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CONTENTS

John Bigelow and Robert Pargetter  Acquaintance with qualia  129
Philip Percival  Indices of truth and intensional operators  148
Gyula Klima and Gabriel Sandu  Numerical quantifiers in game-theoretical semantics  173
Kai Nielsen  On there being philosophical knowledge  193
Paul A. Robinson  Review of Torbjörn Tännsjö: Moral realism  226
Peter Pagin  Review of A. Avramidis: Meaning and mind. An examination of a Gricean account of language  232
Numerical Quantifiers in game-theoretical semantics

by

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I. Introduction

The recent literature on numerical quantifiers in English divides the interpretation of the sentence

(2XM6S) Two examiners marked six scripts

into two classes: (i) the distributive interpretations and (ii) the collective interpretations. The former class contains any interpretation of (2XM6S) which is a combinatorial sum of individual acts consisting in one examiner marking one script. The latter involves collective or joint actions. A full discussion of the philosophical aspects of this distinction is contained in Tuomela (1989). It is worth noting that in languages like Georgian this distinction is marked syntactically (Cf Gil, 1987).

Our primary concern in this paper is to offer a uniform game-theoretical treatment of (2XM6S) which yields all its distributive and non-distributive interpretations. The game-theoretical treatment we develop here was suggested to us by the mediaeval theory of reference, the theory of supposition. (For modern literature on supposition theory see e.g Kretzmann-Pinborg-Kenny, 1982, or Ashworth, 1978.) Indeed, we regard this treatment as a quite natural generalization of the mediaeval theory, and, at the same time, we also find the mediaeval theory to provide the best informal motivation for the game-theoretical approach in this context.

Our game-theoretical approach to (2XM6S) will be compared with other approaches, which without exception rely on classical second-order or generalized quantification theory, such as those presented by Kempson and Cormack (1981), Davis (1989), van Benthem (1989) or May (1989).
Of all these authors only Davis offered a formal analysis of numerically quantified sentences which accounts for both the distributive and the collective interpretations. However, in view of the arguments given by Gil (1987), the number of possible non-distributive interpretations exceeds by far the number of those provided by Davis’ analysis. As a matter of fact, our approach to non-distributive interpretations will be somewhat different from the one received in the recent literature. Instead of the above-mentioned twofold distinction between distributive and collective interpretations, we shall adopt a threefold distinction, suggested again by mediaeval logicians, which beyond these two interpretations acknowledges also a third, so-called divisive interpretation. (For more on this see sect. 3, below.) All the above authors agree that (2XM6S) has at least two distributive interpretations. Van Benthem and May acknowledge a third interpretation which they call the “resumptive interpretation” (see below). Finally, Kempson and Cormack, in addition to the resumptive interpretation which they call the “incomplete group interpretation”, give still a fourth one called the “complete group interpretation”. Since Kempson’s and Cormack’s analysis yields the greatest number of distributive interpretations for (2XM6S), we shall adopt it below as the basis of comparison with the game-theoretical treatment we develop in this paper.

2. The theory of Kempson & Cormack

Kempson and Cormack (1981) claim that (2XM6S) has the following four distributive interpretations (it is assumed throughout that E2 is some two-member set of examiners and S6 is some six-member set of scripts):

(A) Each one of the examiners marked six scripts

(A*) EX2 VXx EX6 Sx EX6 (x marked s)

(“the object-phrase with larger scope interpretation”, or the (six, two) interpretation)

(C) Two examiners marked the same set of six scripts

(C*) ES6 EX2 VXx EX6 Sx EX6 (x marked s)

(“the complete group interpretation”).

(D) One examiner marked certain scripts and the other the rest of the six scripts

(D*) EX2 ES6 (VXx EX2 ES6 (x marked s) & VXx Sx EX6 (x marked s))

(the “incomplete group interpretation”, or “the resumptive interpretation” of May and van Benthem)

The authors think that all the four interpretations should be derived from a unique underlying logical form of (2XM6S). The proposed logical form is:

(*) EX2 VXx EX6 ES6 Sx EX6 (x marked s) & ES6 VXx Sx EX6 (x marked s)

Kempson and Cormack derive (A*)-(D*) from (*) using two syntactical operations:

Replace ‘Ex’ by ‘Vx’

Replace ‘VxEx2 ES6’ by ‘ES6 VXx EX6’

‘generalising’

