Consequence

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1. The limitations of Aristotelian syllogistic, and the need for non-syllogistic consequences

Medieval theories of consequences are theories of logical validity, providing tools to judge the correctness of various forms of reasoning. Although Aristotelian syllogistic was regarded as the primary tool for achieving this, the limitations of syllogistic with regard to valid non-syllogistic forms of reasoning, as well as the limitations of formal deductive systems in detecting fallacious forms of reasoning in general, naturally provided the theoretical motivation for its supplementation with theories dealing with non-syllogistic, non-deductive, as well as fallacious inferences. We can easily produce deductively valid forms of inference that are clearly not syllogistic, as in propositional logic or in relational reasoning, or even other types of sound reasoning that are not strictly deductively valid, such as enthymemes, probabilistic arguments, and inductive reasoning, while we can just as easily provide examples of inferences that appear to be legitimate instances of syllogistic forms, yet are clearly fallacious (say, because of equivocation). For Aristotle himself, this sort of supplementation of his syllogistic was provided mostly in terms of the doctrine of "immediate inferences"[1] in his *On Interpretation*, various types of non-syllogistic or even non-deductive inferences in the *Topics*, and the doctrine of logical fallacies, in his *On Sophistical Refutations*. Taking their cue primarily from Aristotle (but drawing on Cicero, Boethius, and others as well), medieval logicians worked out in systematic detail various theories of non-syllogistic inferences, sometimes as supplementations of Aristotelian syllogistic, sometimes as merely useful devices taken to be reducible to syllogistic, and sometimes as more comprehensive theories of valid inference, containing syllogistic as a special, and important, case.

2. A brief survey of historical sources

Accordingly, the characteristically medieval theories of non-syllogistic inferences were originally inspired by Aristotle's logical works other than his *Analytics*. Aristotle's relevant ideas were handed down to medieval thinkers by Boethius' translations of and commentaries on Porphyry's *Isagoge* and Aristotle's *Categories* and *Peri Hermeneias*, along with Boethius' own logical works, the most relevant to the development of consequences being his *De Hypotheticis Syllogismis* and *De Topicis Differentiis*.

As Christopher Martin has convincingly argued, it was not until Abelard's "discovery of propositionality", that is, the applicability of truth-functional logical operators (in particular, propositional negation and conjunction) to propositions of any complexity, that medieval logicians found the conceptual resources to develop what we would recognize as propositional logic. (Martin, 2009 and 2012) However, Abelard's own project, retaining certain elements of Boethius' non-truth-functional treatment of conditionals, was proven to be inconsistent by

[1] In this chapter, I will use this phrase broadly, to refer to medieval doctrines covering logical relations between two categorical propositions sharing both of their terms, *viz* the doctrine of the Square of Opposition and its expansions as well as the doctrine of conversions.

Alberic of Paris (sometime in the 1130s), leading to a great controversy in the middle of the twelfth century (see the chapter on the "Latin period up to 1200" in this volume). A number of schools, each gathered around a famous master (see the same chapter) provided a number of different solutions to the problem. Eventually, the solution of the Parvipontani prevailed, endorsing the claim that from an impossible proposition anything follows, *ex impossibili quodlibet* (and the complementary claim that a necessary proposition follows from anything, *necessarium ex quolibet*).

It is against this background that by the fourteenth century the literature specifically devoted to consequences crops up and flourishes, either in specific smaller works (such as Burley's, Buridan's or Billingham's treatises on consequences) or as parts of larger works (such as Ockham's treatment of consequences in his *Summa Logicae*, or the treatment of consequences provided by Buridan in his *Summulae de Dialectica*). During the fourteenth century, two doctrinally quite clearly separable traditions developed. One of these is the Parisian tradition, represented by John Buridan, Albert of Saxony, Marsilius of Inghen, and others. The other is the English tradition, represented by Richard Billingham, Robert Fland, Ralph Strode, Richard Lavenham, and others. The main doctrinal difference in question is that whereas the Parisian tradition tied the notion of formal validity to truth-preservation under all substitutions of non-logical terms, the English tradition (in line with the earlier, pre-14th-century Parisian tradition) required a containment-principle, often described in psychological terms (requiring that the understanding of the antecedent should contain the understanding of the consequent). Several authors of the fifteenth century, such as Paul of Pergula (1961, pp. 88-89), attempted to combine these traditions in terms of further distinctions, distinguishing between "formally formal" (*consequentia formalis de forma*) and "materially formal" (*consequentia formalis de materia*) consequences. (See the chapter on "Late medieval logic" in this volume.)

However, for a better understanding of these doctrinal developments, we should first clarify more precisely what these authors were talking about: what are consequences, and what are their main kinds and properties?

3. What are consequences?

Perhaps, a usefully non-committal way of characterizing consequences in general would be the following: a consequence is a propositionally complex expression, i.e., one that has parts that taken without the rest would constitute a proposition, such that one of its propositional parts is designated as its consequent and the other or others as its antecedent, connected in such a way (by means of conjunctions like 'if' and 'therefore' and their stylistic variants) that the whole expression indicates that the antecedent warrants the consequent.

