

# DIMENSIONS OF SPACES OF SIEGEL MODULAR FORMS OF LOW WEIGHT IN DEGREE FOUR.

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ABSTRACT. We calculate the dimensions of  $M_4^{12}, M_4^8, S_4^{12}, S_4^8, S_4^6$  using Erokhin's work on Niemeier lattices and geometric methods involving the hyperelliptic locus.

## §0. Introduction.

In this note we calculate some dimensions of spaces of Siegel modular forms and of cusp forms. We obtain the results that  $\dim M_4^{12} = 6$ ,  $\dim M_4^8 = 2$ ,  $\dim S_4^{12} = 2$ ,  $\dim S_4^8 = 1$ , and  $\dim S_4^6 = 0$ . Explicit generators for these spaces of cusp forms are also given. The dimensions of the  $M_g^k$  are known for  $g \leq 3$  [14]; previously it was known [6, pg. 50] only that  $S_4^k = 0$  for  $1 \leq k \leq 5$ . Calculations of Erokhin for the Niemeier lattices play an essential role in our arguments, as does a theorem of Igusa that elements of  $S_4^k$  for even  $k \leq 8$  must vanish on the hyperelliptic locus. The admittedly special nature of these calculations in weights less than or equal to 12 has a natural origin. The Type II lattices in dimension 24 are 24 in number and have been classified by Niemeier, whereas the Type II lattices in dimension 32 number in excess of 80 million and will never be classified in the same detail. A complete classification is essential for applications to Siegel modular forms if Böcherer's result (Theorem 1.2) is to be applied. On the other hand, these dimensions are notoriously difficult to calculate by any means (see [13, pp. 60–61]), and this note provides more data toward this famous problem.

The results on cusp forms may be summarized as follows. The theta series for the Niemeier lattices provide a basis for  $S_4^{12}$  and the study of  $S_4^8$  and  $S_4^6$  is reduced to  $S_4^{12}$  via the inclusions  $M_4^4 S_4^8 \subseteq S_4^{12}$  and  $S_4^6 S_4^6 \subseteq S_4^{12}$ . Let  $f_4 \in M_4^4$  be the theta series of the  $E_8$  lattice and let  $j_8 \in S_4^8$  be Schottky's modular form vanishing on the Jacobian locus. Then  $S_4^{12}$  has a basis  $\{f_4 j_8, \psi_{12}\}$  such that  $\psi_{12}$  does not vanish on  $\bigoplus^4 \mathcal{H}_1$ . For any  $e \in S_4^8$  and  $f \in S_4^6$  we then have  $f_4 e = a f_4 j_8 + b \psi_{12}$  and  $f^2 = \alpha f_4 j_8 + \beta \psi_{12}$ . Since  $e$  and  $f$  necessarily vanish on the hyperelliptic locus we can evaluate the coefficients  $a, b, \alpha, \beta$  by restriction to  $\bigoplus^4 \mathcal{H}_1$ . We obtain  $S_4^8 = \mathbb{C} j_8$  and  $S_4^6 = 0$  by this procedure. This note is another example using geometric information to calculate the dimensions of spaces of cusp forms.

## §1. Notation.

Let  $\mathcal{H}_g$  be the Siegel upper half space of degree  $g \geq 1$  [12, pg. 2] and let  $\Gamma_g = \mathrm{Sp}_g(\mathbb{Z})$  denote the full Siegel modular group which acts on  $\mathcal{H}_g$ . Let  $M_g^k$  be the complex vector

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space of Siegel modular forms of weight  $k$  on  $\mathcal{H}_g$  [12, pg. 43],  $\Phi_g : M_g^k \rightarrow M_{g-1}^k$  be the Siegel map [12, pg. 54], and  $S_g^k = \ker \Phi_g$  be the subspace of cusp forms.

To any integral lattice  $\Lambda \subseteq \mathbb{R}^n$  we may define the *theta series* of  $\Lambda$  (the analytic class invariant)  $\vartheta_\Lambda : \mathcal{H}_g \rightarrow \mathbb{C}$  as follows: for  $\Omega \in \mathcal{H}_g$  let

$$(1.1) \quad \vartheta_\Lambda(\Omega) = \sum_{\ell_1, \dots, \ell_g \in \Lambda} \exp\left(i\pi \sum_{j,k=1}^g \Omega_{jk} \langle \ell_j, \ell_k \rangle\right).$$