‘uniformising’

By applying generalisation to ‘Es’ in the first conjunct of (*), we obtain (A*), the second conjunct of (*) becoming redundant. Similarly, by applying generalisation to ‘E’ in the second conjunct of (*), we obtain (B*). Further, if we apply uniformisation as the next step in either case, we get (C*). Uniformising both conjuncts of (*) yields:

(**) EX2 ES6 VXx EX6 ES6 (x marked s) & VXx Sx EX6 (x marked s)

This is claimed by the authors to be identical with (D*) if the following additional constraint holds: any single noun phrase which is taken to have a referent can only be understood to have a single assignment of reference (i.e., if X2 and S6 are supposed to pick up the same sets in both conjuncts as their values).
Although we take the idea of deriving the several possible readings of (2XM6S) from a single underlying form to be appealing—after all, something like such a derivation should occur when someone hearing this sentence interprets it some way—still, or rather precisely for this reason, we find the above analysis arbitrary, devoid of any semantic intuitions. We propose below an alternative analysis in terms of semantic games. The interpretations (A)-(D) above and even more will turn out to correspond to different ways of playing games with sentence (2XM6S). As we mentioned in the introduction, these games were suggested by and may be regarded as generalizations of the analyses of quantified sentences provided by the mediavol theory of reference, the theory of supposition. As mediavol logic arguably is a kind of “natural logic” (in the sense of Lakoff, 1972, cf. de Libera, 1986), this in our belief just underscores the naturalness of our approach.

3. Supposition theory and a generalization of the idea of suppositional descents

Supposition theory, as it appears in mediavol logic textbooks from the 12th century up to the 17th, usually begins with a series of definitions and divisions exhibiting sometimes considerable variations from author to author, or even explicit disagreements among the authors. Nevertheless, from our present point of view we may afford to neglect minor historical details, and in any case, most of what we will say applies quite well to mediavol authors of logic texts in general.

Supposition was commonly characterized by our authors as a property of terms in propositions, namely the taking of a term for something in a proposition, that is, as we would put it, its referring function. Since supposition was taken to be a property of terms within propositions, different types of supposition were distinguished depending on the different propositional contexts in which the terms in question may occur.

These different types, and correspondingly the different subdivisions of supposition, were characterized by late mediavol logicians by so-called suppositional descents, descensus ad inferiorea, that is to say, by certain types of inferences in which the common term of which the mode of supposition is being characterized is replaced by singular terms falling under it, appearing in either nominal or propositional conjunctions or disjunctions. These several types of conjunctions and disjunctions of singular terms, or of propositions formed with these singular terms, served then both to characterize the mode of supposition of the original common term under which the descent was made and to give the truth conditions of quantified sentences in terms of the truth or falsity of several singular ones.

The descents, as we said, lead to conjunctive and disjunctive terms or propositions. The conjunctive forms result from what we would call universally quantified terms, while the disjunctive forms from existentially quantified ones. Descent to nominal conjunctions or disjunctions corresponds to what we would call narrow scope of a quantified term, while descent to propositional conjunctions or disjunctions to wider scope.

For example, if we take word order to determine scope-relations in a sentence with the two standard quantifiers, we may have the following possible descents:

1. Some boy loves every girl, therefore this boy loves every girl or that boy loves every girl . . . and so on (enumerating every boy) and conversely.
2. Some boy loves every girl, therefore some boy loves this girl and that girl . . . and so on (enumerating every girl) and conversely.
3. Every girl is loved by some boy, therefore this girl is loved by some boy and that girl is loved by some boy . . . and so on (enumerating every girl) and conversely.
4. Every girl is loved by some boy, therefore every girl is loved by this boy or that boy or . . . and so on (enumerating every boy) and conversely.

