Public Debt Structure and Liquidity Provision☆

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Abstract

This study examines the optimal public debt structure when governments can issue multiple assets, the state's fiscal capacity is uncertain, public debt has a liquidity purpose for domestic investors, and public debt markets are open to foreign investors. The optimal policy is to tranch the risky fiscal capacity and then to issue state-contingent assets that pay only when the need for aggregate liquidity in the economy is high. In this way, governments can minimize wasted liquidity of public assets: unused returns on public assets not required by domestic investors. This minimizes the cost of liquidity hoarding for the domestic private sector, which increases investment and welfare in the economy. This study also provides a framework to analyze relevant comparative statics regarding the ownership of state-contingent public assets. In particular, the model assesses the effects of changes in the domestic liquidity needs, the state's fiscal capacity, and the foreign willingness to pay for public debt on the ownership of state-contingent public assets.

Keywords: Public Debt, Liquidity, Collateral Constraints, Tranching,

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1. Introduction

In modern economies, investors value public debt for its liquidity (Krishnamurthy and Vissing-Jorgensen (2012), Bolton and Jeanne (2011), Gennaioli et al. (2014)). This is particularly true in emerging economies where, owing to limited access to international capital markets, investors have fewer options in terms of storing liquidity (Asonuma et al. (2015), Gennaioli et al. (2018)).

Many researchers have studied public liquidity provision using a single bond (Holmstrom and Tirole (1998), Woodford (1990), Aiyagari and McGrattan (1998), Holmstrom and Tirole (2011), Guerrieri and Lorenzoni (2009), Angeletos et al. (2016)), focusing on the optimal quantity of debt. However, relatively less is known about the optimal public debt structure when governments issue multiple debt instruments. Therefore, this is the focus of this study.

Studying the optimal public debt structure with multiple debt instruments is relevant because, in practice, governments issue a wide variety of assets in public debt markets.

To examine the optimal public debt structure, this study uses a model with the following three key elements. First, domestic investors use public debt as collateral. Second, public debt markets are open to risk-neutral foreign investors, who demand public debt as a saving vehicle. Third, the government commits to repaying the assets it issues, but is constrained by its risky fiscal capacity, rendering public debt risky.

In this environment, the optimal policy is for the government to tranch its risky fiscal capacity and then to issue state-contingent assets. Under the optimal public debt structure, the government issues assets that pay only when the need for aggregate liquidity in the economy is high. In doing so, they minimize unused returns on public assets that are not required by domestic investors (assets' wasted liquidity). This minimizes the cost of liquidity hoarding for the domestic private sector, which increases investment and welfare in the economy.

The introduction of state-contigent public assets has long-standing support in academic and policy circles (Fischer (1983), Gale (1990), Shiller

(1993), Allen and Gale (1994), Caballero (2002), Blanchard et al. (2016), IMF (2017)). The primary motivation for state-contingent public assets, according to this literature, is to increase risk-sharing opportunities. Instead, in this study, the motivation for state-contingent public assets is the optimal public provision of liquidity.

The most relevant state-contingent public assets are index-linked securities, bonds whose payments are linked to an inflation index, a commodity price (e.g. oil, gold), or to a reference interest rate (Allen and Gale (1994)). Other state-contingent public assets indexed, for example, to GDP, government revenues or nominal wages are quantitatively minor (IMF (2017)).

The current study provides a framework to analyze relevant comparative statics regarding the ownership of state-contingent public assets. In particular, the model assesses the effects of changes in the domestic liquidity needs, the state's fiscal capacity, and the foreign willingness to pay for public debt on the ownership of state-contingent public assets.

The rest of the paper is structured as follows. Section 2 lays out the model and the key result on wasted liquidity. Section 3 presents the normative analysis, discusses the optimal public debt structure with multiple assets, and examines the comparative statics in the model with multiple assets. Lastly, Section 4 concludes the paper.

1.1. Related Literature

This study is related to several strands of literature. First, it is related to research on the public provision of liquidity and the references cited in the previous subsection. These studies all examine the optimal quantity of debt and rule out constraints on fiscal capacity or multiple debt instruments. In contrast, this study abstracts from the quantity of debt and focuses on the fiscal capacity dimension, thus showing how the issuance of different debt instruments can improve the liquidity provision for a given quantity of debt.

Second, it is related to the literature on the shortage of safe assets (Caballero (2010), Caballero and Krishnamurthy (2009), Caballero and Farhi (Forthcoming), Gourinchas and Jeanne (2012), Gorton (2016)). In particular, it is closely related to the ESBies proposal by Brunnermeier et al. (2016), which identifies the lack of safe assets as the source of the problems in the European Union. The latter study proposes using a European debt agency, which would buy a portfolio of member nations public bonds, and then issue senior and junior tranches from this portfolio. In the current study, the optimal structure includes Arrow–Debreu securities, which can perform better than a senior (safe) and a junior (risky) tranche can. Moreover, the optimal public debt structure in the proposed model does not require being implemented by a supranational entity.

Third, it is related to the literature on state-contingent assets in public debt and the references cited in the previous subsection. The current study finds countercyclical features in the optimal public debt structure; the government issues assets that pay only when the need for aggregate liquidity in the economy is high. Thus, it is related to previous work studying countercylical state-contingent assets in public debt (Ebrahim and Tavakoli (2016)). This policy report highlights the various benefits of countercyclical state-contingent assets in public debt and gives the two operative example of this asset category in public debt; namely, Grenada's hurricane clauses and the Agence Française de Développement countercyclical loan portfolio. In contrast, the current study solves for the optimal public debt structure and concentrates on the liquidity benefit of this asset category.

Finally, this study contributes to the literature on public debt ownership (Broner et al. (2010), Erce (2012), Gennaioli et al. (2014), Broner et al. (2014), Brutti and Saure (2016)). Most other studies concentrate on the composition of the investor base and on default incentives, especially with regard to creditor discrimination. Instead, this study focuses on how heterogeneous investors influence the optimal public debt structure, and introduces heterogeneous debt instruments into the analysis.

2. Model

This section describes a stylized economy where public debt provides liquidity to the domestic private sector, the government's repayment ability is risky, and foreign investors demand public debt. Here, investment and domestic and foreign demand for public debt are described, and key comparative statics are presented.

2.1. Environment with Risky Fiscal Capacity

Consider a three period economy, with time indexed $t = 0, 1, 2$, and a single good. The domestic economy is populated by a unit mass of consumers and a unit mass of entrepreneurs. The latter have an investment opportunity, are risk-neutral, and do not discount future payoffs. Abroad, foreign investors are also risk-neutral and their discount factor, denoted by β^*, satisfies $\beta^* > 1$.

The timing is as follows. At period 0, entrepreneurs have wealth A and choose an initial investment scale, I, in a project of variable scale. They borrow from consumers using a standard debt contract to start a project of scale $I > A$.

At period 1, the projects are hit by an aggregate liquidity shock, which leaves the projects requiring an injection of σ units of good per unit of initial investment to be able to continue. The liquidity shock can take two values: $\{\sigma_L, \sigma_H\}$, where $\sigma_L < \sigma_H$. After the projects are hit by the liquidity shock, the entrepreneurs decide the continuation scale of the projects $\chi \in [0, 1]$. Because all entrepreneurs are identical and the uncertainty is aggregate, all projects and continuation scales are the same.

At period 2, the projects give a private return to entrepreneurs of $R > 1$ for each unit of investment carried through to period 2. Of this return, only $\rho < R$ is pledgeable to consumers.