A lattice is called ‘‘Type II’’ [2, pg. 48] if it is even and self dual. For  $\Lambda$  of Type II, we have  $\vartheta_\Lambda \in M_g^{n/2}$  for each  $g \geq 1$  [6, pg. 17]. We specify lattices in the notation of [2, pp. 119, 120, 407] and use the further designations:

$$\begin{aligned} f_4 &= \vartheta_{E_8}, \\ f_8 &= \vartheta_{D_{16}^+}, \\ j_8 &= f_4^2 - f_8. \end{aligned}$$

Also, for the 24 Niemeier lattices, we denote their theta series by

$$\vartheta_i \in M_g^{12} \text{ for } i \in \{0, 1, \dots, 23\},$$

where we index the Niemeier lattices by 0 through 23 in some manner with the Leech lattice being the 0<sup>th</sup> one; that is,  $\vartheta_0$  is theta series of the Leech lattice. Let  $h_i$  be the Coxeter number [2, pg. 407] of the  $i^{\text{th}}$  Niemeier lattice and let  $\tau_i = 24h_i$ . Except for the Leech lattice, where  $\tau_0 = 0$ ,  $\tau_i$  is the kissing number of the  $i^{\text{th}}$  Niemeier lattice. The following Theorem is due to Böcherer [1, pp. 22, 44].

**1.2 Theorem.** (Böcherer) *For  $k > 2g$  and  $k \equiv 0 \pmod{4}$ , the theta series of all Type II lattices span  $M_g^k$ .*

This Theorem assures us, for example, that the  $\vartheta_i$  span  $M_g^{12}$  for  $1 \leq g \leq 5$ .

We now recall some geometrically defined subsets of  $\mathcal{H}_g/\Gamma_g$  that we will use. View  $\mathcal{A}_g = \mathcal{H}_g/\Gamma_g$  as the moduli space of principally polarized abelian varieties. The Torelli map sends a compact Riemann surface of genus  $g$  to its Jacobian’s class in  $\mathcal{A}_g$ . The closure in  $\mathcal{A}_g$  of the image of the Torelli map is called the *Jacobian locus*,  $\text{Jac}_g$ . Similarly, the closure of the image of the restriction of the Torelli map to hyperelliptic Riemann surfaces is called the *hyperelliptic locus*,  $h_g$ . We call the image of  $\bigoplus^g \mathcal{H}_1$  in  $\mathcal{A}_g$  the *diagonal locus*,  $\text{Diag}_g$ . The following inclusions hold:

$$\mathcal{A}_g \supseteq \text{Jac}_g \supseteq h_g \supseteq \text{Diag}_g.$$

The final inclusion follows easily from the techniques for the degeneration of curves in [5]. The following theorems of Igusa relate the above loci to the ring of Siegel modular forms. We say that a Siegel modular form  $f$  vanishes on  $h_g$  if for all  $\Omega \in \mathcal{H}_g$  such that  $[\Omega] \in h_g$  we have  $f(\Omega) = 0$ .

**1.3 Theorem.** (Igusa, [9, pg. 845]) *A cusp form in  $S_g^k$  of even weight  $k < 8 + 4/g$  necessarily vanishes on the hyperelliptic locus  $h_g$ .*

**1.4 Theorem.** (Igusa, [10][11]) *The ideal of Siegel modular forms in  $\bigoplus_{k=1}^\infty M_4^k$  that vanish on the Jacobian locus,  $\text{Jac}_4$ , is principal and is generated by the irreducible element  $j_8$ .*

## §2. Dimension Calculations.

We begin by constructing certain cusp forms of weight 12 for  $1 \leq g \leq 4$  with simple behavior on direct sums of  $\mathcal{H}_1$ . We let  $\Delta \in S_1^{12}$  denote the usual generator of  $S_1^{12}$  [2, pg. 105]. We also use the following notation:

$$\det_{i,j,k,l}(\vartheta, \tau^2, \tau, 1) = \begin{vmatrix} \vartheta_i & \vartheta_j & \vartheta_k & \vartheta_l \\ \tau_i^2 & \tau_j^2 & \tau_k^2 & \tau_l^2 \\ \tau_i & \tau_j & \tau_k & \tau_l \\ 1 & 1 & 1 & 1 \end{vmatrix}.$$

