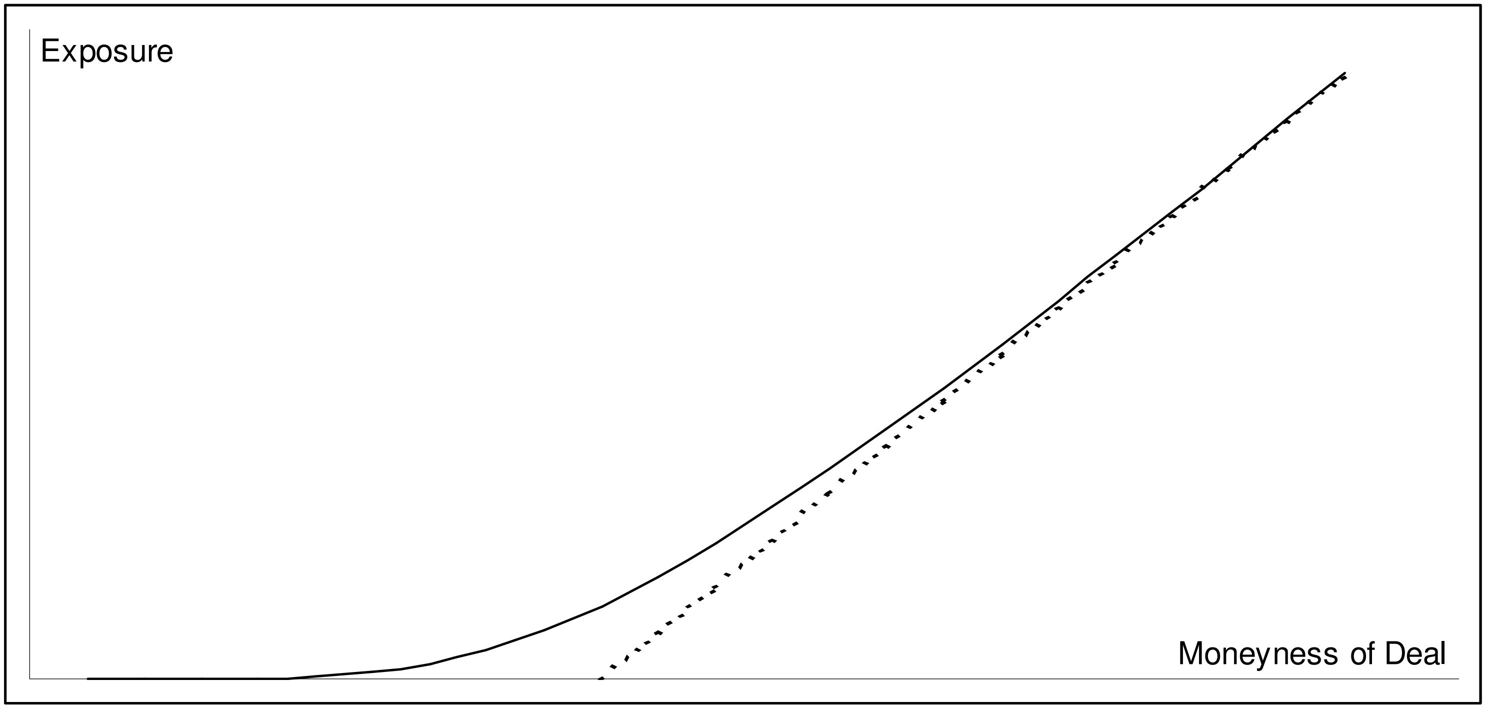
- Skewed loss distribution
- Term structure of CVaR (Basel II)
  - no more Gaussian, no more scalability
  - highly dependent on models
- Difficulties to use models on banks
  - Banks are highly levered
  - Case study of Lehman (CCIS 2014)

CVA

- Became important after the crisis
- quantifies counterparty risk
- trading desk (transfer pricing)
- DVA – CVA to the counterparty
  - worsening credit (DVA falls) helps NI Ironic?!
  - correlation of credit and other products

CVA

- An old method: exposure (call option)



CVA

- Valuation
  - = CDS protection value
- An example (IRS)
  - A pays fixed 4% Receives floating
  - B is BBB rated; CDS spread is 200 bps
  - A needs to hedge for B’s default
  - A books the trade at 6% -- true cost
  - IRS matches with CDS

CVA

- A different example (bond)
  - A buys a Treasury bond from B ($100 face)
  - B CDS spread is 200 basis points
  - PV 5 years of 200 bps is, say 6.2% ($6.2)
  - cost to A is $106.2
- Complexities
  - – correlation among counter parties
  - – correlation between assets and counter parties

CVA

- EAD (exposure at default)
  - exp'ted recoveries fr counterparties
  - exp'ted exposures fr CPs (IRS values)
  - correlations among defaults
    - Similar to FTD/NTD