The borrowing at period 1 warrants further discussion. Entrepreneurs borrow from consumers to cover their liquidity needs at period 1. Throughout this paper, the following parametric condition holds:

$$\sigma_L < \rho < \sigma_H < R. \tag{1}$$

This condition has two important implications. First, if the liquidity shock is low, $\sigma = \sigma_L$, then the projects are self-financeable using the pledgeable part of the return, known as inside liquidity. Entrepreneurs can write a (senior) debt contract with consumers and promise to repay σ_L per unit of investment at period 2 once the projects reach completion.

Second, if the liquidity shock is high, $\sigma = \sigma_H$, entrepreneurs can only raise funds up to $\rho\chi I$ using a senior debt contract. To raise the remaining $(\sigma_H - \rho)\chi I$, entrepreneurs need collateral, ℓ, known as outside liquidity. Hence, the collateral constraint is given by:

$$(\sigma_H - \rho)\chi I \leq \ell, \tag{2}$$

The higher the need for outside liquidity, $\sigma_H - \rho$, the more collateral consumers require from entrepreneurs before lending to them at period 1.

As collateral, entrepreneurs use the returns from one-period public bonds issued by the government at date 0. There are no other securities available for entrepreneurs to use as collateral. This acts as a simplifying assumption in the model. All the results still hold if entrepreneurs have access to other assets, including foreign assets, as long as these do not completely fulfill entrepreneurs' liquidity needs. In other words, if there is room for public provision of liquidity using public debt, there is room to study the optimal structure of public debt.

The bond supply is fixed and normalized to one. The public debt market is open to foreign investors, who buy bonds as a saving vehicle. Unlike entrepreneurs, they do not have a liquidity motive to hold public debt.

The government issues the bond at period 0 and receives the bond price q, which it transfers to domestic consumers. It commits to repay the debt, and redeems the bond at period 1 by taxing consumers and repaying bondholders the bond's face value. The government's taxation power or fiscal capacity at period 1, denoted as η, is uncertain, and can take two values: $\{\eta_L, \eta_H\}$, where $\eta_L < \eta_H$[2].

Furthermore, to ensure that both types of investors hold part of the public debt, η_L satisfies the following parametric condition:

$$\eta_L > \frac{(\sigma_H - \rho)\left[A - (\beta^\star - 1)\mathbb{E}\left[\eta(\omega)\right]\right]}{1 - \mathbb{E}\left[\rho - \sigma(\omega)\right]} \tag{3}$$

where ω denotes each of the four states of the world in table 1 and the expectations are with respect to period's 1 uncertainty described next.

Uncertainty in the model only occurs at period 1, when the liquidity shock and the fiscal capacity shocks are realized. There are four states of the world, ω, depending on whether σ is equal to σ_H or σ_L, and whether η is equal to η_H or η_L. Table 1 summarizes the possibilities. The associated probabilities for each state are $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$. The model imposes no conditions on the magnitudes of these probabilities.

2.2. Demand from Foreign and Domestic Investors

The demand for bonds from foreign investors, z^F, is perfectly elastic at $q = \beta^* \Pi$, which is their valuation of the bond. Under parametric condition (3), foreign investors hold part of the bond issued and their valuation determines the bond price.

[2] Equivalently the government can issue b bonds that pay $\frac{\eta(\omega)}{b}$ in each state of the world ω. Nothing in the study changes in that case. However, because entrepreneurs use the period 1 returns from the bonds as collateral the optimal amount of bonds b is indeterminate. In other words, issuing one bond that repays $\eta(\omega)$ in each state of the world ω is equivalent to issuing b bonds, where each repays $\frac{\eta(\omega)}{b}$ in each state of the world ω.

	η_H	η_L
σ_H	State 1	State 2
σ_L	State 3	State 4

Table 1: **Uncertaninty in the Model** There are four states of the world $\omega = \{\omega_{HH}, \omega_{HL}, \omega_{LH}, \omega_{LL}\}$. The first letter in the subindex refers to the liquidity shock, the second letter refers to the fiscal capacity.

Entrepeneurs maximize their expected net return from the project, which is the illiquid return $R - \rho$ of the investment scale carried through. To maximize the initial investment scale of the project, entrepreneurs assign consumers all pledgeable returns ρ.

Their problem is given by:

$$\begin{aligned} max_{\{I,\chi(\omega),z\}} \quad & \mathbb{E}\{(R-\rho)\chi(\omega)I\} && (4) \\ s.t \quad \mathbb{E}\{(\rho - \sigma(\omega))\,\chi(\omega)I\} + \Pi z \quad & \geq \quad I - A + qz && (5) \\ (\sigma_H - \rho)\,\chi_1 I \quad & \leq \quad \eta_H z && (6) \\ (\sigma_H - \rho)\,\chi_2 I \quad & \leq \quad \eta_L z && (7) \end{aligned}$$

where $\chi(\omega) \in [0, 1]$ is the state ω continuation scale chosen at period one, χ_1 and χ_2 are the continuation scales in states 1 and 2, Π is the expected payoff of the public bond, equal to $\Pi = \mathbb{E}\eta(\omega)$, q is the bond price, and z is the quantity of bonds demanded.

The first constraint is the consumers' participation constraint at period 0. Consumers lend entrepreneurs funds to start the project $I - A$ and purchase collateral, qz. For consumers to be willing to do so, they must expect to at least break even. Consumers receive the net pledgeable return and the return from the bond, given by the left-hand side of equation (5).

The next two constraints of the problem are the collateral constraints. They state that the amount borrowed from consumers at period 1 must be collateralized by the liquid returns of the public bond. Collateral constraints are only relevant when the project is hit by the high liquidity shock, because it is only then that pledgeable returns from the project are not sufficient to raise fresh funds at period 1.

It is possible that constraint (6) is binding and (7) is slack. This happens if entrepreneurs decide to downsize the project in state 2 and continue at full scale only when the fiscal capacity is high in state 1. Appendix A studies this case.

This section focuses on the case when entrepreneurs always continue at full scale. Here, constraint (7), and not (6), is binding, and the domestic demand for bonds is given by the following expression:

$$z = \frac{(\sigma_H - \rho)\,\chi_2 I}{\eta_L}. \tag{8}$$

Because the bond price $q = \beta^{\star}\Pi$ is positive, the consumers' participation constraint is also binding. Using the demand for bonds and the bond price in (5) gives the initial level of the investment, as follows:

$$I = \frac{A}{1 - (\rho - \sigma_L)(\lambda_3 + \lambda_4) - (\rho - \sigma_H)(\lambda_1 + \lambda_2\chi_2) + \frac{\chi_2(\sigma_H - \rho)}{\eta_L}(\beta^{\star} - 1)\Pi}. \tag{9}$$

This model features the scale-liquidity trade-off present in Holmstrom and Tirole (1998). Indeed, $I'(\chi_2) < 0$, which implies that entrepreneurs who, at period 0, want to hold more liquidity in order to withstand future liquidity shocks, must choose a lower initial investment scale.

Given the optimal continuation scale χ_2, equations (8) and (9) denote the demand for bonds and the initial investment scale, respectively. To find χ_2, we set up the entrepreneurs' problem as a unit cost minimization problem, as follows:

$$min_{\chi_2}\, c(\chi_2; \Phi) \equiv \frac{1 + \sigma_L(\lambda_3 + \lambda_4) + \sigma_H(\lambda_1 + \lambda_2\chi_2) + \frac{\chi_2(\sigma_H - \rho)}{\eta_L}(\beta^{\star} - 1)\Pi}{\lambda_1 + \lambda_2\chi_2 + \lambda_3 + \lambda_4}, \tag{10}$$

where Φ is a vector of parameters $\Phi \equiv (\sigma_H, \sigma_L, \eta_H, \eta_L, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \beta^{\star})$. This is a linear problem. Hence, entrepreneurs choose to downsize or continue at full scale, depending on the cost of liquidity hoarding. They continue at full scale in states 1 and 2 ($\chi_1 = \chi_2 = 1$) if $\beta^{\star} \leq \bar{\beta}^{\star}$, and prefer to downsize in state 2 ($\chi_2 = 0$) otherwise. The threshold $\bar{\beta}^{\star}$ is equal to:

$$\bar{\beta}^{\star} = 1 + \frac{\eta_L\eta_H\lambda_2}{(\sigma_H - \rho)\Pi}\left[\frac{1 + (\sigma_L - \sigma_H)(\lambda_3 + \lambda_4)}{(\eta_H - \eta_L)(\lambda_1 + \lambda_3 + \lambda_4) - \eta_L\lambda_2}\right] \tag{11}$$

The next subsection characterizes investment when the above threshold is satisfied. Appendix A characterizes the equilibrium in the opposite case.