**2.1 Lemma.** *For all  $i, j, k, l, m \in \{0, 1, \dots, 23\}$  we have*

$$\begin{aligned} \det_{i,j}(\vartheta, 1) &\in S_1^{12} \text{ and } \det_{i,j}(\vartheta, 1) = \det_{i,j}(\tau, 1) \Delta, \\ \det_{i,j,k}(\vartheta, \tau, 1) &\in S_2^{12} \text{ and } \det_{i,j,k}(\vartheta, \tau, 1)|_{\oplus^2 \mathcal{H}_1} = \det_{i,j,k}(\tau^2, \tau, 1) \Delta \otimes \Delta, \\ \det_{i,j,k,l}(\vartheta, \tau^2, \tau, 1) &\in S_3^{12} \text{ and } \det_{i,j,k,l}(\vartheta, \tau^2, \tau, 1)|_{\oplus^3 \mathcal{H}_1} = \det_{i,j,k,l}(\tau^3, \tau^2, \tau, 1) \Delta \otimes \Delta \otimes \Delta, \\ \det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1) &\in S_4^{12} \text{ and } \det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)|_{\oplus^4 \mathcal{H}_1} \\ &= \det_{i,j,k,l,m}(\tau^4, \tau^3, \tau^2, \tau, 1) \Delta \otimes \Delta \otimes \Delta \otimes \Delta. \end{aligned}$$

*Proof.* Recall that for  $\Omega \in \mathcal{H}_1$ , the coefficient of  $e^{2\pi i\Omega}$  in the theta series  $\vartheta_i(\Omega)$  is  $\tau_i$ , and the coefficient of  $e^{2\pi i\Omega}$  in  $\Delta(\Omega)$  is 1. Since  $\vartheta_i - \vartheta_j \in S_1^{12}$  and  $\Delta$  is a generator of  $S_1^{12}$ , we must have  $\vartheta_i - \vartheta_j = (\tau_i - \tau_j)\Delta$  on  $\mathcal{H}_1$ . Another way of saying this is that  $\det_{i,j}(\vartheta, 1) = \vartheta_i - \vartheta_j = (\tau_i - \tau_j)\Delta = \det_{i,j}(\tau, 1) \Delta$  on  $\mathcal{H}_1$ . Using  $\tau_0 = 0$  we have  $\vartheta_i = \vartheta_0 + \tau_i\Delta$  with  $\vartheta_0$  being the theta series of the Leech lattice. It follows that  $\det_{i,j,k}(\vartheta, \tau, 1)$  is identically zero on  $\mathcal{H}_1$  because the first row is a linear combination of the second and third rows in the determinant; hence it is a cusp form on  $\mathcal{H}_2$  because we have  $\Phi_g(\vartheta_\Lambda$  on  $\mathcal{H}_g) = \vartheta_\Lambda$  on  $\mathcal{H}_{g-1}$ . Restriction to  $\oplus^2 \mathcal{H}_1$  gives  $\vartheta_i = \vartheta_i|_{\mathcal{H}_1} \otimes \vartheta_i|_{\mathcal{H}_1} = \vartheta_0 \otimes \vartheta_0 + \tau_i(\vartheta_0 \otimes \Delta + \Delta \otimes \vartheta_0) + \tau_i^2 \Delta \otimes \Delta$  so that by subtracting multiples of the second and third rows from the first row in the determinant, we have  $\det_{i,j,k}(\vartheta, \tau, 1)|_{\oplus^2 \mathcal{H}_1} = \det_{i,j,k}(\tau^2 \Delta \otimes \Delta, \tau, 1) = \det_{i,j,k}(\tau^2, \tau, 1) \Delta \otimes \Delta$  as claimed.

Without loss of generality and for convenience, we can number the Niemeier lattices so that the first five Coxeter numbers are distinct [2, pg.407] and  $\tau_0 = 0$  still. Then  $\det_{0,1,2}(\tau^2, \tau, 1) \neq 0$ , so that we may let  $\phi = \det_{0,1,2}(\vartheta, \tau, 1) / \det_{0,1,2}(\tau^2, \tau, 1)$ . Since  $\det_{0,1,2}(\vartheta, \tau, 1) = \det_{0,1,2}(\tau^2, \tau, 1) \Delta \otimes \Delta$  on  $\oplus^2 \mathcal{H}_1$ , we have  $\phi = \Delta \otimes \Delta$  on  $\oplus^2 \mathcal{H}_1$ . This implies  $\phi$  is not identically zero on  $\oplus^2 \mathcal{H}_1$ , and hence  $\phi \neq 0$  in  $S_1^{12}$ . Since  $\dim S_2^{12} = 1$ ,  $\phi$  must be a generator. So on  $\mathcal{H}_2$ , we must have

$$(2.2) \quad \det_{i,j,k}(\vartheta, \tau, 1) = s_{ijk} \phi,$$

for some constant  $s_{ijk}$ . We also know that  $\det_{i,j,k}(\vartheta, \tau, 1) = \det_{i,j,k}(\tau^2, \tau, 1) \Delta \otimes \Delta$  on  $\oplus^2 \mathcal{H}_1$ . From (2.2), we also have that  $\det_{i,j,k}(\vartheta, \tau, 1) = s_{ijk} \Delta \otimes \Delta$  on  $\oplus^2 \mathcal{H}_1$ . So we conclude that  $s_{ijk} = \det_{i,j,k}(\tau^2, \tau, 1)$ . So on  $\mathcal{H}_2$ , we have  $\det_{i,j,k}(\vartheta, \tau, 1) = \det_{i,j,k}(\tau^2, \tau, 1) \phi$ .