This characterization, by saying that a consequence is a propositionally complex expression without specifying what kind of expression it is does not pre-judge the issue whether consequences are to be regarded as complex propositions or other complex phrases, such as inferences or arguments, which we would usually take to be sets of distinct propositions. Many medieval authors would provide an explicit characterization of consequences as conditionals, and then use the term to refer to arguments. Indeed, sometimes instead of the term '*consequentia*', they would use the terms '*inferentia*', '*consecutio*' or '*illatio*' equivalently. Actually, even those authors who distinguish conditional propositions from an inference or an argument in terms of whether their propositional components are asserted or unasserted (in obvious awareness of what

Peter Geach would dub "the Frege-point" in Geach, 1980) would subsume both under a broader notion of consequence, as Buridan does in the following passage:

> "…there are two kinds of consequence, the first of which is a conditional proposition that asserts neither the antecedent nor the consequent (e.g., 'if a donkey flies, then it has wings') but asserts only that the latter follows from the former. Such a consequence, therefore, is not an argument, for it does not conclude to anything. The other kind of consequence is an argument, given that the antecedent is known, or is known better than the consequent, and this asserts the antecedent, and from this it assertively infers the consequent. In a conditional we use the conjunction 'if', whereas in an argument we use the conjunction 'therefore'. Furthermore, […] in a conditional the conjunction is attached to the antecedent, whether the antecedent is placed before or after the consequent, as in 'If a donkey flies, then a donkey has wings' and in 'A donkey has wings, if a donkey flies', but in an argument the conjunction is attached to the consequent, as in 'Man is risible; therefore, an animal is risible'." (Buridan, 2001, 7.4.5, p. 575; see also Klima, 2004a)

It is nevertheless generally true that the propositional components of a consequence are such that one of them is designated as the consequent and the others are designated as the antecedent, and that this designation, marked by the conjunctions 'if' or 'therefore' and their stylistic variants (such as 'provided', 'hence', etc.) signifies that the consequent *follows from* the antecedent, or in other words that the antecedent *warrants* the consequent, where the verb 'warrant' is again deliberately vague to allow for a number of more specific interpretations. This is because the warranting in question can be variously interpreted both with regard to *what* it warrants and with regard to the *strength* of the warrant it requires. Of course, the most natural candidate for *what* the warrant in question has to concern would be seem to be the truth of the consequent, to be warranted by the truth of the antecedent. Accordingly, a consequence would naturally be regarded as *valid* (that is, as in fact providing this warrant, which is sometimes expressed by our authors by saying that the consequence is true [*vera*], sometimes by saying that it is good [*bona*]), if the truth of the antecedent would warrant the truth of the consequent by necessity, that is to say, if it is not possible for the antecedent to be true and the consequent not to be true (which is even today the usual definition of the validity of a deductive inference).

However, a simple argument presented by John Buridan shows that interpreting the warrant provided by the antecedent in terms of truth can lead to paradoxical results in a natural language with resources for self-reference. (Cf. Klima, 2004b and Dutilh Novaes, 2005, and see the chapter on "Sophisms and insolubles" in this volume.) Take the proposition 'No proposition is negative; therefore no donkey is running'. The antecedent of this consequence is a negative proposition, whence it cannot be true. But then, it is not possible for the antecedent to be true and the consequent not to be true; therefore, it would seem that the consequence is valid. However, it is certainly a possible situation in which there are no negative propositions (as was actually the case, for example before the first human being formed the first negative proposition in the first human language), in which, however, some donkey is running, which would be precisely the scenario that would have to be excluded by the consequence in question, if it were valid. So, Buridan reformulates the requirement for the validity of a consequence in terms of the correspondence-conditions of the propositions it involves. Defining the validity of a consequence by defining what its antecedent is, he writes as follows:

> "Therefore, some give a different definition [of antecedent], saying that one proposition is antecedent to another, which is such that it is impossible for things to be altogether as it

signifies unless they are altogether as the other signifies when they are proposed together." (Buridan, 1976, p. 22; 2014, p. 67)

This definition now guarantees that even if the antecedent automatically falsifies itself whenever it is formed, its self-falsification does not automatically validate the consequence, for it still leaves open the possibility that the situation signified by the antecedent holds without that signified by the consequent. Of course, since this revision of the definition of the validity of a consequence had to be introduced only because of the possibility of a proposition-token quantifying over itself in a natural language, once one keeps this possibility in mind the definition of validity need not be totally overhauled, as Buridan himself recognized, and he used the definition based on the idea of *necessary truth-preservation* (i.e., on the idea that the truth of the antecedent is "preserved" in the truth of the consequent) without further ado concerning consequences not involving such self-referential propositions.

However, this remark immediately takes us to the other aspect of the warrant the antecedent is signified to provide for the consequent in a consequence, namely, its *strength*. For when we say the idea of the validity of a consequence requires its *necessary* truth-preservation, with the idea of *necessity* we definitely indicate that the warrant excludes the *possibility* of the truth of the antecedent without the truth of the consequent. But now further obvious questions emerge. What sort of necessity is this? What grounds this necessity? Is it absolute or relative to some conditions? And can we have valid consequences with a warrant weaker than necessity, such as *probability*?