2.3. Equilibrium Investment and Wasted Liquidity

Using the fact that the project is always continued in (9), we have the following closed-form solution for investment:

$$I = \frac{A}{1 - \mathbb{E}\left[\rho - \sigma(\omega)\right] + (\sigma_H - \rho)(\beta^\star - 1)\left[(\lambda_2 + \lambda_4) + (\lambda_1 + \lambda_3)\frac{\eta_H}{\eta_L}\right]}. \quad (12)$$

Investment is proportional to entrepreneurs' initial wealth A, and is multiplied by the equity multiplier $k \equiv \frac{1}{1-\mathbb{E}[\rho-\sigma(\omega)]+(\sigma_H-\rho)(\beta^\star-1)\left[(\lambda_2+\lambda_4)+(\lambda_1+\lambda_3)\frac{\eta_H}{\eta_L}\right]} > 1$, which defines the maximum amount by which entrepreneurs can leverage their initial capital.

The maximum leverage per unit of own capital is increasing in the pledgeable return ρ, decreasing in the total expected cost of the project $1 + \mathbb{E}\sigma(\omega)$, and decreasing in the need for outside liquidity $\sigma_H - \rho$. Most importantly, for the purposes of this study, it is decreasing in the cost of liquidity hoarding for the private sector: the last term in the denominator of equation (12).

Two points are worth highlighting with regard to this last term. First, for liquidity hoarding to have an effect on the equity multiplier and to decrease investment, it is key that public debt is sold at a premium. If $\beta^\star = 1$, investment equals $\frac{A}{1-\mathbb{E}[\rho-\sigma(\omega)]}$ and it is independent from the cost of liquidity hoarding, since liquidity is sold at par. However, in reality, rarely is liquidity sold at par, especially in economies where bond prices are high.

Second, the cost of liquidity hoarding is increasing in the ratio of fiscal capacities $\frac{\eta_H}{\eta_L}$. The latter is a novel relationship provided by the proposed model. This ratio parameterizes the amount of *wasted liquidity*, in other words, the magnitude of unused liquidity entrepreneurs are forced to purchase for states in which they do not need it. Entrepreneurs' private investment is decreasing in wasted liquidity: the more expensive liquidity hoarding is for entrepreneurs, the lower is the investment scale they must choose.

The following subsection explores the comparative statics of the most relevant parameters of the model on investment and on domestic and foreign demand for public debt. Investment is given in equation (12), while the following expressions describe domestic and foreign demand for bonds:

$$\begin{aligned} z &= \frac{A(\sigma_H - \rho)}{\eta_L\left[1 - \mathbb{E}\left[\rho - \sigma(\omega)\right]\right] + (\sigma_H - \rho)(\beta^\star - 1)\Pi} \\ z^F &= 1 - z. \end{aligned}$$

2.4. Comparative Statics

An increase in σ_H and a simultaneous decrease in σ_L such that the total expected cost of the project $1 + \mathbb{E}\sigma(\omega)$ remains unchanged, decreases investment, increases domestic demand for public debt, and decreases foreign demand for public debt. This scenario captures an increase in domestic needs for outside liquidity, keeping the expected cost of productive investments constant. The higher the need for outside liquidity, the more collateral entrepreneurs need to hoard and, thus, the lower is the initial investment scale they choose. An increase in the need for outside liquidity increases the need for public liquidity, increasing the demand for bonds at home. By market clearing, an increase in domestic demand for public debt implies a decrease in foreign demand for public debt.

An increase in foreign investors' patience $\beta^\star$ decreases investment, decreases domestic demand for public debt, and increases foreign demand. This scenario captures an increase in foreign willingness to pay for public debt. An increase in foreign willingness to pay increases the cost of liquidity hoarding, forcing entrepreneurs to choose a smaller initial investment scale. The integrated public debt market assumption implies that a higher $\beta^\star$ increases the price of public bonds for domestic entrepreneurs, crowding out domestic demand for public debt. By market clearing, less demand by domestic investors implies greater demand by foreigners.

Finally, an increase in the low fiscal capacity, η_L, increases investment, decreases the public debt held by domestic investors, and increases that held by foreigners. This scenario captures a decrease in the variability of fiscal capacity. Investment increases because a higher η_L decreases wasted liquidity. As described in the previous section, lower wasted liquidity decreases the cost of liquidity hoarding for entrepreneurs, which allows them to increase their initial investment scale. Intuitively, if public debt now gives a higher return, entrepreneurs need less of it to save for the liquidity shock. By market clearing, the remainder is held by foreign investors.

The following derivatives summarize the comparative statics in this section: (i) $\frac{\partial z}{\partial(\sigma_H-\rho)} > 0$, $\frac{\partial z^F}{\partial(\sigma_H-\rho)} < 0$, and $\frac{\partial I}{\partial(\sigma_H-\rho)} < 0$ for a given total expected cost of the project $1 + \mathbb{E}\sigma(\omega)$; (ii) $\frac{\partial z}{\partial\beta^\star} < 0$, $\frac{\partial z^F}{\partial\beta^\star} > 0$, and $\frac{\partial I}{\partial\beta^\star} < 0$; and (iii) $\frac{\partial z}{\partial\eta_L} < 0$, $\frac{\partial z^F}{\partial\eta_L} > 0$, and $\frac{\partial I}{\partial\eta_L} > 0$.

Because investment decreases with wasted liquidity, there is room for the government to improve the provision of liquidity in the economy. If the government issues one or more assets that lower wasted liquidity, private investment in the economy will increase. The next section studies this possibility, and solves for the optimal structure of public debt in this environment.

3. Optimal Structure of Public Debt

In this section, the government can issue more than one asset as part of its public debt issuance. Because there are four states of the world, the government issues four assets, denoted by $j = \{1, 2, 3, 4\}$. The return of asset j in state of the world i is denoted by x_{ji}.

3.1. Entrepreneur's Problem with More than One Asset

The entrepreneurs' problem with more than one asset is similar to the one in the previous section:

$$\begin{aligned} & max_{\{I,\chi(\omega),z_j\forall j\}} \quad \mathbb{E}\left\{(R-\rho)\chi(\omega)I\right\} && (13) \\ s.t \quad & \mathbb{E}\left\{\left(\rho-\sigma(\omega)\right)\chi(\omega)I\right\}+\sum_{j=1}^{4}\Pi_j z_j \;\geq\; I-A+\sum_{j=1}^{4}q_j z_j && (14) \\ & \left(\sigma_H-\rho\right)\chi_1 I \;\leq\; \sum_{j=1}^{4}x_{j1}(\omega)z_j && (15) \\ & \left(\sigma_H-\rho\right)\chi_2 I \;\leq\; \sum_{j=1}^{4}x_{j2}(\omega)z_j && (16) \end{aligned}$$

where q_j denotes the price of asset j, z_j the quantity of asset j demanded by the entrepreneurs, and $\Pi_j = \mathbb{E}\left[x_j(\omega)\right]$ is the expected payoff of asset j.