So, schematically, we have four possibilities for determining scope relations within a sentence with the two standard quantifiers (taking \( x \) ranging over boys, \( y \) over girls, and \( b_1, . . ., b_t \) to be names of boys, while \( g_1, . . ., g_s \) to be names of girls):

\[
(1) \ (\exists x)(\forall y)(R(x)(y)) \leftrightarrow (\forall y)(R(b_1)(y)) \lor (\forall y)(R(b_2)(y)) \lor . . .
\]
(2) \((\exists x)(\forall y)(R(x)(y)) \Leftrightarrow (\exists x)(\forall y)(g_1 \land g_2 \land \ldots)\)

(3) \((\forall y)(\exists x)(R(x)(y)) \Leftrightarrow (\forall y)(\exists x)(R(g_1)(x)) \land (\forall y)(\exists x)(R(g_2)(x)) \land \ldots\)

(4) \((\forall y)(\exists x)(R(y)(x)) \Leftrightarrow (\forall y)(\exists x)(y = b_1 \lor y = b_2 \lor \ldots)\)

But notice here that in each of these cases we could descend even further on the other term too, of course, with the same result in (1) and (2) and in (3) and (4). The lesson of which is that scope relations are determined not only by the mode (nominal or propositional) but also by the order of descents. Indeed, the “undescended” common noun phrases may be regarded as being uniformly equivalent with the corresponding nominal conjuncts or disjunctions (depending on the “quantity” of the determiner), their scope relations depending on whether we descend further propositionally first on the one or on the other side. So if we do not regard word order as strictly determining scope relations, i.e., we regard, say, ‘Every girl loves some boy’ as scope-ambiguous, then we can take it to have the same import as:

\((g_1 \land g_2 \land \ldots) L(b_1 \lor b_2 \lor \ldots)\)

the scope-relations of which are determined by whether we descend further propositionally first on the left or on the right side. Now as these descents provide precisely the truth-conditions of the original sentence by arriving at several singular ones, just as in game-theoretical semantics, where by playing several semantic games with a quantified sentence we determine its truth or falsity in a model by arriving at several singular ones, it is quite natural to view these descents as prescribing several ways of playing semantic games with the original sentence.

But before turning to game-theory we need some further generalization of the idea of suppositional descents to cover also numerically quantified noun phrases, on the one hand, and the introduction of a threefold distinction of the possible readings of nominal conjuncts, on the other, to account also for non-distributive interpretations of our problem-sentence (or sophisma, as such sentences were called by mediaeval logicians).

It is quite easy to see that if ‘every A’ is equivalent to ‘this A and that A and . . .’ (enumerating every A) and ‘some A’ is equivalent to ‘this A or that A or . . .’ (enumerating every A), then ‘two A’s’

should be equivalent to ‘this A and that A or that A and that A or . . .’ (enumerating all pairs of A’s) and ‘three A’s’ should be equivalent to ‘this A and that A and that A or that A and that A and that A or . . .’ (enumerating all triples of A’s) and so on, for each natural number.

Along these lines, in general, we can treat all common noun phrases with numerical determiners as nominal conjuncts of nominal conjuncts having as many members as the cardinality of the numerical determiner, while we can determine their scope relations by allowing further descents to disjunctive and conjunctive propositions. Semantically, we can determine the import of such a complex nominal phrase by saying that a complex predicative is true of a nominal disjunction if and only if it is true of at least one of its members, while it is true of a nominal conjuction, if and only if it is true of each of its members.

But this latter holds only of the distributive reading of nominal conjuncts: further ambiguities can be accounted for by distinguishing between distributive, collective and divisive readings of nominal conjuncts, or rather of argument places of predicates in which these conjuncts occur, just as the mediaevals did. (For detailed account and references to primary sources concerning this distinction see Klima, 1990.) In this way from the general nominal descent scheme of an ambiguous numerically quantified sentence we can get specifications of its possible readings by the further, possible propositional descents and these distinctions.

So e.g. the general nominal descent scheme of ‘Two examiners marked six scripts’ may be given as follows:

\((e_1 \land e_2 \land \ldots) M(s_1 \land s_2 \land s_3 \land s_4 \land s_5 \land s_6 \land \ldots)\)

or, in general, for any terms S and P, and any relation R,

\((GND) \ (s_1 \land s_2 \land s_3 \land s_4 \land \ldots) R(p_1 \land p_2 \land p_3 \land \ldots)\)

