Sample Examination Questions and Answers

I. CHAPTER ONE: INTRODUCTION

1. Consider the following argument:

   1. All men are authors.
   2. Mark Twain is a man.

   /∴ 3. Mark Twain is an author.

   a. Are all of the premises true?
   b. Is the conclusion true or false?
   c. Is the argument deductively valid or deductively invalid?
   d. Is the argument sound? Explain, giving the definition of a sound argument.

2. Consider the following argument:

   1. If George Bush is a Republican, then he is not a Democrat.
   2. George Bush is not a Democrat.
   3. George Bush is a Republican.

   a. Are all of the premises true?
   b. Is the conclusion true or false?
   c. Is the argument deductively valid or deductively invalid?
   d. Is the argument sound? Explain, giving the definition of a sound argument.

3. Suppose you know of an argument only that it is valid and has a true conclusion. What, if anything, can you tell about its premises? (Defend your answer, including examples.)

4. Suppose you know of an argument only that it is valid and has a false conclusion. What, if anything, can you tell about its premises? (Defend your answer.)

5. Suppose you know that an argument is sound. What can you determine about its conclusion? (Defend your answer.)

6. Suppose you know of an argument only that it has all true premises and a true conclusion. Can you tell from that whether it is valid or invalid? (Defend your answer.)

7. Suppose you know that an argument is invalid. Can you tell from that whether its conclusion is true or false? (Defend your answer.)

8. Suppose you know that an argument is valid. Can you tell from that whether its conclusion is true or false? (Defend your answer.)

9. Suppose you know that a set of sentences is consistent. Can you tell from that whether every set member is actually true? (Defend your answer.)

10. Suppose you know every member of a set of sentences is false. Can you tell from that whether the set is inconsistent? (Defend your answer.)

I. *ANSWERS*

1. a. No, the first premise is false.
   b. True.
   c. Valid.
   d. No, since a sound argument is an argument that is valid and has all true premises, but this argument has a false premise.

2. a. Yes.
   b. True.
   c. Invalid, since it is possible that the first two premises are true but the conclusion is false (for example if George Bush were an independent).
   d. No, since a sound argument is an argument that is valid and has all true premises, but this argument is invalid.

3. You can't tell anything about its premises. For instance, the valid argument "If Socrates is a man then Socrates is mortal. Socrates is a man. Therefore, Socrates is mortal", has a true conclusion and all true premises. But the valid argument "If Koko (the gorilla) is a man, then Koko is mortal. Koko is a man. Therefore, Koko is mortal", has a true conclusion, and one true and one false premise.

4. At least one of the premises must be false since a valid argument cannot have all true premises and a false conclusion.

5. If an argument is sound, then it is valid and has all true premises. If an argument is valid then it is impossible for it to have all true premises and a false conclusion. Therefore, the conclusion of a sound argument must be true.

6. No, both valid and invalid arguments can have all true premises and a true conclusion. You would need to determine if it is also possible for the argument to have all true premises and a false conclusion.

7. No, the conclusion of an invalid argument could be true or false. We only know that in either case it will be possible for it to have all true premises and a false conclusion.

8. No, the conclusion of an valid argument could be true or false. We only know that in either case it will be impossible for it to have all true premises and a false conclusion.

9. No, a person can have a consistent set of beliefs where one or more are false. If the set is consistent, it only follows that it is possible for all set member to be true at the same time.

10. No. You would need to determine whether it is impossible for all members of the set to be true at the same time.

II. CHAPTER TWO: SYMBOLIZING IN SENTENTIAL LOGIC

A. *General Theory*

For 1-3, circle one of a-d:

1. A compound sentence is truth–functional if and only if:
   a. the truth value of the compound sentence is determined by the truth values of its component sentences
   b. each of its component sentences is either true or false
   c. the truth values of the component sentences are determined by the truth values of the compound
   d. none of these

2. The sentence "It will rain only if the temperature drops" is correctly symbolized (using obvious abbreviations) as:
   a. $R \supset D$
   b. $D \supset R$
   c. $R \equiv D$
   d. none of these

3. The sentence "It will rain unless the temperature drops" is correctly symbolized (using obvious abbreviations) as:
   a. $R \supset D$
   b. $D \supset R$
   c. $R \equiv D$
   d. none of these

4. a. Give an original example of a *truth–functional* use of a sentence connective in ordinary English, and defend your view that it is truth–functional.
b. Give an original example of a *non–truth–functional* use of a sentence connective in ordinary English, and defend your view that it is non–truth-functional.