There are two key differences between this problem and that described in equations (4) to (7). First, entrepreneurs now have four assets instead of only one. This appears in the participation constraint and in the collateral constraints above. Second, and most importantly, asset j's expected payoff Π_j depends on payoffs x_{ji}, and these can be smaller than the fiscal capacities in the various states of the world.

The model's solution proceeds as follows. The next three subsections analyze the case in which entrepreneurs continue the project fully in all states and foreign investors hold a part of all assets. Appendix C shows the parametric conditions under which these two conditions are met, and solves for the optimal public debt structure when they are violated. Subsection 3.4 below discusses the results in Appendix C.

If the project is continued fully in all states, entrepreneurs need to hoard their purchases of collateral for states 1 and 2. Denote by $c\ell_i$ the unit cost of liquidity in state i. Then, entrepreneurs choose assets j that minimize $c\ell_i$; that is, $c\ell_i = min\{\frac{q_j - \Pi_j}{x_{ji}}\} \forall j$. Suppose that assets j and j' minimize $c\ell_i$ and provide enough liquidity for these states. In this case, because entrepreneurs do not wish to purchase more collateral than necessary, the demand for the other assets is zero.

The collateral constraint, with equality, gives the local demand for the relatively cheap assets:

$$z_j = \frac{I(\sigma_H - \rho)}{x_{j1}} \tag{17}$$

$$z_{j'} = \frac{I(\sigma_H - \rho)}{x_{j2}}. \tag{18}$$

Substituting these equations into the budget constraint gives a closed-form solution for investment:

$$I = \frac{A}{1 - \mathbb{E}(\rho - \sigma(\omega)) + (c\ell_1 + c\ell_2)(\sigma_H - \rho)}. \tag{19}$$

which depends only on the cost of liquidity, $c\ell_1$ and $c\ell_2$, and on the parameters. As in the previous section, investment is decreasing in the cost of liquidity hoarding.

Finally, the last piece of the model is the foreign valuation of assets. The price of each asset j must satisfy $q_j \geq \beta^\star \Pi_j$, with strict equality, if foreign investors hold part of asset j.

Next, we incorporate the economy's behavior described in this subsection in a planner's problem. The planner chooses the payoffs of each asset in each state of the world in order to maximize total welfare in the economy.

3.2. Government's Problem

The government's objective is to maximize domestic welfare, which is equal to the sum of entrepreneurs' and consumers' utility of consumption in the three periods. Both have linear utility of consumption, and neither of them discount future payoffs.

Entrepreneurs consume the expected rent from their investment at period 2, $(R-\rho)I$. At period 0, consumers lend $A - I + \sum_{j=1}^{4} q_j z_j$ of their endowment E to entrepreneurs. Consumers also lend to entrepreneurs at period 1. Then, consumers' expected return is equal to $\mathbb{E}\left[(\rho-\sigma(\omega)I\right]+\sum_j \Pi_j z_j$. In addition, at period 0, they obtain the proceeds from the total asset issuance $\sum_j q_j$, and are taxed the face value of each asset j in order to repay asset holders.

Thus, the utility of entrepreneurs, consumers, and total welfare are given by:

$$\begin{aligned} U^E &= (R-\rho)I \\ U^C &= E-I+A-\sum_j q_j z_j+\sum_j \Pi_j z_j+\mathbb{E}\left[(\rho-\sigma(\omega)I\right]+\sum_j (q_j-\Pi_j) \\ W &= E+A+\left[R-\mathbb{E}\sigma(\omega)-1\right]I+\sum_j (q_j-\Pi_j)(1-z_j), \end{aligned}$$

respectively.

The following expression for welfare is identical to that above:

$$W=E+A+\left[R-\mathbb{E}\sigma(\omega)-1\right]I+\sum_j (q_j-\Pi_j)z_j^F, \tag{20}$$

where $1-z_j$ becomes z_j^F. This expression is intuitive. The government wants to maximize the total net surplus from the investment and the premiums $\sum_j q_j-\Pi_j$ obtained from foreign investors. The liquidity premium paid by entrepreneurs to consumers is a transfer across agents, which cancels out in the welfare calculation, and only the premiums coming from abroad matter to the welfare in the economy.

The government's issuance of each asset is fixed and normalized to one. The government is constrained by its fiscal capacity. The sum of the payoffs of all assets in each of the states cannot be larger than the government's fiscal capacity in that state:

$$\begin{aligned} \sum_j x_{ji} &\leq \eta_H \;\; if \;\; i=1,3 \\ \sum_j x_{ji} &\leq \eta_L \;\; if \;\; i=2,4. \end{aligned}$$

The government solves the following problem:

$$max_{c\ell_1,c\ell_2,x_{ji}\,\forall i,j} \quad \left[R-\mathbb{E}\sigma(\omega)-1\right]I(c\ell_1,c\ell_2)+\textstyle\sum_j (q_j-\Pi_j)z_j^F(c\ell_1,c\ell_2) \tag{21}$$

$$s.t \qquad \textstyle\sum_j x_{ji} \leq \eta_H \;\; if \;\; i=1,3 \tag{22}$$

$$\sum_j x^{ji} \leq \eta_L \;\; if \;\; i = 2, 4 \tag{23}$$

$$I(c\ell_1, c\ell_2) = \frac{A}{1 - \mathbb{E}(\rho - \sigma(\omega)) + (\sigma_H - \rho)(c\ell_1 + c\ell_2)} \tag{24}$$

$$c\ell_1 = min\{\frac{q_j - \Pi_j}{x_{ji}}\} \;\; \forall j \tag{25}$$

$$c\ell_2 = min\{\frac{q_j - \Pi_j}{x_{ji}}\} \;\; \forall j \tag{26}$$

$$0 \leq z_j^F(c\ell_1, c\ell_2) \leq 1 \tag{27}$$

$$q_j \geq \beta^\star \Pi_j \;\; with \;\; inequality \;\; only \;\; if \;\; z_j^F(c\ell_1, c\ell_2) = 0, \tag{28}$$

where endowment $E + A$ drops from the objective function because it cannot be changed by the government. The first and second constraints are the fiscal capacity constraints. The third constraint internalizes the entrepreneurs' investment decision, which is decreasing in the unit of liquidity $c\ell_1$ and $c\ell_2$. The fourth and fifth constraints define the unit cost of liquidity in states 1 and 2. The sixth constraint imposes the market-clearing and short-selling constraints: the foreign demand for asset j cannot be larger than the supply of asset j, and demand cannot be negative. The last constraint prices the assets. The foreign investors' valuation determines the asset prices if they hold the asset, or is strictly above if they do not.

The approach I take to solving this problem is to solve a slightly modified version of it, first, with a modified objective and, second, expressed in terms of fiscal capacity allocations at home and abroad in each state of the world. This greatly simplifies the analysis. Then, I present a combination of assets that implements the optimal fiscal capacity allocation found in the modified problem. Finally, I verify that the proposed combination of assets satisfies constraints (22) to (28), and that the objectives in both problems take the same value.

The modified objective in terms of fiscal capacity allocations is an upper bound on welfare. The government, when choosing the liquidity premiums received from abroad, is constrained by (28). The maximum the government can obtain from foreigners for each asset j is $(\beta^\star - 1)\Pi_j$, which happens when all assets are held exclusively by foreign investors.