where the number of conjuncts is that of the numerical determinant, the range of the numerical subscripts relative to a model is identical with the cardinality of the extensions of the original terms (in our example the terms: ‘examiner’ and ‘script’) in that model, while the number of disjuncts is to be such that the set of referents of the
singular terms should be identical with the extension of the original common term in this model, if the set of singular terms occurring in the conjunctions varies from disjunct to disjunct, and arbitrary, if the same set of terms makes up the conjunctions in each disjunct. Indeed, we may take this as a degenerate case, and take here a one-member disjunction instead of one with several members, that is, one conjunction alone. As a matter of fact, this treatment of degenerate cases shows us that noun phrases with the standard quantifiers can be regarded as degenerate cases of the above general scheme. A universally quantified noun phrase may be regarded as a one-member disjunction of conjunctions in which all members are different and their referents together exhaust the extension of the quantified term. An existentially quantified noun phrase may be regarded as a disjunction of one-member conjunctions (that is conjunctions with the same members), but such that the referents of the disjuncts are different, and together exhaustive of the extension of the quantified term. In our view, if anything, then it is only (GNDS) that can justly be regarded as the “unique form” sought by Kempson and Cormack underlying a doubly quantified ambiguous sentence. Note, however, that (GNDS) is not a syntactically unambiguous formula of some formal theory: it is rather a general scheme standing for natural language sentences that are materially equivalent with the original sentence, exhibiting the same ambiguities as the original sentence, but showing in a more explicit manner the source of these ambiguities.

Let us now see how can get from the above general nominal descent scheme the possible different readings of the same ambiguous sentence.

As we could see from the four types of descent schemata (1)–(4) above, descent to propositional disjunctions and conjunctions expresses the larger scope of a noun phrase in comparison with descent with nominal conjunctions and disjunctions. So descending to disjunctive propositions once under the left and once under the right side argument of M gives us two scope-differentiated readings of the restricted quantifier analysis of ‘Two examiners marked six scripts’, satisfiable either by a situation possibly involving two examiners and twelve scripts, each of them being marked by one of the examiners, or by a situation involving six scripts and twelve examiners each of the scripts being marked by two examiners and each examiner marking exactly one script.

However, we can descend by disjunctive propositions also on both sides, so that neither of the noun phrases of the original sentence gets wider scope than the other like this: ‘this and that examiner marked this and that . . . and so on’, that is, *these* six scripts, or that and that examiner marked *those* six scripts, and so on, in which case our sentence says that we have some set of two examiners and some set of six scripts each of which was marked by each of the examiners, which is the branching quantifier or ‘complete group’ reading of this sentence.

But we can get even further possible readings if we consider the collective and divisive interpretations of nominal conjunctions, or rather of argument places of predicables in which these conjunctions occur, as we have said. For, as is well-known, certain predicables can apply only to groups of individuals without applying to the members of these groups. For example, even if we can truly say that six wolves surrounded two deer, it is not true of any of these wolves that it surrounded two deer, or for that matter, any number of deer. So in this case we cannot think of the predicable ‘surrounded two deer’ as applying to the conjunction enumerating six wolves if and only if it applies to all of its members, but as applying to what the conjunction as a whole applies to, namely the six wolves enumerated by it together. In general, we can say that a predicable is true of a nominal conjunction taken collectively if and only if it is true of what the conjunction as a whole applies to, namely of the collection of the individuals enumerated in the conjunction. Note here that while the first argument place of ‘surrounded’ is necessarily collective, the other may be taken either as collective or as distributive. In the latter case the sentence ‘Six wolves surrounded two deer’ may be true in a situation in which one deer is surrounded by six wolves and another by other six wolves.

But it may also be the case that six wolves so surround two deer that three of them surround one deer and the other three the other one. In this case neither six wolves taken one by one, nor six wolves taken together can be said to have surrounded two deer, rather we...
can say that some subgroups of a sum total of six wolves surrounded a sum total of two deer. In general, we can say that if we attribute divisive readings to the two argument places of the relation R, then the truth-condition of (the particular formula instantiating in a particular model) the general descent scheme of a sentence (NS)R(MP) is that there be some together exhaustive subconjunctions of some of the N-member and M-member conjunctions of (the formula instantiating in that model) the general scheme such that R holds of all of these subconjunctions either collectively or distributively. For example, on a divisive reading of ‘Twelve wolves surrounded three deer’ this sentence may be true in a situation in which, say, six wolves surrounded one deer and six others surrounded two other deer.

Returning to our original example for a while, it is worth noting here that the so-called “incomplete group” interpretation of (2XM6S) can be got by attributing divisive reading to both of its noun-phrases, descending simultaneously on both sides to the “dividends” of the nominal conjunctions of the general nominal descent scheme and reading these dividends distributively. (For more on this see the next section.)

