

11. If John Madden has a pleasant personality and is not a political expert, then he's no TV newscaster.

EXERCISE 7-8:

1. $(Fa \cdot Ga) \cdot (Fb \cdot Gb)$
3. $[Fa \supset (Ga \vee Ha)] \cdot [Fb \supset (Gb \vee Hb)]$
5. $\sim (Fa \supset Ga) \cdot \sim (Fb \supset Gb)$
7. $\sim [(Fa \supset Ga) \cdot (Fb \supset Gb)]$
9. $[Fa \supset (Ga \supset Ha)] \cdot [Fb \supset (Gb \supset Hb)]$
11. $[(Fa \cdot Ga) \vee (Ha \cdot Ka)] \vee [(Fb \cdot Gb) \vee (Hb \cdot Kb)]$
13. $\sim [(Fa \supset Ga) \vee (Fa \supset Ha)] \vee \sim [(Fb \supset Gb) \vee (Fb \supset Hb)]$

EXERCISE 7-9:

(using obvious abbreviations)

- | | |
|---|---|
| 1. $(x)[(Sx \cdot Fx) \supset \sim Rx]$ | 7. $\sim (x)(Dx \supset Sx)$ |
| 3. $(x)(Gx \supset Px)$ | 9. $\sim (x)(Ex \supset Px) \supset (x)(Ex \supset Dx)$ |
| 5. $(x)(Dx \supset Sx)$ | 11. $(x)[(Rx \cdot Tx) \supset Cx]$ |

EXERCISE 7-10:

(using obvious abbreviations)

- | | |
|---|---|
| 1. $\sim (\exists x)(Fx \cdot Lx)$ | 13. $(\exists x)[(Sx \cdot Bx) \cdot \sim Cx]$ |
| 3. $(x)(Wx \supset Bx)$ | 15. $(x)\{[Sx \cdot \sim (Bx \vee Cx)] \supset \sim Lx\}$ |
| 5. $\sim (x)(Gx \supset Lx)$ ($Lx = x$ glitters) | 17. $\sim (\exists x)[(Px \cdot Bx) \cdot (Sx \vee \sim Lx)]$,
or $(x)[(Px \cdot Bx) \supset \sim (Sx \vee \sim Lx)]$ |
| 7. $(x)(Cx \supset Dx) \cdot (x)(\sim Cx \supset Tx)$ | 19. $(x)(Px \supset Sx) \supset (x)(Px \supset \sim Lx)$ |
| 9. $(x)(Ex \supset Ox)$ | |
| 11. $(\exists x)[(Px \cdot Bx) \cdot Sx]$ | |

EXERCISE 8-1:

- | | |
|---|---|
| <p>1. True:
 <i>Domain:</i> Unrestricted
 $Ax = x$ is an NBA basketball player
 $Bx = x$ is more than 6 feet tall
 <i>English:</i> Some NBA basketball players are more than 6 feet tall.</p> <p>False:
 <i>Domain:</i> Positive integers
 $Ax = x$ is even
 $Bx = x$ is odd
 <i>English:</i> Some positive integers are both even and odd.</p> | <p>3. True:
 <i>Domain:</i> Unrestricted
 $Ax = x$ is a bachelor
 $Bx = x$ is married
 <i>English:</i> No bachelors are married.</p> <p>False:
 <i>Domain:</i> positive integers
 $Ax = x$ is even
 $Bx = x$ is divisible by four
 <i>English:</i> No even numbers are divisible by four.</p> |
|---|---|

5. True:
Domain: People
 $Ax = x$ is a man
 $Bx = x$ is a woman
 $Cx = x$ is mortal
English: All men and women are mortal.

False:
Domain: Unrestricted
 $Ax = x$ is a whale
 $Bx = x$ is a shark
 $Cx = x$ is a fish
English: All whales and sharks are fish.

7 True:
Domain: Whales
 $Gx = x$ is a fish
 $Bx = x$ is male
English: If some whale is a fish, then not all male whales are fish.

False:
Domain: Unrestricted
 $Gx = x$ is a dog
 $Bx = x$ is a poodle
English: If something is a dog, then not all poodles are dogs.

EXERCISE 8-2:

(For our answers assume the domain of discourse to be positive integers.)