The government's modified problem is given as follows:

$$max_{c\ell_1, c\ell_2, FC_{Ei}, FC_{Fi} \forall i} \;\; [R - \mathbb{E}\sigma(\omega) - 1]\, I(c\ell_1, c\ell_2) + (\beta^\star - 1) \sum_i \lambda_i FC_F^i \tag{29}$$

$$s.t \quad FC_E^i + FC_F^i \leq \eta_H \;\; if \;\; i = 1, 3 \tag{30}$$

$$FC_E^i + FC_F^i \leq \eta_L \;\; if \;\; i = 2, 4 \tag{31}$$

$$I(c\ell_1, c\ell_2) = \frac{A}{1 - \mathbb{E}(\rho - \sigma(\omega)) + (\sigma_H - \rho)(c\ell_1 + c\ell_2)} \tag{32}$$

$$I(\sigma_H - \rho) \leq F_{Ei} \;\; if \;\; i = 1,2 \tag{33}$$

$$c\ell_1 \geq \lambda_1(\beta^\star - 1) \tag{34}$$

$$c\ell_2 \geq \lambda_2(\beta^\star - 1), \tag{35}$$

where FC_{Ei} and FC_{Fi} denote the fiscal capacities allocated to entrepreneurs and foreigners, respectively, in each state of the world i. The first two constraints state that the sum of the two fiscal capacities cannot be larger than the fiscal capacity available in each state of the world. The next constraint gives the expression for investment. The fourth constraint rewrites the collateral constraint in terms of the new variables, and is isomorphic to equation (2). The last two constraints give the lower bound for the liquidity premium in both states of the world. They require further discussion.

Constraint (28) imposes a lower bound on the unit cost of liquidity in state 1 $c\ell_1$, namely $c\ell_1 \geq \frac{(\beta^\star-1)\Pi_j}{x_j^1}$ for all assets j. If foreigners hold part of asset j, this expression holds with equality. Using the expression for Π_j, we have $c\ell_1 = (\beta^\star - 1)(\lambda_1 + \sum_{i=2,3,4} \frac{\lambda_i x_{ji}}{x_{ji}})$. This expression has a minimum value when $x_{ji} = 0$, for states $i = 2,3,4$. This is the same as constraint (34). Similar reasoning applies to (35).

Written in this way, the solution to the problem is simple. It proceeds in four steps. First, because investment is decreasing in the unit cost of liquidity, the government chooses the minimum $c\ell_1$ and $c\ell_2$, and constraints (34) and (35) hold with equality. This determines the optimal investment, as follows:

$$I = \frac{A}{1 - \mathbb{E}(\rho - \sigma(\omega)) + (\sigma_H - \rho)(\beta^\star - 1)(\lambda_1 + \lambda_2)} \equiv I^\star. \tag{36}$$

Second, the government's objective is to provide a necessary level of liquidity only to entrepreneurs and to maximize the financial flows coming from abroad. Therefore, constraint (33) holds with equality in both states of the world, and $FC_{E1} = FC_{E2} = I^\star(\sigma_H - \rho)$. Third, the fiscal capacity constraints hold with equality, or else fiscal capacity would be wasted. Hence, $FC_{F1} = \eta_H - I^\star(\sigma_H - \rho)$ and $FC_{F2} = \eta_L - I^\star(\sigma_H - \rho)$.

Finally, for states $i = 3,4$, the government allocates all fiscal capacity abroad in order to maximize foreign financial flows. Thus, $FC_{E3} = FC_{E4} = 0$, $FC_{F3} = \eta_H$, and $FC_{F4} = \eta_L$.

To summarize, the optimal fiscal capacity allocation is given by:

(i) In state 1, $FC_{E1} = I^\star(\sigma_H - \rho)$ and $FC_{F1} = \eta_H - I^\star(\sigma_H - \rho)$

(ii) In state 2, $FC_{E2} = I^\star(\sigma_H - \rho)$ and $FC_{F2} = \eta_L - I^\star(\sigma_H - \rho)$

(iii) In state 3, $FC_{E3} = 0$ and $FC_{F3} = \eta_H$

(iv) In state 4, $FC_{E4} = 0$ and $FC_{F4} = \eta_L$.

Arrow–Debreu (AD) securities for states 1 and 2 implement the optimal fiscal capacity allocation for the states when entrepreneurs need liquidity. Then, assets that pay only when liquidity needs are low implement the above optimal fiscal capacity allocation:

$$\begin{aligned}
(x_{11}, x_{12}, x_{13}, x_{14}) &= (\eta_H, 0, 0, 0) \quad (37)\\
(x_{21}, x_{22}, x_{23}, x_{24}) &= (0, \eta_L, 0, 0) \quad (38)\\
(x_{31}, x_{32}, x_{33}, x_{34}) &= (0, 0, x_{33}, x_{34}) \quad (39)\\
(x_{41}, x_{42}, x_{43}, x_{44}) &= (0, 0, x_{43}, x_{44}), \quad (40)
\end{aligned}$$

where it must be the case that $x_{33}+x_{43} = \eta_H$ and $x_{34}+x_{44} = \eta_L$. Indeed, the returns in states 3 and 4 of the assets held abroad only are not determined. Because foreign investors are risk-neutral and do not have a liquidity motive to hold foreign public debt, they will hold these assets regardless of their payoffs in the two states.

The demands for assets 1 and 2 that pay in the states of the world when entrepreneurs need collateral are:

$$z_1 = \frac{I^\star(\sigma_H - \rho)}{\eta_H} \qquad z_2 = \frac{I^\star(\sigma_H - \rho)}{\eta_L} \quad (41)$$

$$z_1^F = 1 - \frac{I^\star(\sigma_H - \rho)}{\eta_H} \qquad z_2^F = 1 - \frac{I^\star(\sigma_H - \rho)}{\eta_L}, \quad (42)$$

respectively.

Multiplying these by the assets' returns yields the fiscal capacity allocations in (i) and (ii) above. The domestic and foreign demands for assets 3 and 4 are $z_2 = z_3 = 0$ and $z_3^F = z_4^F = 1$, respectively, because they pay only in those states in which entrepreneurs do not need collateral. Multiplying these by the asset returns yields (iii). Furthermore, the unit cost of liquidity for assets 1 and 2 are $c\ell_1 = (\beta^\star - 1)\lambda_1$ and $c\ell_2 = (\beta^\star - 1)\lambda_2$, respectively, because they are AD securities.

This is the solution to the initial problem in equations (21) to (27). Indeed, AD securities satisfy constraints (22) to (28). Moreover, because all assets are held, at least partly, abroad, $q_j - \Pi_j = (\beta^\star - 1)\Pi_j$, for all assets j.

In addition, the original objective (21) takes the same value as the modified objective (29) under AD securities.

The next subsection explores comparative statics of the most relevant parameters of the model on the domestic and and foreign holdings of assets 1 and 2, given in equations (41) and (42), respectively.

3.3. Comparative Statics of the Model with More than one Asset

An increase in σ_H and a simultaneous decrease in σ_L such that the total expected cost of the project $1 + \mathbb{E}\sigma(\omega)$ remains unchanged, increases the domestic holdings of assets 1 and 2 and decreases the foreign holdings of these assets. This scenario captures an increase in domestic needs for outside liquidity, keeping the expected cost of productive investments constant. Intuitively, an increase in domestic needs for outside liquidity increases the domestic demand for the assets that provide this outside liquidity. By market clearing, the share of these assets held by foreign investors decreases.

An increase in fiscal capacity η_L decreases the domestic holdings of asset 2, because z_2 is decreasing in η_L. By market clearing, an increase in η_L increases the foreign holdings of those assets. The reason why domestic holdings decrease is that each asset now pays more and, thus, a lower share of asset is needed at home. The same reasoning applies to an increase in fiscal capacity η_H for the domestic and foreign holdings of asset 1. These scenarios capture improvements in the state's fiscal capacity.

Finally, an increase in international patience $\beta^\star$ decreases the domestic demand for assets 1 and 2 and increases the share of these assets held abroad. This scenario captures an increase in foreign willingness to pay for public debt. Crowding-out is still present, even if the government issues more than one asset. A higher $\beta^\star$ increases the cost of liquidity hoarding, which forces entrepreneurs to choose a lower initial investment scale, which, in turn, means they need a lower amount of liquidity.