So in this way from the general nominal descent scheme of a numerically quantified ambiguous sentence by means of the further possible propositional descents and by distinguishing the three possible readings of nominal conjunctions we can generate apparently all possible readings of these sentences.

As we said in the framework of game-theoretical semantics these several possible descents may be interpreted as prescribing different ways to play semantic games with the original sentence, thereby producing the different truth-conditions of the corresponding readings. So let us now turn to the formulation of the appropriate game-rules.

4. The treatment of numerical quantifiers in game-theoretical semantics (GTS)

The basic ideas of GTS have been developed by Jaakko Hintikka and his collaborators starting from the seventies. A basic exposition of them is contained in Saarinen (1979) and Hintikka-Kulas (1983).

In GTS, a sentence of English receives an interpretation via a semantic game played by two players, Myself (the verifier) and Nature (the falsifier). Every quantifier phrase and connective prompts a move in the game by one of the two players. The nature of this move is specified by the rules of the game.

The rules for the standard quantifiers are already well-known: roughly, a universal quantifier prompts a choice of an individual by the falsifier and an existential quantifier prompts a similar move by the verifier. In an analogous manner, conjunctions prompt a move on the part of the falsifier (the choice of one conjunct), while disjunctions do the same on the part of the verifier. Clearly, each play of the game G(S), where S is an arbitrary sentence of the fragment of English under consideration, will come to an end after a finite number of moves, reaching an atomic sentence or its negation A. If A is true, then player I (the verifier) wins the play. Otherwise, player II (the falsifier) wins the play. A strategy for a player is a set of functions FI (FII), each containing functions fi for every move M_i made by player I (II). Each function fi corresponding to the move M_i of the game is defined in the following way: if the opponent of player I (II) made n moves before M_i and each choice was from D_m (1 ≤ m ≤ n), then f_i: Π_{m≤n}D_m ↠ D_i. (Where Π_{m≤n}D_m: D_1 × ... × D_n) A strategy is a winning one in a game if a player, using it, wins a play of the game against any one of the opponent’s strategy.

A semantic game is called a “game of perfect information” if each player “knows” all the moves made by his opponent before him. Otherwise the game is played in conditions of imperfect information. In this case a move of a player, e.g. the move M_i prompted by the symbol Q_i may be informationally independent from a move of his opponent, say M_j prompted by the symbol Q_j. Intuitively, this means that the player who makes M_i ignores the choice made by his opponent at the move M_j. Formally speaking, informational independence introduces the following difference: if the move M_i of player I (II) is informationally independent from the move M_j of player II (I), then, provided that the opponent of player I (II) made
n moves before \( M_i \), each resulting in a choice from \( D_{1 \leq m \leq m'} \), then the function \( f_j \) is not anymore defined on \( \Pi_{m=n} D_m \) but on \( \Pi_{m=n} D_m \) (that is, the possible choices from \( D_j \) are dropped out).

**Definition:** Suppose our fragment of English L is interpreted on the model \( M \) and \( S \) is a sentence of L. Then, \( S \) is true in \( M \) if and only if the verifier has a winning strategy in the game \( G(S) \).

With these preliminaries at hand we are now in a position to formulate the game rules for numerical quantifiers. As we said, disjunctions prompt a move on the part of the verifier, while conjunctions do the same on the part of the falsifier. Now as numerically quantified noun phrases in the light of the above considerations may be regarded as (nominal) disjunctions of (nominal) conjunctions, a numerically quantified noun phrase of the form 'n \( X \)' should prompt first a move by the verifier, choosing an n-member conjunction of names, while this latter should prompt a move by the falsifier choosing one of these names. However, as such a conjunction taken alone may be regarded as just a transitive function of an 'every'-phrase restricted to the set of individuals denoted by the members of the conjunction we may formulate our first rule producing the distributive reading of a numerically quantified noun-phrase as follows:

\[(G \text{n dis}) \quad \text{If the game has reached the sentence} \quad n \quad X - Z \quad \text{then let the verifier choose a set} \quad X_n \quad \text{(a subset of the extension of the term} \quad X) \quad \text{such that} \quad \text{Card}(X_n) = n. \quad \text{And let the game continue with the sentence} \quad (\text{dis}) \quad \text{Every}_{X_n} - Z \quad \text{where} \quad \text{"Every}_{X_n} \text{" means that the rule applicable to} \quad (\text{dis}) \quad \text{is the following}: \quad (G. \text{ Every} X_n) \quad \text{If the game has reached the sentence} \quad \text{Every}_{X_n} - Z \quad \text{then let the falsifier choose a name, say} \quad b, \quad \text{such that the referent of} \quad b \quad \text{is an element of} \quad X_n. \quad \text{The game continues with} \quad b - Z \quad \text{Note: if the cardinality of the extension of} \quad X \quad \text{is less than} \quad n, \quad \text{then let} \quad X_n = 0. \quad \text{In this case in the application of} \quad (G. \text{ Every}_{X_n}) \quad \text{the falsifier should choose an empty name, i.e., a name with no referent. The truth-value of the resulting sentence and, accordingly, of the original one then depends on our treatment of sentences with empty names. (For the consequences of a classical two-valued treatment regarding the reconstruction of the Square of Opposition and Aristotelian syllogistic see Essays II and III of Klima, 1988.)}

**Notice** also the peculiarity of the rule \((G. \text{ Every}_{X_n})\). It is the same as the rule for universal quantifiers in general, except that the possible choices of the falsifier are restricted by it to a certain subset of the domain, called a 'choice set' by Hintikka and Kulas. They used the idea of choice sets for explaining anaphora and definite descriptions in English. The same idea was used by G Sandu for analysing discourse quantifiers. (Sandu, 1990) Thus, with this treatment of numerically quantified noun phrases we found evidence for the ubiquity of a game-theoretical category in natural languages again, just as Hintikka as Sandu (1989) with respect to informational independence.

There are several ways to apply the above rule to the sentence \((2X M \text{6} S)\) whereby we get its three distributive readings.

**Distributive Readings**

**Case 1.** We start from left-to-right with an application of \((G. \text{ 2})\). Let the verifier pick up the set \( X_2 = \{e_1, e_2\} \) which is a subset of the set of examiners. The game continues with

\[ \text{Every}_{X_2} \text{ marked six scripts.} \]

Now, let the falsifier choose a person named, say, \( e_1 \) from the set \( X_2 \), corresponding to the universal quantifier 'every'. The game continues with

\[ e_1 \text{ marked six scripts.} \]
Now, let us apply (G. 6) and let the verifier choose a set \( S_0 = \{S_1, \ldots, S_6\} \) which is a subset of the set of scripts. The game will continue with

\[
e_1 \text{ marked every } s_{56}.
\]

Finally, the falsifier will choose an individual from the set \( X_6 \), say \( s_1 \). The game ends with

\[
e_1 \text{ marked } s_1.
\]

If this sentence is true, the verifer won the game. Otherwise the falsifier won.

If in the above game we represent the choices of the verifer by an existential quantifier and the choices of the falsifier by a universal quantifier, we get the following as the logical form of (2XM6S):

\[
(A^*) \quad \text{EX}_2 \forall x \in X_2 \exists S_0 \forall s \in S_0 (x \text{ marked } s).
\]

This is the subject-phrase with larger scope interpretation.

Case 2. Analogous with case 1, except that we start the game with an application of (G. 6) instead of (G. 2). The logical form of (2XM6S) will turn out to be:

\[
(B^*) \quad \text{ES}_6 \forall s \in S_6 \exists x \forall x \in X_2 (x \text{ marked } s)
\]

This is the object-phrase with larger scope interpretation.

Case 3. The same as case 1 or 2 with one exception: the choice of the second set \( S_0 \) by the verifier is informationally independent from the move made by the falsifier corresponding to 'Every'. This means that the move made by the verifier could have been made before the move made by the falsifier. Accordingly, the game in this case can be redescribed as follows: the verifier starts the game by choosing the two sets, \( X_2 \) and \( S_0 \). (Of course, this corresponds to the case of simultaneous propositional descent on both sides in the framework of suppositional descents.) The game continues with

\[
\text{Every } s_2 \text{ marked every } s_{56}.
\]

Thus, the logical form of (2XM6S) in this case is:

\[
(C^*) \quad \text{EX}_2 \text{ES}_6 \forall x \in X_2 \forall s \in S_0 (x \text{ marked } s).
\]

Collective Readings

If we admit that marking is something that may be carried out also by joint action, then (2XM6S) may have at least one collective interpretation, according to which the joint efforts of two examiners resulted in six scripts' being marked by them. If, on the other hand, six papers may in some way be imagined to receive a common marking, this possibility would increase the number of collective readings to three. (If one finds attribution of collective reading to the object-argument of this verb unnatural, then one may take just another example; quia non quaeritur exemplorum verificatio, as the mediaevals said echoing Aristotle's remark in the Prior Analytics.)