Expanding the determinant by cofactors using the second row ( $\tau^2$ ), we have on  $\mathcal{H}_2$ ,

$$\begin{aligned} \det_{i_1, i_2, i_3, i_4}(\vartheta, \tau^2, \tau, 1) &= \sum_{k=1}^4 (-1)^{k+1} \tau_{i_k}^2 \det_{\substack{i_1, \dots, i_4 \\ \text{no } i_k}}(\vartheta, \tau, 1) \\ &= \sum_{k=1}^4 (-1)^{k+1} \tau_{i_k}^2 \det_{\substack{i_1, \dots, i_4 \\ \text{no } i_k}}(\tau^2, \tau, 1) \phi \\ &= \det_{i_1, i_2, i_3, i_4}(\tau^2, \tau^2, \tau, 1) \phi \\ &= 0. \end{aligned}$$

Therefore,  $\det_{i,j,k,l}(\vartheta, \tau^2, \tau, 1)$  is a cusp form on  $\mathcal{H}_3$ . Restriction to  $\bigoplus^3 \mathcal{H}_1$  gives  $\vartheta_i = \vartheta_i \otimes \vartheta_i \otimes \vartheta_i = \vartheta_0 \otimes \vartheta_0 \otimes \vartheta_0 + \dots + \tau_i^3 \Delta \otimes \Delta \otimes \Delta$  so that  $\det_{i,j,k,l}(\vartheta, \tau^2, \tau, 1)|_{\bigoplus^3 \mathcal{H}_1} = \det_{i,j,k,l}(\Delta \otimes \Delta \otimes \Delta \tau^3, \tau^2, \tau, 1) = \det_{i,j,k,l}(\tau^3, \tau^2, \tau, 1) \Delta \otimes \Delta \otimes \Delta$ .

Since  $\dim S_3^{12} = 1$  [14, pg. 832] and since we have four distinct Coxeter numbers  $\tau_0, \tau_1, \tau_2, \tau_3$ , we may employ the same linear algebra techniques as above to deduce that  $\det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)$  is identically zero on  $\mathcal{H}_3$  and is a cusp form on  $\mathcal{H}_4$ . Restriction to  $\bigoplus^4 \mathcal{H}_1$  gives  $\det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)|_{\bigoplus^4 \mathcal{H}_1} = \det_{i,j,k,l,m}(\tau^4, \tau^3, \tau^2, \tau, 1) \Delta \otimes \Delta \otimes \Delta \otimes \Delta$  in the same manner as above.  $\square$

The simple pattern of this Lemma does not continue because  $\dim S_4^{12} = 2$ , as we will deduce from a Theorem of Erokhin [3][4].

**2.3 Theorem.** (Erokhin) *We have  $\dim_{\mathbb{C}} \text{Span}\{\vartheta_i \text{ on } \mathcal{H}_4 : i \in \{0, 1, \dots, 23\}\} = 6$ .*

*Proof.* In the notation of [4] the assertion of the Theorem is  $\dim \text{Im } \phi_4 = 6$ , where  $\phi_g : \mathbb{C}^{24} \rightarrow M_g^{12}$  is defined by  $c \mapsto \sum_i c_i \vartheta_i$ . Let  $V_g = \{v \in \mathbb{C}^{24} : \forall c \in \ker \phi_g, {}^t v c = 0\}$ , then we have  $\dim V_g = \dim \text{Im } \phi_g$ . Corollary 1 of [4, pg. 1017] and Theorem 2 of [4, pg. 1018] assert that  $V_4$  has a basis of six elements. (These are  $\{1, v_{A_1}, v_{A_1}^2, v_{A_1}^3, v_{A_1}^4, v_{D_4}\}$  in the notation of [4].)  $\square$