Despite the crowding-out effect, capital controls that ban financial flows from foreigners are welfare-reducing. As Appendix D shows, under capital controls, entrepreneurs are better off and consumers are worse off. Overall, welfare decreases under capital controls. Intuitively, although investment increases, the economy gives up the premiums it receives from selling public assets abroad. This is similar to the findings in Bolton and Jeanne (2011), regarding the gains from financial integration.

3.4. Discussion of Assumptions

To conclude the optimal structure of public debt, this subsection discusses the two key assumptions made so far in this section.

The first assumption, that entrepreneurs continue the projects in all states, ensures that they require public assets to hoard liquidity. Appendix C shows that if condition (C.1) is violated, liquidity is too expensive and entrepreneurs downsize whenever the projects are hit by the high liquidity shock, $\sigma = \sigma_H$. Evidently, if entrepreneurs do not use public assets to hoard liquidity, then there is no role for optimal liquidity provision. However, in reality, investors value public debt for its liquidity, which makes studying the optimal structure of public debt for liquidity purposes relevant.

The second assumption, that foreign investors hold a part of all assets, ensures that wasted liquidity is priced in public assets' prices. Appendix C shows that if conditions (C.2) or (C.3) are violated, only entrepreneurs hold public assets and the cost of liquidity, $c\ell$, only depends on valuable returns for entrepreneurs, rendering wasted liquidity unpriced. However, in reality, public debt is held by a large variety of investors, particularly foreign investors (Arslanalp and Tsuda (2014)), whose valuation of assets' returns impact asset prices. Thus, studying the optimal structure of public debt in open debt markets is of practical relevance.

4. Conclusion

This study examines the optimal public debt structure when governments can issue multiple assets, the state's fiscal capacity is uncertain, public debt has a liquidity purpose for domestic agents, and public debt markets are open to foreign investors. The optimal policy is to tranch governments' risky fiscal capacity, and then to issue state-contingent assets that pay only when the aggregate liquidity needs in the economy are high.

By issuing state-contingent assets, governments bring wasted liquidity to zero. Arrow–Debreu securities, which pay zero when the aggregate liquidity needs in the economy are low, minimize the cost of liquidity hoarding for the private sector. This allows the private sector to increase its investment, which increases welfare in the economy.

The welfare gains from issuing state-contingent assets are larger the more expensive liquidity is. Thus, the benefits from state-contingent assets are highest, for example, in low-interest rate environments, in which debt prices are high or in countries where foreign valuation of public debt drives its price up.

The optimal structure of public debt exploits differences in the composition of the market's investors by catering to the different motives for holding public debt. The Arrow–Debreu securities satisfy the demand for cheap liquidity at home. The remaining assets cater to foreigner investors' risk-neutrality and their saving motive for holding public debt.

Moreover, the study shows that in order to provide optimal liquidity at home using public debt, the government does not need to segment markets or tax foreigners. Instead, an appropriate asset design, which results in partial market segmentation, maximizes domestic investment and welfare.

This study provides support for the introduction of state-contingent assets in public debt and offers a framework to analyze relevant comparative statics regarding ownership of these assets. In particular, the model assesses the effects of changes in the domestic liquidity needs, the state's fiscal capacity, and the foreign willingness to pay for public debt on the ownership of public state-contingent assets.

There are many examples of state-contingent assets in public debt. However, with the exception of inflation-linked bonds, their quantitative importance in public debt markets is minor. Furthermore, many of them, including GDP-linked bonds and oil-linked bonds, are procyclical.

However, the optimal public debt structure in this study has countercyclical features; assets pay only when domestic investors' inside liquidity is insufficient. The only two examples of countercyclical state-contingent assets in public debt are Grenada's hurricane clauses and the Agence Française de Développement's countercyclical loan portfolio (Ebrahim and Tavakoli (2016)).

The results presented in this paper point to a number of promising avenues for future research. Understanding the reasons behind the relatively few countercyclical state-contingent assets in public debt markets is of practical relevance to inform public debt management. Allowing for a lack of commitment and for sovereign default will add relevant trade-offs to the optimal public debt structure problem. Finally, in this study, the amount of public debt and the associated fiscal capacity are exogenous. Introducing a variable quantity of debt and endogenizing fiscal capacity will certainly add an important dimension to the optimal public debt structure.

Appendix A. Full Characterization of the Benchmark Model with One Asset

The main text of the paper focuses on the case when entrepreneurs's projects are always fully continued. This happens if:

$$\beta^\star \leq 1 + \frac{\eta_L \eta_H \lambda_2}{(\sigma_H - \rho)\Pi}\left[\frac{1 + (\sigma_L - \sigma_H)(\lambda_3 + \lambda_4)}{(\eta_H - \eta_L)(\lambda_1 + \lambda_3 + \lambda_4) - \eta_L \lambda_2}\right] \tag{A.1}$$

which ensures that $c(\chi_1 = \chi_2 = 1) < c(\chi_1 = 1, \chi_2 = 0)$, where c is unit cost given in equation (10).

This condition is in the main body of the paper and is reproduced here for convenience. If this condition does not hold, $\chi_1 = \chi_2 = 1$ does not minimize entrepreneurs's unit cost of investment. Indeed the solution to the problem in equation (10) is $\chi_2 = 0$ and entrepreneurs prefer to downsize in one state of the world. The unit cost of liquidity then equals:

$$\frac{1 + \sigma_L(\lambda_3 + \lambda_4) + \sigma_H \lambda_1 \chi_1 + \frac{\chi_1(\sigma_H - \rho)}{\eta_H}(\beta^\star - 1)\Pi}{\lambda_1 \chi_1 + \lambda_3 + \lambda_4}$$

Entrepreneurs choose χ_1 to minimize this cost.

The following cutoff for the foreign discount factor $\beta^\star$ determines the χ_1 decision:

$$\beta^\star \leq 1 + \frac{\lambda_1 \eta_H}{(\sigma_H - \rho)\Pi}\left[\frac{1}{\lambda_3 + \lambda_4} - (\sigma_H - \sigma_L)\right] \equiv \bar{\bar{\beta}}^\star \tag{A.2}$$

For $\beta^\star$ lower than $\bar{\bar{\beta}}^\star$, entrepreneurs choose $\chi_1 = 1$ and $\chi_2 = 0$ instead of $\chi_1 = \chi_2 = 0$ and for $\beta^\star$ higher than this cutoff entrepreneurs choose $\chi_1 = \chi_2 = 0$. Combining this with the condition for $\bar{\beta}^\star$ in the main body of the paper gives the following continuation levels:

$$\begin{aligned}
\chi_1 = \chi_2 = 1 \quad & if \quad \beta^\star \leq \bar{\beta}^\star \\
\chi_1 = 1, \chi_2 = 0 \quad & if \quad \bar{\beta}^\star < \beta^\star \leq \bar{\bar{\beta}}^\star \\
\chi_1 = \chi_2 = 0 \quad & if \quad \beta^\star > \bar{\bar{\beta}}^\star
\end{aligned}$$

The main body of the paper concentrates on the first case. This Appendix studies the other two cases. Let's start considering when $\chi_1 = 1$ and $\chi_2 = 0$. If $\chi_2 = 0$ the liquidity needed in state $i = 1$ pins down the entrepreneurs's demand for public debt: $z = I(\sigma_H - \rho)/\eta_H$. Two cases arise: one if foreign investors hold part of the public debt, the other one if only entrepreneurs buy the public debt. The following cutoff on fiscal capacity η_H distinguishes both cases:

$$\eta_H > \frac{(\sigma_H - \rho)(A - (\beta^\star - 1)\Pi)}{1 - \mathbb{E}(\rho - \chi(\omega)\sigma(\omega))} \tag{A.3}$$

When (A.3) holds entrepreneurs and foreign investors hold public debt. The valuation of the latter pins down its price: $q = \beta^\star\Pi$. Investment equals:

$$I = \frac{A}{1 - \mathbb{E}(\rho - \chi(\omega)\sigma(\omega)) + (\beta^\star - 1)(\sigma_H - \rho)\left[\lambda_1 + \lambda_3 + \frac{\eta_L}{\eta_H}(\lambda_2 + \lambda_4)\right]} \tag{A.4}$$

Investment depends on the ratio of fiscal capacities, so there is room for the government to improve investment by changing the structure of public debt.