- | | |
|--|---|
| <p>1. $Ax = x$ is an even number
 $Bx = x$ is an integer
 $Cx = x$ is an odd number</p> <p>3. $Ax = x$ is even
 $Bx = x < 2$
 $Cx = x < 3$
 $Dx = x$ is odd</p> | <p>5. $Px = x > 2$
 $Qx = x > 3$
 $Rx = x > 4$</p> <p>7. $Px = x < 0$
 $Qx = x > 4$
 $Rx = x > 3$</p> <p>9. $Ax = x > 2$
 $Bx = x$ is even
 $Cx = x$ is odd</p> |
|--|---|

EXERCISE 8-3:

- (1) 1. $(Aa \cdot Ba) \vee (Ab \cdot Bb)$
T T T T F F T
2. $(Ba \cdot Ca) \vee (Bb \cdot Cb) \therefore (Aa \cdot Ca) \vee (Ab \cdot Cb)$
T F F T T T T T F F F F F T
- (3) 1. $(Aa \cdot \sim Ba) \vee (Ab \cdot \sim Bb)$
F FT F T T TT F
2. $(Aa \cdot \sim Ca) \vee (Ab \cdot \sim Cb)$
F F T F T T T T F
3. $(\sim Ba \cdot Da) \vee (\sim Bb \cdot Db)$
T F T T T T F F F
- $\therefore [Aa \cdot (\sim Ba \cdot Da)] \vee [Ab \cdot (\sim Bb \cdot Db)]$
F F T F T T F T F T F F F

$$(5) \quad 1. (Pa \cdot \sim Qa) \vee (Pb \cdot \sim Qb)$$

$$\quad \quad \quad T \quad T \quad T \quad F \quad T \quad T \quad T \quad T \quad F$$

$$2. (Ra \supset Pa) \cdot (Rb \supset Pb)$$

$$\quad \quad \quad F \quad T \quad T \quad T \quad F \quad T \quad T$$

$$\therefore (Ra \cdot \sim Qa) \vee (Rb \cdot \sim Qb)$$

$$\quad \quad \quad F \quad F \quad T \quad F \quad F \quad F \quad F \quad T \quad F$$

$$(7) \quad 1. (Pa \supset Qa) \cdot (Pb \supset Qb)$$

$$\quad \quad \quad F \quad T \quad T \quad T \quad F \quad T \quad T$$

$$2. (Qa \supset Ra) \cdot (Qb \supset Rb)$$

$$\quad \quad \quad T \quad T \quad T \quad T \quad T \quad T \quad T$$

$$\therefore (Pa \cdot Ra) \cdot (Pb \cdot Rb)$$

$$\quad \quad \quad F \quad F \quad T \quad F \quad F \quad F \quad T$$

$$(9) \quad 1. (Aa \cdot Ba) \vee (Ab \cdot Bb)$$

$$\quad \quad \quad T \quad T \quad T \quad T \quad T \quad F \quad F$$

$$2. (\sim Ba \vee \sim Ca) \cdot (\sim Bb \vee \sim Cb)$$

$$\quad \quad \quad F \quad T \quad T \quad T \quad F \quad T \quad T \quad F \quad T \quad F \quad T$$

$$\therefore (\sim Aa \vee \sim Ca) \cdot (\sim Ab \vee \sim Cb)$$

$$\quad \quad \quad F \quad T \quad T \quad T \quad F \quad F \quad F \quad T \quad F \quad F \quad T$$

EXERCISE 8-4:

(For our answers assume the domain of discourse to be positive integers.)

$$1. \quad Ax = x \text{ is divisible by 4}$$

$$\quad \quad Bx = x \text{ is even}$$

$$\quad \quad Cx = x > 3$$

$$5. \quad Rx = x \text{ is prime}$$

$$\quad \quad Tx = x \text{ is even}$$

$$\quad \quad Sx = x \text{ is odd}$$

$$3. \quad Lx = x < 0 \text{ (this guarantees the}$$

$$\quad \quad \text{truth of the first two premises)}$$

$$\quad \quad Mx = x \text{ is even}$$

$$\quad \quad Nx = x \text{ is odd (or anything you}$$

$$\quad \quad \text{like)}$$

$$7. \quad Fx = x > 2$$

$$\quad \quad Gx = x < 0$$

$$\quad \quad Dx = x > 1$$

EXERCISE 9-1:

(1) 4. $(x) \sim Gx$	1, 2 DS	OR	4. $Dx \supset Gx$	3 UI
5. $\sim Gx$	4 UI		5. $\sim Dx \vee Gx$	4 Impl
6. $Dx \supset Gx$	3 UI		6. $(\exists x)(\sim Dx \vee Gx)$	5 EG
7. $\sim Dx$	5, 6 MT			
8. $\sim Dx \vee Gx$	7 Add			
9. $(\exists x)(\sim Dx \vee Gx)$	8 EG			