In contemporary logic, when we talk about logical validity, we primarily mean formal, deductive validity with reference to an artificial, formal language. This notion of validity is either spelled out syntactically, in terms of *deducibility* by means of deduction rules, or semantically, in terms of all *possible interpretations* of the primitive, non-logical symbols of the language (or, equivalently, provided our language has sufficient resources to express all possible evaluations, all *possible substitutions* thereof) determining all possible evaluations of our formulae, to see whether there is a possible interpretation under which the premises are true and the conclusion is false. In fact, this is what many contemporary logicians take to constitute the *logical necessity* of a valid inference: the impossibility of the truth of the premises and the falsity of the conclusion under any possible interpretation (or substitution) of their non-logical components, as opposed to their logical components that have a fixed interpretation, constituting the logical form of the formulae or the corresponding natural language sentences in question. (Tarski, 1983, pp. 409–420)

One might think that this is precisely the same idea that is indicated by Aristotle's use of schematic letters in his syllogistic. However, Aristotle never quite spelled out the idea in this way, and he certainly did not apply this notion of validity to what became treated as consequences in general in the medieval literature. Furthermore, even if some of his ancient commentators did distinguish form and matter in syllogisms in the way that tied the notion of logical validity to logical form, nevertheless, in medieval theories of consequences, this was not the primary notion of validity or of the corresponding notion of the necessity of the warrant provided by the antecedent for the consequent. (Thom, 2012; Dutilh Novaes, 2012; Read, 1994) For Boethius, the necessity of what he called a *natural consequence* is grounded in a causal relation between what is signified by the antecedent and the consequent, but he also accepts true *accidental consequences*, the truth of which simply rests on the co-occurrence of what is signified by the antecedent and the consequent. (Boethius, 1969, 1.3.6) Boethius' distinction

between these two types of consequences persisted until it came to be replaced by the idea of *formal* as opposed to *material* validity in the fourteenth century.

Although the latter distinction already had its anticipation in Abelard (as is the case with so many other important philosophical ideas), in his remark to the effect that only that inference is "perfect with regard to the construction of the antecedent" (*perfecta quantum ad antecedentis constructionem*) in which no substitution of the terms will be "able to abrupt the consecution" (*cessari valet consecutio*); nevertheless, he would still regard a consequence as equally necessary if it is based on "the nature of the terms", even if it is not perfect with regard to construction. (Abelard, 1956, p. 255)

So, the necessity of a consequence for medieval thinkers is not always the necessity *we* would recognize as logical or formal necessity (based on syntactic deducibility or semantic validity, i.e., truth-preservation under all possible interpretations/substitutions of non-logical primitives), but it can also be based on our understanding of causal, metaphysical connections of the nature of things signified by the non-logical terms involved or on the conceptual containment relations of the concepts whereby we conceive of them.

Furthermore, the necessity in question may not even be some absolute necessity, but possibly dependent on actual conditions that obtain at a given time. Indeed, William of Sherwood is willing to entertain true consequences that are not necessary whether absolutely or conditionally (Sherwood, 1966, pp. 34-35), and he is also satisfied with mere probability in the case of topical inferences, although he always attempts to reduce them to syllogisms with at least probably true premises, the probable truth of which rests on the probability of the topical maxim. (For further details, see section 5.1 below.) Others, on the other hand, such as Peter of Spain, would claim that all true conditionals are necessary and all false ones are impossible, that is, that there are no contingent conditionals. (Peter of Spain, 2014, pp. 114-115, n. 17.) But then again, he would also admit merely probable topical consequences.

By the fourteenth century, authors also regularly distinguished between different types of consequences based on the different strengths of the warrant provided by the antecedent, although they would make the distinction not in terms of the natural or metaphysical necessity of the consequence, but rather in terms of a consistently applied criterion separating *formal* and *material* consequences, while they would also distinguish between *simple* (*simplex*) and *as of now* (*ut nunc*) consequences, which were taken to hold at all times or just for a given time, respectively.

These two distinctions could be variously related, depending on the author. For Walter Burley, for instance, the latter distinction is the primary, and it is simple consequences that he divides into natural and accidental on the basis of whether they hold in virtue of an intrinsic or an extrinsic topic (a distinction that will be explained later), respectively. The distinction between formal and material consequences comes up for Burley only in the context of solving a problem, but not as a primary distinction of basic types of consequences *per se*. (Burley, 2000, pp. 85-86) For Buridan, on the other hand, the primary distinction is that between formal and material consequences, and it is only among material consequences that he draws the distinction between *simple* and *as of now* consequences. However, as we shall see in more detail, this difference is due to their interpretation of what constitutes formal validity.

For Burley, whether a consequence is formal is based on the containment principle that allows a consequence to be formal either "by reason of the form of the whole structure" or "by

reason of the form of incomplex terms", as in the case of arguing affirmatively from the inferior to the superior term, for instance, 'This is a man; therefore, this is an animal'. (Burley, 2000, p. 173) But for Buridan, the formal validity of a formal consequence is dependent solely on the form of the propositions involved, where the form of a proposition is identified as its syncategorematic structure, whereas its matter is constituted by its categorematic terms. Therefore, for Buridan a formally valid consequence is one in which the truth of the antecedent guarantees the truth of the consequent under any substitution for its categorematic terms, whence those terms can be represented by schematic letters, leaving the formal structure of the argument immediately recognizable, pretty much in the same way as in modern formal logic.

So, let us deal first with those non-syllogistic consequences that both Buridan and the Parisian tradition following him and the English tradition would have deemed to be formal on account of their logical structural features, which would not, however, fit into any syllogistic form. Then we shall consider formal consequences that only the English tradition would have deemed formal, which the Parisian tradition would have taken to be enthymematic, but reducible to a formal consequence by the addition of some further premise. Next, we shall deal with irreducibly, but still valid material consequences and the issue of what separates the two kinds, and conclude with a systematic survey of the various criteria for validity proposed by the various authors considered here, in comparison with our modern notion of logical validity.