Domestic demand for public debt equals $z = \frac{(\sigma_H - \rho)I}{\eta_H}$ and $z^F = 1 - z$.

If (A.3) does not hold all public debt is hold domestically, investment equals $I = \frac{\eta_H}{(\sigma_H - \rho)}$, and the premium $q - \Pi = A - \frac{\eta_H[1 - \mathbb{E}(\rho - \sigma(\omega)\chi(\omega))]}{\sigma_H - \rho}$.

Finally, $\chi_1 = \chi_2 = 0$ happens when (A.2) is not satisfied. In that case, $z = 0$ and foreign investors hold all public debt $z^F = 1$. The bond price $q = \beta^\star\Pi$ but it does not affect investment which equals $I = \frac{A}{1 - (\rho - \sigma_L)(\lambda_3 + \lambda_4)}$. Indeed, when hedging liquidity is expensive enough, entrepreneurs prefer to downsize in the states of the world when the high liquidity shock hits. Hence, they do not hedge liquidity and its cost does not affect them.

The main body of the paper also makes the following assumption on η_L:

$$\eta_L > \frac{(\sigma_H - \rho)\left[A - (\beta^\star - 1)\Pi\right]}{1 - \mathbb{E}\left[\rho - \sigma(\omega)\right]} \tag{A.5}$$

which ensures that public debt is held by domestic and foreign investors.

If this condition is not satisfied, then two cases arise. If the high liquidity shock is low enough, that is:

$$\sigma_H \leq \left[\frac{A}{\eta_L} - (\lambda_1 + \lambda_2)\right]^{-1}\left[\frac{\rho}{\eta_L}A - \rho + 1 + \frac{\lambda_2}{(\lambda_1 + \lambda_3 + \lambda_4)}\right] \tag{A.6}$$

then entrepreneurs continue the project at full scale in all states of the world $\chi_i = 1$ for all i. Hence they need to purchase bonds to cover the liquidity shock. Because fiscal capacity is small only domestic entrepreneurs hold public debt: $z = 1$, $z^F = 0$. They price the asset and the premium equals $q - \Pi = A - \frac{\eta_L[1-\mathbb{E}(\rho-\sigma(\omega))]}{\sigma_H-\rho}$. Investment is given by: $I = \frac{\eta_L}{\sigma_H-\rho}$. The liquidity premium and investment do not depend on wasted liquidity, they only depend on η_L, which is the assets' return that entrepreneurs use. Because foreign investors do not hold public debt the condition on $\beta^\star$ becomes irrelevant. The relevant condition is (A.6).

Finally, this section characterizes the equilibrium if this condition does not hold. In that case, the liquidity shock is too high and entrepreneurs prefer to not hedge. Hence they downsize in the states of the world when $\sigma = \sigma_H$: $\chi_1 = \chi_2 = 0$. Debt is held exclusively by foreign investors, $z = 0$ and $z^F = 1$ and the foreign asset valuation prices public debt $q = \beta^\star\Pi$. Investment equals $I = \frac{A}{1-(\rho-\sigma_L)(\lambda_3+\lambda_4)}$.

Appendix B. Closed Economy Equilibria

The problem for entrepreneurs is identical to the one in Section 2. In the closed-economy case, foreign investors cannot buy public debt but domestic consumers can. Domestic consumers' demand public debt is perfectly elastic at $q = \Pi$. This demand takes a similar form to the one from foreign investors. The only difference is that now the domestic discount rate equals 1.

Market clearing imposes that $z + z^C = 1$, where z^C is the demand from domestic consumers. Domestic consumers hold part of the public debt only if $z < 1$, which happens when the return η_L is high enough:

$$\eta_L > \frac{(\sigma_H - \rho)A}{1 - \mathbb{E}(\rho - \sigma(\omega))} \tag{B.1}$$

When domestic consumers hold part of the public debt $q = \Pi$ and liquidity does not sell at a premium. This implies that, if domestic consumers hold part of the public debt, entrepreneurs always fully continue their project: in both states 1 and 2, so $\chi_1 = \chi_2 = 1$. In that case, the equilibrium investment and demand for public debt equals:

$$I = \frac{A}{1 - \mathbb{E}(\rho - \sigma(\omega))} \tag{B.2}$$

$$z = \frac{(\sigma_H - \rho)I}{\eta_L} \tag{B.3}$$

$$z^C = 1 - z \tag{B.4}$$

In this model, when domestic consumers drive the liquidity premium to zero there is no room for the government to lower the cost of public liquidity provision further. If domestic consumers' discount factor were above 1, then it would be similar to the case considered in the main body of the paper with foreign investors.

Next, the appendix studies the case when condition (B.1) does not hold. In that case, only entrepreneurs hold public debt. Furthermore, entrepreneurs continue the project at full scale in both states of the world when σ_H realizes, $\chi_1 = \chi_2 = 1$ if the following condition on σ_H holds:

$$\sigma_H < \left[\frac{\eta_L}{A - (\lambda_1 + \lambda_2)}\right] \left[1 + \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_4} - (\rho - \sigma_L)(\lambda_3 + \lambda_4)\right] \tag{B.5}$$

In this case, entrepreneurs' demand drive a premium on public debt and $q - \Pi > 0$. Equilibrium investment, demand for public debt, and the price of debt are given by:

$$I = \frac{\eta_L}{\sigma_H - \rho} \tag{B.6}$$

$$z = \frac{(\sigma_H - \rho)I}{\eta_L} \tag{B.7}$$

$$q = \Pi + A - \frac{\eta_L(1 - \mathbb{E}(\rho - \sigma(\omega)))}{\sigma_H - \rho} \tag{B.8}$$

$$z^C = 0 \tag{B.9}$$

If condition (B.5) does not hold, then entrepreneurs downsize in state 2, $\chi_2 = 0$ and continue at full scale in state 1, $\chi_1 = 1$. The equilibrium is characterized by the following expressions:

$$I = \frac{\eta_H}{\sigma_H - \rho} \tag{B.10}$$

$$z = \frac{(\sigma_H - \rho)I}{\eta_H} \tag{B.11}$$

$$q = \Pi + A - \frac{\eta_H(1 - (\rho - \sigma_L)(\lambda_3 + \lambda_4) - (\rho - \sigma_H)\lambda_1)}{\sigma_H - \rho} \tag{B.12}$$

$$z^C = 0 \tag{B.13}$$

In this model, when entrepreneurs are the only buyers of public debt there is no room for lowering the cost of liquidity hoarding either. As equations (B.8) and (B.12) show, to lower the cost of liquidity hoarding the government should increase η_L in equation (B.8) or η_H in equation (B.12). However, this is not an option for the government, which is constrained by the given fiscal capacity.