CVA

$$CVA_{A,CP}(t_0) = 1/N \sum_{s=1}^{N} \sum_{j=1}^{T} \boxed{\hat{E}_s(t_j)} \cdot \boxed{\hat{q}_{s,d}(t_{j-1}, t_j)} \cdot \boxed{(1-\hat{r})} \cdot \boxed{d_s(t_i, t_j)}$$

Component one: Exposure Measure

Component two: Probability of Default – Risk Neutral marginal default probability

Component three: Recovery Rate per Instrument

Component four: discount factor

CVA

- CVA - DVA

$$CVA = E_A s_A - E_B s_B$$

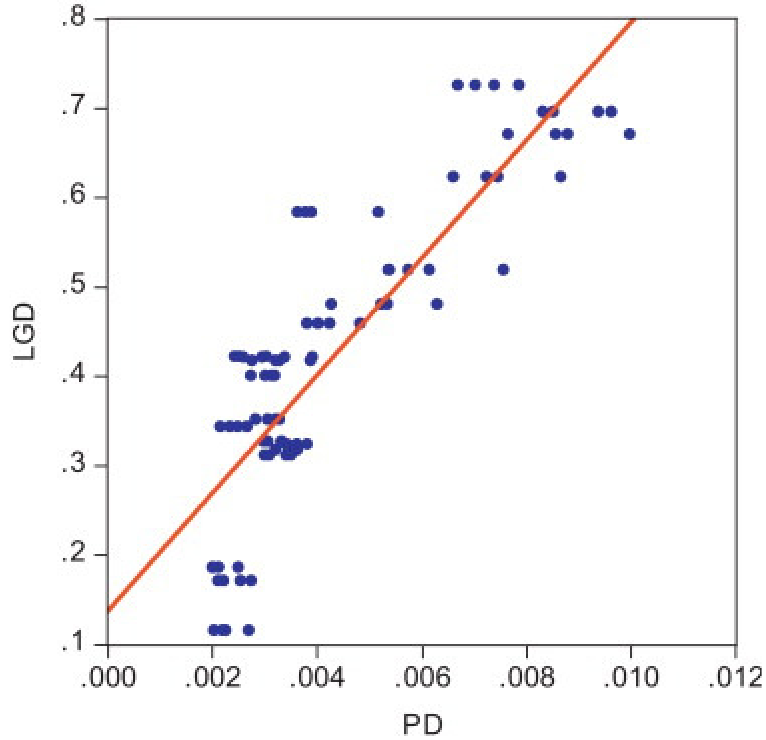
$\mathbf{E_A}$ is the present-valued expected exposure faced by counterparty B with respect to Bank A;
$\mathbf{s_A}$ is the market loss rate (i.e. the product of risk-neutral PD and risk neutral LGD) of A
$\mathbf{E_B}$ is the present-valued expected exposure faced by A with respect to B;
$\mathbf{s_B}$ is the market loss rate of B.

CVA

- Wrong Way Risk (WWR)
  - In general, the exposure with a CP is not independent of the CP's credit quality
  - Wrong Way Risk is cases where the exposure increases when the credit quality of the CP deteriorates – i.e. exposure tend to be high when PDs are high

CVA

- WWR
  - Negative correlation between PD and LDG (source: Altman)



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CVA

- Two types of WWR
  - General WWR: the CP’s credit quality is for correlated with macroeconomic factors which also affect the value of the derivatives (e.g. correlation between declining corporate credit quality and high (or low) interest rates causing higher exposures – such as long IRS) rates high, expo high, CP quality low
  - Specific WWR: CP’s exposure is highly correlated with CP’s PD. (e.g. a company writing put options on its own stock, derivatives collateralized by own shares)

CVA

- WWR quantification is still an open challenge other than the self referencing specific WWR due to:
  - Difficulty to separate statistical noise from systematic correlation
  - Challenge of dynamic forward looking adjustment to historical calibration

CVA Sensitivity and Hedging

- CVA is based upon a set of exposures X (vector) which are random
- X are functions of a set of CP-specific and macro risk factors y (vector)
- y can be modeled as (e.g. PCA):

$$y_i(t) = a_i + \sum_{k=1}^{K} b_{i,k} F_k(t) + e_i(t)$$

- This is a common modeling structure.

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CVA Sensitivity and Hedging

- CVA Sensitivity is straight forward by definition:

$$\text{First order}: \frac{\partial CVA}{\partial \vec{X}_{i-j}} \approx \frac{CVA(\vec{X}^{+}_{i-j}) - CVA(\vec{X}^{-}_{i-j})}{2\delta} \quad (6)$$

$$\text{Diagonal second order}: \frac{\partial^2 CVA}{\partial \vec{X}_{i-j}{}^2} \approx \frac{CVA(\vec{X}^{+}_{i-j}) - 2CVA(\vec{X}_0) + CVA(\vec{X}^{-}_{i-j})}{\delta^2} \quad (7)$$

- CCDS (contingent CDS)
  - CVA securitized
  - perfect hedge of CVA

CVA

- Meaningful hedging and P&L explain: a long way to go
  - Common hedges on the street
    - Counterparty credit spread delta, FX delta, IR delta, FX vega
  - In progress and outstanding
    - IR Vega, FX/IR Gamma, Cross Gamma - WWR
  - Additional note
    - CVA hedging and P&L explain gap, Debt/Liability DVA management
    - Direction of CVA desk's roles and performance factors – real PnL, VaR capital mitigation, counterparty risk capital mitigation
    - Difficulty to draw clear line between hedging to limit risk and hedging for profit

CVA

- Practical challenges
  - Path dependent impact: early exercise, Barrier, Bermudan
  - Recalibration noise: bucketed FX Vega/IR Delta/IR Vega
  - IR Vega and cross gamma for meaningful hedging/P&L explain.