**2.4 Corollary.** *We have  $\dim M_4^{12} = 6$ ,  $\dim S_4^{12} = 2$ .*

*Proof.* From Theorem 1.2, and  $12 > 2 \cdot 4$ , we see that the theta series of the Niemeier lattices span  $M_4^{12}$  and so  $\dim M_4^{12} = 6$  using the previous Theorem of Erokhin. From the surjectivity of  $\Phi_g : M_g^k \rightarrow M_{g-1}^k$  for even  $k > 2g$  [12, pg. 68] the following sequence of complex vector spaces is exact:  $0 \rightarrow S_4^{12} \rightarrow M_4^{12} \rightarrow M_3^{12} \rightarrow 0$ . Since  $\dim M_3^{12} = 4$  [14, pg. 835] we have  $\dim S_4^{12} = 2$ .  $\square$

**2.5 Proposition.** *For any five indices  $i, j, k, l, m \in \{0, 1, \dots, 23\}$  such that the Coxeter numbers  $\tau_i, \tau_j, \tau_k, \tau_l, \tau_m$  are distinct, we have that  $S_4^{12}$  is spanned by  $f_4 j_8$  and  $\det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)$ .*

*Proof.* Since  $\dim S_4^{12} = 2$  by Corollary 2.4 it suffices to show that the cusp forms  $f_4 j_8$  and  $\det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)$  are linearly independent. By Lemma 2.1 the cusp form  $\det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)$  does not vanish on the diagonal locus whereas  $f_4 j_8$  does because  $j_8$  vanishes on the Jacobian locus and hence on the diagonal locus.  $\square$

**2.6 Theorem.** *We have  $\dim S_4^8 = 1$  and  $j_8$  spans  $S_4^8$ .*

*Proof.* Since there exist five Niemeier lattices with distinct Coxeter numbers [2, pg.407], let  $\psi_{12} = \det_{i,j,k,l,m}(\vartheta, \tau^3, \tau^2, \tau, 1)$  be such that  $f_4 j_8$  and  $\psi_{12}$  span  $S_4^{12}$  as in Proposition 2.5. Take any  $f \in S_4^8$ . By Theorem 1.3  $f$  must vanish on the hyperelliptic locus and hence on the diagonal locus. We also have  $f_4 f \in S_4^{12}$  and so  $f_4 f = a f_4 j_8 + b \psi_{12}$  for some  $a, b \in \mathbb{C}$ . Upon restriction to  $\bigoplus^4 \mathcal{H}_1$  we obtain  $0 = b \det(\tau^4, \tau^3, \tau^2, \tau, 1) \Delta \otimes \Delta \otimes \Delta \otimes \Delta$  by Lemma 2.1. Therefore  $b = 0$  and  $f_4 f = a f_4 j_8$ . We conclude that  $f = a j_8$  because the ring of Siegel modular forms has no zero divisors.  $\square$

**2.7 Corollary.** *We have  $\dim M_4^8 = 2$ .*

*Proof.* Since  $M_3^8$  is spanned by  $f_4^2$  [9, pg. 854], the map  $\Phi_4 : M_4^8 \rightarrow M_3^8$  is onto and the sequence  $0 \rightarrow S_4^8 \rightarrow M_4^8 \rightarrow M_3^8 \rightarrow 0$  is exact. Hence we have  $\dim M_4^8 = 2$ .  $\square$

*Remark.* We see that the theta series also span  $M_4^8$  in this particular case where  $k = 2g$ .

**2.8 Theorem.** *We have  $S_4^6 = 0$ .*

*Proof.* Let the notation be as in the proof of Theorem 2.6. Any  $f \in S_4^6$  vanishes on the hyperelliptic locus by Theorem 1.3 and hence on the diagonal locus. Since  $f^2 \in S_4^{12}$  we have  $f^2 = a f_4 j_8 + b \psi_{12}$ , for some constants  $a$  and  $b$ . Considering the restriction to  $\bigoplus^4 \mathcal{H}_1$ , since  $f$  and  $j_8$  both vanish there and  $\psi_{12}$  does not, we must have  $b = 0$ . Thus we have  $f^2 = a f_4 j_8$ . There are two ways to show that  $a = 0$  and  $f = 0$ , completing the proof. First, the ring of Siegel modular forms is a unique factorization ring for  $g \geq 3$  [7][8] and  $j_8$  is irreducible by Theorem 1.4. Second, since  $f^2 = a f_4 j_8$  vanishes on the Jacobian locus, so must  $f$  itself. But since  $j_8$  generates the ideal of forms vanishing on the Jacobian locus by Theorem 1.4, we must have that  $f = f_{-2} j_8$  for some modular form  $f_{-2}$  of weight  $6 - 8 = -2$ , which is necessarily 0.  $\square$

*Remark.* Since  $\dim M_3^6 = 1$  [9, pg. 852] this Theorem shows that either  $\dim M_4^6 = 0$  or  $\dim M_4^6 = 1$ .

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