Appendix C. Full Characterization of Optimal Public Debt Structure

Section 3 considers the optimal public debt structure when the following parametric conditions are met:

$$\beta^\star \leq 1 + \frac{1}{\sigma_H - \rho}\left[\frac{1}{(\lambda_3+\lambda_4)} + (\sigma_L - \sigma_H)\right] \tag{C.1}$$

$$\eta_H > (\sigma_H - \rho) I^\star \tag{C.2}$$

$$\eta_L > (\sigma_H - \rho) I^\star \tag{C.3}$$

where $I^\star = \frac{A}{1-\mathbb{E}(\rho-\sigma(\omega))+(\sigma_H-\rho)(\beta^\star-1)(\lambda_1+\lambda_2)}$.

The upper bound on $\beta^\star$ ensures that liquidity hoarding is not too expensive, making entrepreneurs willling to hoard liquidity for states 1 and 2 and continue at full scale in all states of the world. The lower bound on η_H ensures that both entrepreneurs and foreign investors hold asset 1: the asset that provides liquidity for state 1. The lower bound on η_L ensures the same for asset 2: the asset that provides liquidity for state 2.

If condition (C.1) is violated, liquidity hoarding is too expensive. In that case, downsizing the project in states 1 and 2, $\chi_1 = \chi_2 = 0$, minimizes the unit cost:

$$c(\chi_1, \chi_2) = \frac{1 + \sigma_L(\lambda_3+\lambda_4) + \sigma_H(\lambda_1\chi_1 + \lambda_2\chi_2) + (\sigma_H - \rho)(\chi_1 c\ell_1 + \chi_2 c\ell_2)}{\lambda_1\chi_1 + \lambda_2\chi_2 + \lambda_3 + \lambda_4} \tag{C.4}$$

Indeed if (C.1) is violated, $c(\chi_1 = \chi_2 = 0) < c(\chi_1 = 1, \chi_2 = 0)$ and $c(\chi_1 = \chi_2 = 0) < c(\chi_1 = \chi_2 = 1)$, and entrepreneurs do not hedge liquidity for the states in which the liquidity shock is high, $\sigma = \sigma_H$. Evidently, in this case, there is no role for optimal provision of liquidity, since entrepreneurs use their inside liquidity to continue the project only when $\sigma = \sigma_L$.

Next, if condition (C.2) is violated, the cost of liquidity hoarding for state 1 is pinned down by entrepreneurs' demand, because foreign investors do not hold that asset. The problem looks identical to the one in equations (21)-(28), but where the last constraint holds with strict inequality for the asset that provides liquidity in state 1 (asset j), as it is bought exclusively by entrepreneurs. Condition (C.3) still holds, implying that $c\ell_2 = (\beta^\star - 1)\lambda_2$.

Because all of asset j is held by entrepreneurs, equating demand equation (17) to 1 pins down the level of investment, which equals $\frac{x_{j1}}{\sigma_H - \rho}$. The government makes x_{j1} as big as possible to maximize investment, subject to its fiscal capacity constraints. Thus, $x_{j1} = \eta_H$. Next, equating the expression for investment (19) to the expression for investment obtained before, $\frac{x_{j1}}{\sigma_H - \rho}$, makes evident that $c\ell_1$ only depends on asset j's return in state 1, η_H, and on other non-fiscal capacity parameters:

$$c\ell_1 = \frac{A}{\eta_H} - \frac{1 - \mathbb{E}(\rho - \sigma(\omega))}{(\sigma_H - \rho)} - (\beta^\star - 1)\lambda_2 \tag{C.5}$$

The cost of liquidity and the investment do not depend on wasted liquidity, they only depend on η_H, which is the asset's return that entrepreneurs use. This implies that the government cannot improve the cost of liquidity provision, investment nor welfare. The intuition for this conclusion is the following. Because all of asset j is held by entrepreneurs, they do not value returns in states different to state 1. Hence, their demand for asset j does not price unused returns, x_{j2}, x_{j3} and x_{j4}.

The same conclusion arises if condition (C.3) is violated, $c\ell_2$ only depends on η_L. Investment equals $I = \frac{x_{j'2}}{\sigma_H - \rho}$. Like before, the government chooses $x_{j'2} = \eta_L$ and the cost of liquidity in state 2, $c\ell_2$, is independent of asset j''s return in other states different to state 2:

$$c\ell_2 = \frac{A}{\eta_L} - \frac{1 - \mathbb{E}(\rho - \sigma(\omega))}{(\sigma_H - \rho)} - (\beta^\star - 1)\lambda_1 \tag{C.6}$$

Appendix D. Capital Controls

This appendix studies whether capital controls would be welfare-improving. Under the parametric conditions in section 2, capital controls imply that $q = \Pi$ because domestic consumers are the marginal buyers of public debt. Under open public debt markets, the bond price, $q = \beta^\star \Pi$ and the economy receives financial flows from abroad.

The welfare expressions for the economy with capital controls and open public debt markets are:

$$\begin{aligned} W^{CC} &= [R - E(\sigma(\omega)) - 1] I^{CC} \\ W^{OPEN} &= [R - E(\sigma(\omega)) - 1] I^{OPEN} + (\beta^\star - 1) \left[1 - \frac{\sigma_H - \rho}{\eta_L} I^{OPEN} \right] \end{aligned}$$

where investment levels are given by:

$$\begin{aligned} I^{CC} &= \frac{A}{1 - E\{\rho - \sigma(\omega)\}} \\ I^{OPEN} &= \frac{A}{1 - E\{\rho - \sigma(\omega)\} + \frac{(\beta^\star - 1)(\sigma_H - \rho)\Pi}{\eta_L}} \end{aligned}$$

respectively and satisfy $I^{CC} > I^{OPEN}$.

Welfare under open public debt markets is larger than under capital controls, if the gain in investment from bringing the cost of liquidity to zero is smaller than the forfeited financial flows from abroad. In other words, if the following condition holds:

$$[R - E(\sigma(\omega)) - 1](I^{CC} - I^{OPEN}) < (\beta^\star - 1) \left[1 - \frac{\sigma_H - \rho}{\eta_L} I^{OPEN} \right] \quad \text{(D.1)}$$

Dividing this expression by I^{OPEN} gives:

$$\begin{aligned} [R - E(\sigma(\omega)) - 1] \left[\frac{(\beta^\star - 1)(\sigma_H - \rho)\Pi}{\eta_L(1 - E\{\rho - \sigma(\omega)\})} \right] &< \\ (\beta^\star - 1) \left[\frac{1 - E\{\rho - \sigma(\omega)\}}{A} + \frac{(\beta^\star - 1)(\sigma_H - \rho)\Pi}{A\eta_L} - \frac{\sigma_H - \rho}{\eta_L} \right] & \end{aligned}$$

Multiplying both sides by $\frac{A\eta_L}{\beta^\star - 1}$ and rearranging, gives the following inequality:

$$\eta_L(1-\mathbb{E}\{\rho-\sigma(\omega)\}) > [R-\mathbb{E}(\sigma(\omega))-1]\frac{(\sigma_H-\rho)\Pi}{1-\mathbb{E}\{\rho-\sigma(\omega)\}}+(\sigma_H-\rho)(A-(\beta^\star-1)\Pi) \tag{D.2}$$

According to equation (3):

$$\eta_L\left[1-\mathbb{E}\{\rho-\sigma(\omega)\}\right] > (\sigma_H-\rho)\left[A-(\beta^\star-1)\Pi\right] \tag{D.3}$$

Because the lower bound for $\eta_L(1-\mathbb{E}\{\rho-\sigma(\omega)\})$ is larger in equation (D.2) than in equation (D.3), welfare under capital controls is always smaller than under open public debt markets.

Recall, equation (D.2) gives a lower bound on $\eta_L(1-\mathbb{E}\{\rho-\sigma(\omega)\})$ for which $W^{CC} < W^{OPEN}$. Under the parametric assumption on η_L made in the main body of the paper, which ensures foreign investors hold part of the public debt, this condition is always satisfied. Therefore, if foreign investors are to hold public debt, capital controls are welfare-reducing.

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