In any case, the general game-rule for the collective reading of a numerically quantified noun phrase is quite easy to formulate as follows:

\[
(G. \text{n col}) \quad \text{If the game has reached the sentence } \quad n \text{ X-Z then let the verifer choose a set } X_n \text{ (a subset of the extension of the term X) such that Card}(X_n) = n. \text{ And let the game continue with the sentence (col) Every } y \in X_n \text{ then let the verifer choose a name, say } A, \text{ such that } A \text{ is the name of } X_n. \text{ The game continues with } A \text{ Z}
\]

Note: Again, if the cardinality of the extension of X is less than n, then let \( X_n = 0 \). In this case, however, the application of (G. Every) should cause no problem, since in this case the falsifier only has to choose a name for the empty set. Of course, this rule presupposes that predicables with argument places prompting collective reading of the noun-phrases filling them should be so interpreted that their extensions relative to these argument places may contain subsets of the domain of interpretation as their ele-
ments. We may also stipulate that the empty set is never an element of these extensions.

The logical form of (2XM6S) according to this rule may be given in the following ways (depending on whether we read the left, the right, or both noun phrases collectively):

(C1) \( \exists X \exists S \exists \alpha \mathbf{v} \in S \alpha (X \text{ marked } s) \)

(C2) \( \exists X \exists S \exists x \in X \alpha (x \text{ marked } s) \)

(C3) \( \exists X \exists S (X \text{ marked } s) \)

**Divisive Readings**

To get the divisive readings of (2XM6S) the players should follow a somewhat more complicated rule. In keeping with our previous informal considerations the rule may be stated as follows:

(G.n.div.) If the game has reached the sentence

\[ n \cdot X \cdot Z \]

then let the verifier choose a set \( X_n \) (a subset of the extension of the term \( X \)) such that \( \text{Card}(X_n) = n \). And let the game continue with the sentence

\[ \text{Every}^d_{X_n} Z \]

where “\( \text{Every}^d_{X_n} \)” means that the rule applicable to (div) is the following:

let the verifier choose a series of nonempty proper subsets of \( X_n, X^1, \ldots X^i \), such that \( X^1 \cup \ldots \cup X^i = X_n \), that is, \( X^1, \ldots X^i \) together provide an exhaustive division of \( X_n \).

Let the game continue with the sentence:

\[ \text{Every}^e_{X^1} Z \& \text{Every}^e_{X^2} Z \& \ldots \& \text{Every}^e_{X^i} Z \]

Here, of course, the conjunction prompts a move on the part of the falsifier, who has to choose one conjunct, say \( \text{Every}^{(e)}_{X^j} Z \). Where “\( \text{Every}^{(e)}_{X^j} \)” means that the game continues with either

\[ \text{Every}_{X^j} Z \]

or

\[ \text{Every}^{(e)}_{X^j} Z \]

This rule yields the “incomplete group” interpretation for (2XM6S) as a special case, namely when the game is played with it under the following conditions:

1) both of its noun phrases are read divisively
2) the moves of both players are made simultaneously on both sides of the verb (i.e. we ‘descend’ to the two- vs. six-member nominal conjuctions simultaneously on both sides and, further, we descend simultaneously on both sides again to the ‘dividends’ of these conjuctions)
3) the ‘dividends’ are read distributively, i.e., after the simultaneous application of (div) and the choices from the resulting conjuncts by the falsifier on both sides, the game continues with the relevant instances of \( \text{Every}^e_{X^n} Z \) (instead of \( \text{Every}^{(e)}_{X^n} Z \)) on both sides.