4. Formal consequences

4.1. Syllogisms with oblique terms

Standard modern histories of logic used to make the claim that Aristotelian syllogistic was incapable of handling "relational reasoning", that is, deductively valid inferences that involved propositions with relational terms. However, medieval logicians were quite aware of forms of reasoning the validity of which depends not on the connection of complete syllogistic terms, which may be of any complexity, but rather on the connection of the parts of such complex terms, which in Latin are usually in some oblique case. Hence, they treated such forms of reasoning under the heading of "syllogisms with oblique terms" (*de syllogismis ex obliquis*). (See, e.g., Parsons 2014, §§5.3-5.7)

Here is just a simple example from Buridan to show how this is supposed to work. Consider the following argument 'A donkey sees every man; every king is a man; therefore, a donkey sees every king'. Clearly, the predicate of the minor premise is only a part of the predicate of the "canonical", (subject)-[copula]-{predicate}, form of the major premise (using the matching parentheses to mark out the relevant parts of the major): 'A (donkey) [is] {something seeing every man}', where the predicate term 'something seeing every man' (*videns omnem hominem*) contains 'man' (*hominem*) as a distributed term in the oblique, accusative, case (that is, within the scope of a universal quantifier), but the two premises do not share an entire complex term whether in their subject or predicate position, which a valid syllogistic form would require.

Thus, to account for the validity of this argument, Buridan distinguishes between the syllogistic terms and the terms of the propositions of the syllogism: "in syllogisms with oblique or with complex terms, it is not necessary that the syllogistic terms, namely, the middle term and the extremities, be the same as the terms of the premises and the conclusion, namely, their subjects and predicates. This is because it is permissible to carry out a subsumption under a

distributed term not only if it is placed at the beginning of a proposition, but wherever it is placed." (Buridan, 2001, 5.8.2, p. 367)

Therefore, if the oblique term of the major is distributed, then it can be replaced in the conclusion by the oblique form of the subject of the minor, when the predicate in the minor is the nominative form of the distributed oblique term of the major. Accordingly, the syllogistic terms of this syllogism can be marked out in the following manner: {major}; [middle]; (minor); 'A {donkey} sees every [man]; every (king) is a [man]; therefore, a {donkey} sees every (king)'. Buridan treats this as an example of a syllogism resembling the syllogisms of the Aristotelian first figure. He also deals with examples resembling the other Aristotelian figures in a similar manner.

4.2. Syllogisms with ampliated terms

Again, it is easiest to handle the issue of syllogisms with ampliated terms through an example (see the chapter on "Properties of terms" for more on ampliation). Consider the following argument: 'Nothing dead is alive, but some horse (say, Alexander's horse, Bucephalus) is dead; therefore, some horse is not alive', which appears to be a perfect substitution-instance of the valid syllogistic form *Ferio*, the fourth mood of the first figure. (Buridan, 2001, 9.5. First sophism, pp. 914-916) However, the premises are true, and the conclusion is false, since what is actually a horse must be alive. The solution is that despite appearances to the contrary, this is not a valid instance of *Ferio*, because the term 'dead', meaning 'something that was alive but is not' *ampliates*, that is, extends the range of supposition of the term with which it is construed to past entities, so the minor should be analysed as saying 'something that was or is a horse is dead'. But then, since in the conclusion there is no such ampliation, the inference is not valid. To cancel out the ampliative force of the term 'dead', we could instead have as the minor premise, explicitly restricting the subject's reference to present horses, 'something that is a horse is dead'. This would render the argument valid, but it would not cause a problem, since then the minor is false.

4.3. Consequences in propositional logic and immediate inferences

As we could see in our historical survey, in medieval logic it was some time before what we would recognize as truth-functional propositional operations, such as negation, conjunction, disjunction or implication, came to be generally treated as such, as equally applicable to propositions of any complexity. Perhaps, the most obvious reason for this is provided by the fact that natural language negation can take virtually any scope, whence it is not just the simple proposition-forming operation that modern logic acknowledges. But similar observations apply to disjunction and conjunction, which besides being propositional functions, can work as nominal operators as well, forming nominal disjunctions and conjunctions out of nominal expressions, as the medieval theories of suppositional descents clearly acknowledged, using the differences of nominal and propositional disjunctions and conjunctions to distinguish different modes of supposition to make distinctions that we would represent in terms of different quantifier-scopes.[2] Indeed, given the different possible interpretations of nominal conjunctions, namely, distinguishing their distributive, divisive and collective interpretations, nominal

[2] For a diagrammatic summary of the relationships between suppositional descents and quantifier scopes, see Klima, 2009, p. 181. See also the chapter on "Supposition and properties of terms" in this volume.

conjunctions also served in dealing with phenomena of what *we* would describe as plural and numerical quantification. (Klima-Sandu, 1990) The case is somewhat similar with our modern notions of material (Philonian) implication and necessary entailment, which would be recognized only as specific cases of the variety of logical relations that the general notion of consequence was supposed to cover in medieval logic.

In general, dealing directly with the rich expressive resources of a natural language, namely, Latin, medieval logicians recognized and dealt with the variety of ways in which the same expressions can function in different contexts, not by means of a simplified artificial language that represents only certain facets of the various functions of our natural language expressions, but rather in terms of distinguishing the different functions of the same phrases in different contexts, and sometimes just stipulating those functions in a technical, "regimented" Latin, for the sake of simplicity and uniformity.