(3)	4. $\sim Ax \vee (Bx \cdot Cx)$	1 UI
	5. $(Ax \supset Cx) \supset Dx$	2 UI
	6. $Dx \supset \sim Cx$	3 UI
	7. Ax	AP $\therefore \sim Ax$
	8. $Ax \supset (Bx \cdot Cx)$	4 Impl
	9. $Bx \cdot Cx$	7, 8 MP
	10. Cx	9 Simp
	11. $Cx \vee \sim Ax$	10 Add
	12. $\sim Ax \vee Cx$	11 Comm
	13. $Ax \supset Cx$	12 Impl
	14. Dx	7, 13 MP
	15. $\sim Cx$	6, 14 MP
	16. $Cx \cdot \sim Cx$	10, 15 Conj
	17. $\sim Ax$	7-16 IP
	18. $(\exists x) \sim Ax$	17 EG

(5)	3. $\sim Bc$	2 UI
	4. Ab	1, 3 DS
	5. $(\exists x) Ax$	4 EG

(7)	3. $Aa \supset (\sim Ba \supset Ca)$	1 UI
	4. Aa	AP $\therefore Ca$
	5. $\sim Ba \supset Ca$	3, 4 MP
	6. Ca	2, 5 MP
	7. $Aa \supset Ca$	4-6 CP

Exercise 9-2:

(1)	1. $(x)[(Hx \cdot Kx) \supset Mx]$	p
	2. $(\exists x)(Hx \cdot Kx)$	p
	3. $Hx \cdot Kx$	2 EI
	4. Mx	1, 3 MP (<i>invalid, you can't apply MP to part of a line</i>)
	5. $(\exists x)Mx$	4 EG
(3)	1. $(x)[(Gx \vee Hx) \supset Mx]$	p
	2. $(\exists x)(Fx \cdot \sim Mx)$	p
	3. $(Gy \vee Hy) \supset My$	1 UI
	4. $Fy \cdot \sim My$	2 EI (<i>invalid, y is already free in line 3</i>)
	5. $\sim My$	3 Simp
	6. $\sim (Gy \vee Hy)$	4, 5 MT
	7. $(\exists x) \sim (Gx \vee Hx)$	6 EG

- (5) 1. $(\exists x)[(Px \cdot Qx) \vee Rx]$ p
 2. $(x) \sim Rx$ p
 3. $(\exists x)(Px \vee Rx)$ 1 **Simp** (invalid, you can't apply **Simp** to part of a line)
 4. $Px \vee Rx$ 3 **EI**
 5. $\sim Px$ 2,4 **DS** (invalid, you can't apply **DS** to part of a line)
 6. $(x) \sim Px$ 5 **UG** (invalid, x is free in an **EI** line)
 7. $\sim Py$ 6 **UI**
 8. $(z) \sim Pz$ 7 **UG**

EXERCISE 9-3:

- | | | | | | |
|-----|-----------------------------------|-----------------|-----|-------------------------------------|------------------|
| (1) | 3. $\sim Bx$ | 2 EI | (7) | 3. $Ax \cdot Bx$ | 1 EI |
| | 4. $Rx \supset Bx$ | 1 UI | | 4. $Ax \supset Cx$ | 2 UI |
| | 5. $\sim Rx$ | 3,4 MT | | 5. Ax | 3 Simp |
| | 6. $(\exists x) \sim Rx$ | 5 EG | | 6. Cx | 4,5 MP |
| | | | | 7. Bx | 3 Simp |
| (3) | 3. $Ka \supset (y)Hy$ | 2 UI | | 8. $Bx \cdot Cx$ | 6,7 Conj |
| | 4. $(y)Hy$ | 1,3 MP | | 9. $(\exists x)(Bx \cdot Cx)$ | 8 EG |
| | 5. Hy | 4 UI | | | |
| | 6. $(x)Hx$ | 5 UG | (9) | 3. $\sim(\sim Fx \cdot \sim Rx)$ | 2 EI |
| (5) | 3. $Mx \supset Sx$ | 1 UI | | 4. $\sim \sim Fx \vee \sim \sim Rx$ | 3 DeM |
| | 4. $\sim Bx \vee Mx$ | 2 UI | | 5. $Fx \vee Rx$ | 4 DN (2x) |
| | 5. $Bx \supset Mx$ | 4 Impl | | 6. $(Fx \vee Rx) \supset \sim Gx$ | 1 UI |
| | 6. $Bx \supset Sx$ | 3,5 HS | | 7. $\sim Gx$ | 5,6 MP |
| | 7. $\sim Sx \supset \sim Bx$ | 6 Contra | | 8. $(\exists y) \sim Gy$ | 7 EG |
| | 8. $(x)(\sim Sx \supset \sim Bx)$ | 7 UG | | | |