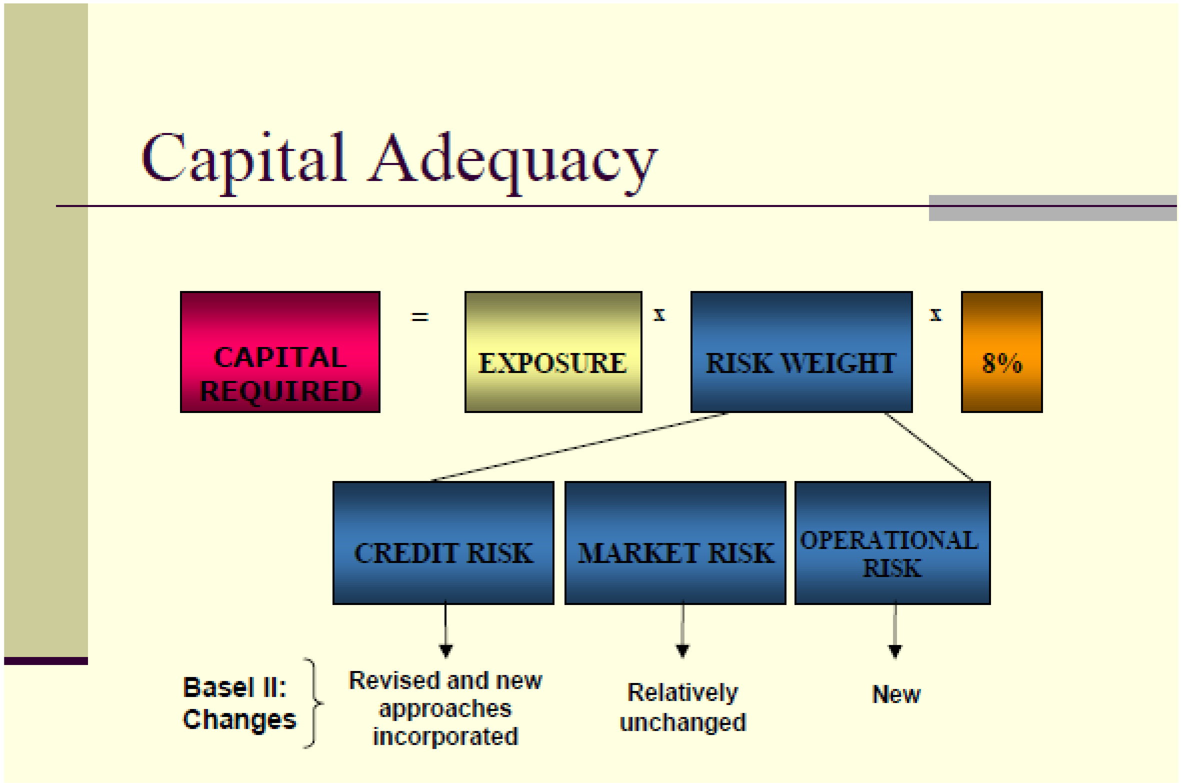
CVA

- xVA
  - CVA
  - DVA
  - FVA (funding)
  - MVA (margin)

Basel II

- Pillar 1: Capital Adequacy
  - Min. of 8% (but now credit, market & operational)
- Pillar 2: Supervisory Review
  - Supervisors responsible for ensuring banks have sound internal processes to assess capital adequacy
- Pillar 3: Market Discipline
  - Enhanced disclosure by banks
  - Sets out disclosure requirements

Reg(ulatory) Capital



Reg(ulatory) Capital

- Risk Weighted Assets are computed by:
  - RWA = Banking Book RWA + Trading Book RWA
  - Banking Book RWA = Position * Risk Weighting
- Trading Book RWA = Market Risk Capital / 8%

Other Topics

- Risk funding
  - Black-Scholes world falls apart (risk-free funding)
    - Borrowing and lending rates are functions of credit risk
    - Counterparty charges
    - Liquidity charges
  - If borrowing cost is risky, then …
    - Law of one price is broken (no unique pricing)
    - Challenge to valuation

Other Topics

- Variable margins
  - Like futures contracts
    - Futures-style margining
      - No up front
      - Margin is like a fee
      - Pay/withdraw as we go
    - Everything is a futures
      - Off balancesheet
  - Constant margin calls …