This is neatly illustrated in the doctrine of so-called "immediate inferences", the doctrine of the relationships among pairs of affirmative and negative, universal and particular categorical propositions sharing both their terms, stemming from Aristotle's *On Interpretation*, usually summarized in the *Square of Opposition* and in "the rules of conversion".[3] Without going into much detail, I would just like to illustrate the previous general remark by a quick comparison of Abelard's treatment of the Square with what became "the standard account" and with Buridan's extension of the Square into an Octagon covering not only simple categorical propositions, but propositions with oblique terms, categorical propositions whose both terms are explicitly quantified, as well as modal propositions with two modalities.

When dealing with the propositions of Aristotle's Square, Abelard noticed that one should distinguish between the negation of their verb-phrase alone (verbal predicate, or copula + nominal predicate) and the negation of the entire proposition including the determiner of the noun phrase (providing the subject term). Thus, he distinguished between the contradictory, what he called "extinctive" or "destructive", negation of 'Every man runs', namely, 'Not every man runs', and the "separative" or "remotive" negation of the same, namely, 'Some man does not run'. Likewise, he also distinguished between the contradictory negation of 'Some man runs', namely, 'No man runs', and its remotive negation: 'Every man does not run'. (Martin, 2009, p. 135) The difference is that the contradictory negation destroys or extinguishes the existential import of the affirmative proposition, which the mere separative negation leaves intact. At any rate, this correctly accounts for the intuition that 'Some man does not run', as opposed to 'Not every man runs' has to entail that there are humans.

However, Abelard's distinction did not really catch on, and gave way to the stipulation that these two forms of negation are equivalent, and both express the contradictory of the corresponding universal affirmative, equally cancelling its existential import. Thus, in the "regimented" Latin of later scholasticism it became a universal rule that affirmative propositions have existential import, whereas their contradictory negations (which may be effected either by pre-positing the negative particle to the entire affirmative proposition, or by negating the verb-phrase of the affirmative proposition after replacing its quantifier with the quantifier's dual) do not. This is how we get the "classic" Square of Opposition. (Parsons, 2014; Klima, 2001)

[3] See the chapter on "Syllogisms" in this volume.

However, further possibilities emerge, as soon as these stipulations are in place, and in accordance with the requirements of syllogistic term-logic, the verb-phrase of categorical propositions is "canonically" analysed into the verbal copula and a further quantifiable noun-phrase, where the affirmative copula is interpreted as expressing identity. For on this analysis, 'Every man runs' becomes 'Every man is some runner', which by way of a further stipulation can be regarded as equivalent to the "unusual" construction in which both terms precede the copula: 'Every man some runner is', which, however, has the advantage of having its quantifiers listed in the order of their decreasing scope left to right (as in "prenex normal form" in modern formal logic).Thus, a negation applied at the front can syntactically "wriggle its way through" the subsequent quantifiers, changing them into their duals, until it lands on the verb, just as it would do with the corresponding quantificational formula: $\sim(\forall x)(\exists y)(x=y)$, yielding $(\exists x)(\forall y)\sim(x=y)$, that is to say, 'Not every man some runner is' (i.e., colloquially, 'Not every man is a runner') would become 'Some man every runner is not' (i.e., colloquially, 'Some man is not any runner', 'Some man does not run').

Since the combination of the two quantifiers and negation can yield eight different types of proposition, Buridan used this "canonical form" to construct an Octagon of Opposition, listing all logical relations among the resulting propositions. Finally, having observed the analogy of the logical behaviour of dual quantifiers, quantified oblique terms and modal operators, Buridan expanded his Octagon to these further types of propositions as well. This way, he basically managed to get as close as anyone can to a formal theory of the logical relations for large classes of strictly defined propositional types. (Buridan, 2001, 1.5.2, pp. 44-45; Read, 2012)

But what is it exactly, one may ask, that renders such a theory strictly "formal"? Obviously, not that we use schematic letters or other symbols for the words or phrases of a natural language, although once the construction of the phrases of the natural language is strictly regulated, it becomes obvious which of those phrases can be replaced by schematic letters that then can be replaced by any natural language phrase of the same type. We know from our artificial languages that what makes a logical theory "formal" is its strict, well defined syntax, precisely specifying the types of its primitive symbols, its rules of construction, and rules of inference (if it is a syntactic theory) or rules of interpretation (if it is a semantic theory) for the types of expression defined in the syntax. But that is exactly what Buridan's "regimentation" also achieved with regard to the several types of propositions discussed above, although, of course, not for the entirety of all possible forms of reasoning with all possible forms of propositions in Latin. For then, at least for these well-defined sets of propositions, we do have those schematic rules that allow us to regard any concrete sentence as a mere substitution instance of the schematic form for which we have effective methods for checking its logical relations with any other sentence of a similarly well-defined schematic form.