For 5 and 6, identify the main connective.

5. $\sim[(A \vee B) \equiv (B \cdot C)] \supset (F \vee G)$

6. $\sim[\sim G \supset (B \vee A)]$

A. *Answers*

1. a
2. a

3. d
4. a. The word "and" in the sentence "Art went to the show and Betsy went to the show" is truth–functional, because the truth value of the sentence formed by this connective is a function of the truth value of the two conjuncts, "Art went to the show" and "Betsy went to the show".
b. The word "because" in the sentence "Art went to the show because Betsy went to the show" is not truth–functional. For example the sentence may be false if both component sentences are true or it may be false. In other words, the truth value of the sentence is not a function of its component parts.
5. The horseshoe.
6. The first tilde.

B. *Symbolizing*

Symbolize the following, letting R = "It will rain", U = "The temperature will go up", D = "The temperature goes down", S = "It will snow":

1. It won't snow, unless the temperature goes down.
2. The temperature will go up only if it doesn't rain, and will go down if it does rain.
3. The temperature will go down just in case it rains.
4. The temperature will go up or down but not both.
5. If it neither snows nor rains, the temperature won't go up and it won't go down.
6. Either it snows and the temperature won't go up, or it rains and the temperature won't go down.
7. If it rains, then if the temperature goes down it will snow.
8. If it rains, then it snows if and only if the temperature goes down.
9. It won't rain, but it will snow unless the temperature goes up.
10. If it rains, the temperature will go up unless, of course, it goes down.
11. It will snow if and only if it does not rain and the temperature does not go up.
12. If it both rains and snows, then the temperature is neither going up nor going down.

B. *Answers* (Note that you may accept sentences logically equivalent to these)

1. $\sim D \supset \sim S$
2. $(U \supset \sim R) \cdot (R \supset D)$
3. $D \equiv R$
4. $(U \vee D) \cdot \sim (U \cdot D)$
5. $\sim (S \vee R) \supset (\sim U \cdot \sim D)$
6. $(S \cdot \sim U) \vee (R \cdot \sim D)$
7. $R \supset (D \supset S)$
8. $R \supset (S \equiv D)$
9. $\sim R \cdot (\sim U \supset S)$
10. $R \supset (\sim D \supset U)$
11. $S \equiv (\sim R \cdot \sim U)$
12. $(R \cdot S) \supset \sim (U \vee D)$

C. *Translating*

Translate the following into more or less colloquial English sentences. Let J = "Art gets a new job"; B = "Art gets a new boss"; A = "Art gets a new apartment"; R = "Art get a new roommate."

1. $\sim (J \vee B)$
2. $J \supset (B \cdot A)$
3. $\sim (A \cdot R)$
4. $\sim J \supset \sim [(B \vee (A \vee R)]$
5. $(\sim A \vee \sim R) \supset \sim J$
6. $(B \equiv J) \cdot (R \equiv A)$

C. *Answers*

1. Art will get neither a new job nor a new boss.
2. If Art gets a new job then he will get a new boss and a new apartment.
3. Art will not get both a new apartment and a new roommate.
4. If Art does not get a new job, then he will get neither a new boss, nor a new apartment, nor a new roommate.
5. If Art either does not get a new apartment or does not get a new roommate, then he does not get a new job.
6. Art gets a new boss if and only if he gets a new job, and a new roommate just in case he gets a new apartment.

III. CHAPTER THREE: TRUTH TABLES

A. *General Theory*

1. Determine the sentence forms of which the following are substitution instances:
   a. $\sim (A \equiv B) \supset C$
   b. $A \vee (B \cdot \sim C)$

2. If a sentence form contains five variables, how many lines or rows must its complete truth table analysis have?

3. Assume you know of an argument only that its premises are *not* consistent. What, if anything, can you tell about the argument's validity? (Defend your answer.)