If the game with (2XM6S) is played under these conditions, then it has the logical form:

\[ \exists X \exists S \exists X^1 \ldots \exists X^i \exists S^1 \ldots \exists S^i \exists x \in X^1 \ldots \exists x \in X^i \exists S^1 \ldots S^i \exists x \in S^1 \ldots S^i \exists x \in S \]

which is equivalent to:

\[ \text{(D* ) } \exists X \exists S (\forall x \in X \exists S (x \text{ marked } s) \& \forall x \in S (x \text{ marked } s)) \]

As a matter of fact, conditions 1) and 2) above seem to be demanded by an intuitively acceptable divisive interpretation of (2XM6S). It is an interesting question whether this may be regarded as a general constraint on the “natural” (intuitively acceptable) divisive interpretations of numerically quantified ambiguous sentences. However, we would rather leave the decision of this question to linguists. In any case, it is easy to see that changing the conditions of the games, the divisive interpretation of the noun phrases along with the possible variations concerning the distributive or collective readings of the “dividends” (especially if we take higher numbers, say, ‘Twelve examiners marked sixty scripts’) would give an enormous number of possible interpretations.
5. Conclusion

We think that the above treatment provides a truly natural, uniform analysis of numerically quantified ambiguous sentences. This treatment, beyond giving the possibility of generating apparently all possible interpretations of these sentences in a systematic manner, also renders quite easy to see the logical relations holding among these possible interpretations by means of the application of some general game-theoretical principles.

Considering the distributive readings of our problem-sentence, we can notice that the game in case 3 is obtainable from either the game in case 1 or case 2, by requiring that the second move of the verifier is informationally independent from the first move of his opponent. The game thus obtained, i.e. the game in case 3, is the game in which the verifier has at his both moves a minimum amount of information, since the falsifier does not make any move before him. That (C*) implies both (A*) and (B*) is thus a corollary of a metalogical principle governing games which can be stated very simply as follows:

If the verifier has a winning strategy in a semantic game G, then he has a winning strategy in any semantic game G* which is exactly like G except that for each move in G* he has more information than in G.

If we take into account also non-distributive readings, then the situation, of course, is more complicated. Since the logical relations holding among the distributive and collective readings are dependent on the meaning of the particular terms in question, there are no generally valid relations among these readings holding in any terms whatsoever.

This does not mean, however, that particular cases cannot be judged correctly on the basis of further game-theoretical principles: since the game yielding (D*) is identical with the game in case 3 except that the choices of the falsifier are restricted now to proper subsets of X instead of the whole X, the fact that (C*) implies (D*) is a corollary of the following principle:

If the verifier has a winning strategy in a semantic game G in which a move M_1 consists in a choice from the set Y, then he has a winning strategy in the semantic game G* which is identical with G except that the move M_1 of the falsifier is from a subset of Y.

As our game-theoretical analysis of (2XM6S) has shown, the logical form of a sentence is the result of the semantic game associated with it. Depending on the way the semantic game associated with (ZXM6S) was played, that is, whether in conditions of perfect or imperfect information, or whether according to a distributive or non-distributive reading of its noun-phrases, we obtained different logical forms of (2XM6S).

However, since neither the distributive/non-distributive distinction nor informational independence are syntactically marked in English, the syntax of an English sentence S does not determine completely the way G(S) is to be played. This is the reason why (2XM6S) and the like are ambiguous, and this is why no formal theories with an unambiguous syntax, like the theories used in the recent literature, are capable of providing “the unique underlying logical form” of such sentences.

References

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On there being philosophical knowledge

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I

Philosophers make claims about the structure of reality, about human nature, about how human beings should live, about what a well-ordered society should look like, about what the mind is and what mental representations are and the like. A systematic philosophy, if we could ever really have such a thing, would yield a better understanding of our fundamental concepts: existence, knowledge, identity, truth and value. In doing this philosophers would elucidate those concepts and, as well, concepts such as space, time, causality, person, mind, morality, the state and the like. Existence, identity, knowledge, truth and value are the central governing concepts, or organizing notions, of any system of thought and action. We could hardly be or act in the world without them. Moreover, they are concepts which in their essence are ahistorical. They are, in partial explanation of this, as vital to a Stone Age person as to a modern Londoner. Our lives, no matter who we are, are necessarily organized around such concepts. No matter how historicist we are we need to realize that philosophy (at least as traditionally conceived) endeavors to give us a more adequate understanding of them and their interrelations. Moreover, at not enfrenegts attempts to show that these concepts are not just concepts of a particular time or place or set of times and places.

Our concepts, of course, are embedded in language. We have no independent access to them apart from language. But at least some of our concepts, such as plainly the first set of concepts mentioned above (mentioned, of course, in a particular language), are, that notwithstanding, language-neutral. The same concepts are expressed by different inter-translatable expressions in different languages. The philosopher's remarks about "knowing," "cause,"