Yet, it is still desirable to have a general notion of formal validity, even without having the effective syntactical or semantical methods for checking it in each and every case. After all, it is only in possession of such a general notion that we can figure out what can even count as a valid consequence, and whether its validity is due to its logical form or some other, more specific considerations. As we have seen earlier, the general intuitive criterion for the validity of a consequence in general was the repugnance or incompatibility of the negation of the consequent (conclusion) with the antecedent (premises). This intuitive idea, however, can be further articulated in a number of different ways: it can be taken to be some metaphysical, natural, causal or conceptual impossibility, which in turn may manifest itself in the absolute or

conditional impossibility of the antecedent and the negation of the consequent obtaining together either on account of the logical form of the propositions in question, under any possible substitution/interpretation of their non-logical components, or on account of some conceptual, natural or metaphysical connection between the semantic values of those components. All this, of course, leaves us with at least two further questions: (1) what exactly is this repugnance or incompatibility that is required for the validity of a consequence, and (2) what, if anything, can be a principled basis for separating the "logical" and "non-logical" sub-components of its propositional components, which would distinguish *formal* consequences from *material* ones?

5. Material consequences

5.1. Material consequences reducible to formal ones

As we have already seen, there is, at least from our modern perspective, an intuitive way to draw the distinction between formal and material consequences, along the lines Buridan and the Parisian tradition following him did: a consequence is formal just in case it is valid on account of its form, where its form is nothing but the syncategorematic structure of its propositional components, whereas its matter is constituted by its categorematic terms, which is why those terms can be represented by schematic letters, to indicate their substitutability with any particular terms of the relevant type.

This is certainly neat and workable, as long as we have a neat and workable distinction between categorematic and syncategorematic terms, and as long as we are willing to "sacrifice" a whole lot of clearly valid inferences on the altar of extra-logical connections, based on our ever-fallible knowledge of the nature of things. This is pretty much the choice Buridan and his "modern" ilk made, but also the choice nearly all of his predecessors, many of his contemporaries and, apparently, the later "English tradition" did *not* want to make.

Take, for instance, Walter Burley. He provides the following primary division of consequences:

> "One kind of consequence is simple and another 'as of now'. A simple consequence is one that holds for every time, so that the antecedent can never be true unless the consequent is true. An 'as of now' consequence is one that holds for a determinate time and not always, such as 'every man runs; therefore, Socrates runs'. For this consequence does not hold for every time, but only holds while Socrates is a man.[4] Simple consequence is of two kinds. One kind is natural. That happens when the antecedent *includes* the consequent. Such a consequence holds through an intrinsic topic. An accidental consequence is one that holds through an extrinsic topic. That happens when the antecedent does not include the consequent but the consequence holds through a certain extrinsic rule. For example, 'If a man is an ass, you are sitting'. This consequence is a good one, and holds through the rule 'Anything follows from the impossible'. The rule relies on the topic 'from the lesser appearance'. For the impossible appears to be less true than anything else. Therefore, if the impossible is true, it follows through the topic 'from the lesser appearance' that anything else will be true." (Burley, 1955, pp. 60-61; Cf. Burley, 2001, p. 146)

[4] After Socrates dies, and so he ceases to be a man, the antecedent of this consequence may be true as long as there are humans all of whom run, but the consequent is false, because then Socrates, no longer being a man, cannot run. But as long as Socrates is alive, the consequence is valid, for its antecedent cannot be true without its consequent.

There are a number of interesting features of these divisions. The first, alluded to earlier, is that it does not contain the division of consequences into formal and material ones; that distinction comes up later in Burley's discussion, in connection with the solution of a problem. The second interesting point is that even among simple consequences, which do not require some further, extrinsic conditions to hold for their validity, there is the distinction between natural and accidental consequences, a distinction based on whether they hold in virtue of an "intrinsic" or "extrinsic" topic. This is the kind of distinction that tends to puzzle modern commentators. After all, the rule that from the impossible anything follows seems to be the direct implication of the understanding of the notion of validity as the impossibility of the simultaneous truth of the antecedent with the negation of the consequent, since if the antecedent cannot be true in itself, then of course it cannot be true together with anything, let alone the negation of the consequent. However, Burley's description of the rule as being based on an extrinsic topic, validating an accidental consequence, as opposed to a natural consequence, which holds in virtue of an intrinsic topic, may suggest that the former should be somehow weaker than the latter, whereas from the point of view of our modern intuitions just the opposite seems to be the case: 'A man runs; therefore, an animal runs' is not even a formally valid consequence, whereas by Burley's lights it is a simple, natural consequence that holds by virtue of an intrinsic topic; so, one would think, it should somehow be "stronger" than 'A man is an ass; therefore, you are sitting'. So, what is going on?

Even if Burley does not quite elaborate, we can get further hints from his thirteenth-century predecessors, such as William of Sherwood, Peter of Spain and the author of the *Summa Lamberti*. A *locus* is described by these authors as "the seat of an argument" (*sedes argumenti*) or that from which an appropriate argument is elicited. Each *locus* contains several maxims, where a maxim is described as "a known general proposition containing and confirming many arguments" (*nota propositio et communis multa continens et confirmans argumenta* – Sherwood, 1983, p. 248). The *loci* are commonly divided into intrinsic, extrinsic, and intermediate. Their distinction is described most succinctly by William of Sherwood in the following way:

> "When there is some doubt about a proposition, we first form it as a question, next we find the middle and we syllogize it affirmatively or negatively. When, therefore, an argument is elicited from an internal property of one of the terms of the question, then the locus is said to be intrinsic, when from an external property, then the locus is called extrinsic, and when from an intermediate property, then the locus is said to be intermediate." (Sherwood, 1983, p. 248)