4. Why is it true that any argument with a conclusion with the form $p \vee \sim p$ is valid?

5. a. We can define a tautologous sentence as one that is a substitution instance of some tautologous sentence form, and a contradictory sentence as one that is a substitution instance of some contradictory sentence form. Why can't we analogously define a contingent sentence as one that is a substitution instance of some contingent sentence form? (Defend your answer, including examples.)
   b. We cannot define a contingent sentence as one that is a substitution instance of some contingent sentence form. But then how *can* we define that term?

6. Suppose one of the premises of an argument is logic equivalent to the conclusion. What, if anything, can you conclude about the argument's validity?

A. *Answers*

1. a. $p, p \supset q, \sim p \supset q, \sim (p \equiv q) \supset r$
   b. $p, p \vee q, p \vee (q \cdot r), \; p \vee (q \cdot \sim r)$

2. There would be $2^5 = 32$ rows of the truth table.

3. If an argument's premises are not consistent then it will be impossible for all of them to be true. Therefore, the argument must be valid, because it will be impossible for the premises to be true and the conclusion false.

4. Any argument with a conclusion of the form $p \vee \sim p$ will have a conclusion that is a tautology, and thus a conclusion that cannot be false. Thus, it will be impossible for that argument to have all true premises and a false conclusion, and so it will be valid.

5. a. Because *every* sentence, tautologous, contradictory, or contingent, is a substitution instance of some contingent sentence form or other. For instance, every sentence is a substitution instance of the contingent sentence form p.
   b. We can define a contingent sentence as a sentence that is not a substitution instance of any tautological or contradictory sentence form.

6. If one of the premises of an argument is logically equivalent to the conclusion, then the premise and the conclusion will always have the same truth value. Thus, it will be impossible for that argument to have all true premises and a false conclusion, and so it will be valid.

B. *Tautologies, Contradictions, and Contingent Sentences*

Determine by truth table analysis which of the following sentence forms are tautologous, which are contradictory, and which are contingent:

1. $(p \vee q) \equiv [(p \cdot \sim q) \vee q]$
2. $(p \supset q) \vee (q \supset r)$
3. $p \equiv (q \equiv p)$
4. $(p \supset \sim q) \supset \sim (q \supset \sim p)$
5. $\sim [(\sim p \vee q) \equiv (\sim q \supset \sim p)]$

B. *Answers*

1. Tautologous
2. Tautologous
3. Contingent
4. Contingent
5. Contradictory

C. *Logical Equivalences*

Use a truth table to determine which of the following pairs of sentence forms are logically equivalent.

1. $\sim (p \supset q), p \cdot \sim q$
2. $\sim (p \equiv q), (\sim p \equiv q) \vee (p \equiv \sim q)$
3. $p \supset (q \cdot r), (p \supset q) \cdot (p \supset r)$
4. $\sim [(p \vee q) \supset p], \sim [(p \supset q) \vee q]$

C. *Answers*

1. Equivalent
2. Equivalent
3. Equivalent
4. Not Equivalent

D. *Proving Validity of Argument Forms*

Determine by truth table analysis which of the following argument forms are valid and which are invalid:

(1) 1. $(\sim p \vee q) \vee (\sim p \cdot q)$
2. $q \supset p$ /∴ $\sim q$

(2) 1. $p \supset \sim (\sim q \vee r)$
2. $\sim r \supset q$ /∴ $\sim p \cdot q$

(3) 1. $(p \cdot q) \equiv (q \cdot r)$
2. $\sim (r \supset \sim q)$ /∴ $\sim r \supset p$

(4) 1. $\sim (p \cdot \sim q) \equiv \sim p$
/∴ $(\sim q \vee p) \vee (\sim q \supset \sim p)$

D. *Answers*

(1) Invalid
(2) Invalid
(3) Valid
(4) Valid

E. *Short Truth Table Test for Invalidity*
Use the short truth table method to show that the following arguments are invalid and provide the truth-value assignments that show invalidity for each:

(1) 1. $A \supset B$
2. $C \supset \sim B$
3. $\sim C$ /∴ A

(2) 1. $A \supset (B \vee C)$
2. $B \supset (C \supset D)$
3. $\sim D$ /∴ $\sim A$

(3) 1. $A \vee (B \cdot \sim C)$

(3) 1. $A \vee (B \cdot \sim C)$
2. $\sim [A \cdot \sim (C \vee B)]$
3. $B \supset C$
4. $\sim B$ /∴ $\sim A$