Here we should realize that the distinction between internal, intermediate and extrinsic properties is closely related to Aristotle's doctrine of the *Categories* (as is his doctrine of the *Topics* in general). Accordingly, the division of loci into intrinsic extrinsic and intermediate is based on whether the middle whereby the terms of the original question are going to be joined in the conclusion is an intrinsic, extrinsic, or intermediate property of the substance of things to which the terms in the question apply. So, when Burley is claiming that a consequence can be formal on account of the form of simple terms, and when such a consequence is natural that holds by virtue of an intrinsic topic, then he refers to such rules of inference that are validated "formally" for entire sets of categorematic terms, those, for instance, that are related to each other as species to genus, which is what validates, among countless others, 'This is a man; therefore, this is an animal'. So, this consequence is "formal", because it concerns not only the particular terms occurring in it (connected by a Carnapian "meaning postulate"), but any number of terms related

in the same way, namely, as species and genus, respectively. Yet, this is not a formal consequence "by reason of the form of the whole structure", but "by reason of the form of incomplex terms", that is, those terms coming under the formal, second-order concepts of genus and species (which is why such concepts were often referred as "logical intentions"), applying to all sorts of simple terms related in the same way. Yet, this consequence would not count as formal by Buridan's criterion, for it does not hold in all terms without any restriction, but only in those terms that would be permissible substituents in the schema: 'if x is an S; then x is a G', where S and G have to be related as species and genus. And this consequence is also simple, since it holds for all times, and natural, because the antecedent *includes* the consequent on account of the intrinsic locus that establishes the intensional inclusion of the predicate of the consequent in the predicate of the antecedent, and hence the intensional inclusion of the total significate of the consequent in the total significate of the antecedent. So, in the end, the intrinsic topics establish the strong intensional connections of terms (whether categorematic or syncategorematic), which in turn establish the intensional inclusion-connections about certain types of propositions that can be formed with them. This is why Burley's criterion can exclude from the realm of such strong (simple, natural, formal) consequences, which *we* might even call "relevant entailments", those consequences that are instances of the rule *ex impossibili quodlibet*, which holds only in virtue of an extrinsic topic (namely, the *locus a minori*).

So, what precisely is the status of these *loci*? If we follow Burley's lead, it might seem that they are certain formal rules of inference establishing logical connections on the basis of the meanings of certain types of categorematic terms, to mark out necessarily valid consequences based on a containment criterion, which is supposed to be stronger than mere necessary truth-preservation. Alas, things are not so simple, though. For there are extrinsic topics, such as the topic from contraries, that might be regarded as validating formal consequences insofar as they would validate consequences with any appropriately related terms, and yet, by Burley's criterion they would not count as natural, although they would still seem to be simple. (For example, 'Socrates is black; therefore, Socrates is not white', where the maxim validating the consequence is the following: "positing one of two contraries in a given subject, the other is removed from the same subject"; of course, along with the knowledge that 'black' and 'white' are contraries.) So, it would seem that Burley would have to accept formal, simple, yet accidental consequences as well.

In any case, it seems to be fairly certain that at least some of the topical maxims did serve as semantic rules to establish consequence relations that are stronger than mere necessary truth-preservation, establishing containment relations between various classes of propositional forms based on the formal logical relations of well-defined classes of their terms. In this sense, topics could function as defining a stronger, more restricted sense of formal validity as compared to Buridan's *syncategoremata*-based notion of formal validity, thereby providing a notion of validity closer to what is sought in modern "relevance logics". All such arguments are, therefore, formal in Burley's sense, but not necessarily in Buridan's sense.

However, they can be rendered formal even in Buridan's sense by adding a "missing" premise, which would be verified by the locus. This is how William of Sherwood would "reduce" topical inferences to syllogisms, and this is the practice that the *Summa Lamberti*, taking its cue from Boethius, would describe by distinguishing between two types of maxims, one that is inside the argument and one that is outside. For instance, if we say 'this is a man and every man is an animal; therefore, this is an animal', then we have just added the missing

premise inside the argument that renders the argument formally valid in Buridan's sense, but the premise itself is justified in terms of the maxim "of whatever a species is predicated, its genus is also predicated", along with the knowledge that 'man' is a species of the genus 'animal'. However, by Burley's and the older tradition's lights, the maxim licenses the inference with the same strength as the added premise would, the only difference being that when we add the proposition verified by the maxim, then the strength of the warrant provided by the antecedent is *transferred* from the strength of the consequence to the strength of the antecedent.

Besides the maxims that are able to provide such a *stronger* notion of validity, several topical rules were also regarded by medieval authors as providing a *weaker* sense of validity, which would consist in a merely *probable*, rather than necessary, preservation of truth, or alternatively, if the maxim is taken to support an additional premise, then such a maxim would warrant a merely probable premise rather than a necessary one. But without the addition of the "reductive" premise, the consequence would have to be formally invalid by Buridan's criterion of formal validity.

5.2. Irreducibly material consequences (such as induction)

However, not all arguments are reducible to formally valid arguments in this way. This is obvious in the case of induction. As Buridan writes, "an induction is not formally valid unless by the addition of another premise it becomes a syllogism", (Buridan, 2001, 6.1.5, p. 398) namely, in the case of finite induction, where we can have a complete enumeration of all singulars. However, he continues, "if an induction cannot be performed over all the singulars, as in the case of our concluding from the singulars that every fire is hot, then such an induction is not reduced to syllogism, nor does it prove its conclusion on account of its being a formally valid consequence, nor because it may be reduced to a formally valid consequence, but because of the intellect's natural inclination toward truth." (Buridan, 2001, 6.1.5, p. 399)

Whatever this "natural inclination toward truth" is and how it is supposed to validate an infinite induction Buridan never tells, but it clearly takes us beyond the realm of formally valid logical consequences in such a way that we cannot tell exactly what additional premise could reduce the consequence to a formal one that is valid in every term. In fact, it is easy to see that an induction can never be logically valid in the case of accidental predicates; so, it can never be valid in all terms. That is to say, 'This S is P and that S is P, … etc.; therefore, all S are P' can be valid only if being P is essential to anything that is an S, insofar as it is an S, but if P is accidental to S, then it is always possible to have an S that is not a P, even if perhaps all previously observed S were P, which at once invalidates the consequence. Accordingly, the enumeration of singulars is not there to provide stronger corroboration with greater numbers (see the flawed logic of Russell's chicken, in Russell, 2008, p. 44); rather, it is there to test whether the predicate is essential to the subject, which it is not a matter of logic to establish. Accordingly, induction should perhaps not even be treated as a consequence; it should rather be called scientific generalization, insofar as it is generalization over essentials, as opposed to rash generalization or prejudice, which is generalization over accidentals. (Cf. Klima, 2005)

6. Conclusion: medieval theories of consequence and modern notions of logical validity

As we have seen, medieval theories of consequence can be viewed as parts of a grand enterprise to map out the domain of logicality for natural language reasoning. The result is not what we could regard as a single, large, unified theory, defining the validity of logical consequence for all possible forms of reasoning, along with a decision procedure to sort out valid from invalid consequences. Rather, the result is a cluster of several theories covering consequences from conditional propositions of various strengths to argument forms of various strengths, ranging from what we would recognize as formal validity to mere probability. Yet, this cluster of theories all relate to the focal idea that a consequence is valid just in case the denial of the consequent is in some way "repugnant" to the antecedent. This idea of "repugnancy" was spelled out in several ways with regard to different forms of consequences. Setting the standard, we find in the centre Aristotle's syllogistic, which is a complete system for a well-defined set of argument-forms (along with a "decision procedure": check whether an actual argument fits into one of the valid forms). However, as the foregoing survey has shown, our medieval predecessors were well aware of the fact that there are huge numbers of valid, non-syllogistic arguments that can be just as strong in themselves, or can be reduced to arguments just as strong as syllogistic arguments or to arguments that actually *are* syllogistic arguments, or even arguments that are not as strong, although they are just as useful as syllogistic arguments are, and are to be counted within a comprehensive account of reasoning.

Actually, all these references to *natural* language reasoning are rather anachronistic from the medieval perspective: after all, the *only* kind of language medieval logicians worked with was a *natural* language (although a highly technical, "regimented" natural language), and are justified only in comparison to contemporary formal logical theories, defining logical validity for an explicitly constructed *artificial* language. So, in conclusion, let us briefly reflect on how the medieval approach to consequences compares to our contemporary enterprise.

Take the idea behind Richard Montague's project (Montague, 1974): given a well-defined part of a natural language, which can be translated into a (sufficiently rich) formal (intensional) logic through automated rules of translation, we can check the validity of our natural language arguments through our formal logic without getting bogged down in "the murky business of formalization". If we look at the medieval enterprise from this perspective, it may well be regarded as an enterprise comparable to Montague's, but with one important difference. Whereas the medieval enterprise used the method of "partial regimentation", namely, regulating certain forms of natural language reasoning and working out criteria of validity *directly* for those regimented forms, indeed *various criteria* for *various kinds of validity* for several forms, Montague's uses the method of "partial formalization", where arguments formulated in the regimented part of natural language are effectively translatable into a formal language, for which we have a *universal validity-checker* for a *uniformly defined notion of formal validity*.

These two different methodologies can quite naturally lead to the idea of two rather different, yet not necessarily incompatible "hypothetical projects" for a "natural logic", that is, a universal logical theory checking the validity of all possible forms of natural language reasoning:

1. The "modern project": to "cannibalize" ever greater portions of all possible forms of natural language reasoning, expand the expressive resources of our formal language(s)

for which we can have a uniform definition of validity, grounding the construction of a universal method for checking validity either in terms of deduction rules or a compositional semantics;

2. The "medieval project": to "regulate" ever greater portions of all possible forms of natural language reasoning, regiment the syntax of our natural language as much as ordinary usage would tolerate, so as to be able to accommodate as many forms of natural language reasoning as possible, and thus to be able to separate valid from invalid consequences in accordance with a range of different criteria of validity.

What the "medieval project" could learn from the "modern project", then, is the use of a recursive definition of syntax and the corresponding semantics to account for validity relations among all possible well-formed expressions of a formal language or among all possible sentences of a similarly well-defined fragment of a natural language, allowing the construction of a universal validity-checking process for a correspondingly well-defined notion of validity. On the other hand, what the "modern project" could learn from the medieval project is the accommodation of the forms of logicality that medieval logicians recognized both in various forms of formal and material consequences, as well as in the different, but not unrelated notions of their validity. In this sense, the study of medieval logic can provide a promising "shortcut" toward a truly comprehensive theory of consequences in a contemporary "natural logic".

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