- (11) \rightarrow
- | | |
|--------------------------------|-----------------|
| 4. Dx | AP/∴ Gx |
| 5. $\sim(Dx \cdot \sim Ex)$ | 3 UI |
| 6. $\sim Dx \vee \sim \sim Ex$ | 5 DeM |
| 7. $\sim Dx \vee Ex$ | 6 DN |
| 8. $\sim \sim Dx$ | 4 DN |
| 9. Ex | 7,8 DS |
| 10. $Ex \supset Fx$ | 2 UI |
| 11. $Fx \supset Gx$ | 1 UI |
| 12. $Ex \supset Gx$ | 10,11 HS |
| 13. Gx | 9,12 MP |
| 14. $Dx \supset Gx$ | 4-13 CP |
| 15. $(x)(Dx \supset Gx)$ | 14 UG |

EXERCISE 9-4:

- | | |
|---|--|
| 1. Incorrect. Correct use would result in $\sim(\exists x)Fx$. | 3. Incorrect. Correct use would result in $\sim(\exists x)(\sim Fx \vee Gx)$. |
|---|--|

5. Incorrect. Correct use would result in $(y) \sim (\sim Ry \cdot \sim Ky)$.

7. Correct.

9. Incorrect. Correct use would be $\sim (\exists y)Fy \supset (\exists z)(Gz \cdot Hz)$.

EXERCISE 9-5:

(1) 3. $\sim (\exists x) Fx$ 2 QN
4. $(\exists x) Gx$ 1,3 DS

(3) 2. $(\exists x) \sim Ax$ 1 QN
3. $\sim Ax$ 2 EI
4. $\sim Ax \vee Bx$ 3 Add
5. $Ax \supset Bx$ 4 Impl
6. $(\exists x) (Ax \supset Bx)$ 5 EG

(5) 3. $(\exists x) Ex \supset (\exists x) Fx$ 2 QN
4. $(\exists x) Ex \supset (x) \sim Gx$ 1,3 HS
5. $(\exists x) Ex \supset \sim (\exists x) Gx$ 4 QN

(7) 3. $(\exists x) \sim (Ax \cdot Gx)$ 2 QN
4. $\sim (Ax \cdot Gx)$ 3 EI
5. $(Fx \vee Hx) \supset (Gx \cdot Ax)$ 1 UI
6. $(Fx \vee Hx) \supset (Ax \cdot Gx)$ 5 Comm
7. $\sim (Fx \vee Hx)$ 4,6 MT
8. $\sim Fx \cdot \sim Hx$ 7 DeM
9. $\sim Hx$ 8 Simp
10. $(\exists x) \sim Hx$ 9 EG

(9) \rightarrow 3. Fx AP/ $\therefore Lx$
4. $Fx \vee Gx$ 3 Add
5. $(Fx \vee Gx) \supset Hx$ 1 UI
6. Hx 4,5 MP
7. $Hx \vee Kx$ 6 Add
8. $(Hx \vee Kx) \supset Lx$ 2 UI
9. Lx 7,8 MP
10. $Fx \supset Lx$ 3-9 CP
11. $(x) (Fx \supset Lx)$ 10 UG

(11) 3. $(y) \sim (Cy \vee Dy)$ 2 QN
4. $\sim (Cy \vee Dy)$ 3 UI
5. $\sim Cy \cdot \sim Dy$ 4 DeM
6. $(Ay \vee By) \supset Cy$ 1 UI
7. $\sim Cy$ 5 Simp
8. $\sim (Ay \vee By)$ 6,7 MT
9. $\sim Ay \cdot \sim By$ 8 DeM
10. $\sim Ay$ 9 Simp
11. $(x) \sim Ax$ 10 UG
12. $\sim (\exists x) Ax$ 11 QN

(13) 4. $(Ab \cdot Bb) \supset Cb$ 1 UI
5. $\sim (Ab \cdot Bb)$ 3,4 MT

(15) \rightarrow 3. $\sim (\exists x) Kx$ AP/ $\therefore (\exists x)Kx$
4. $(\exists x) Hx$ 2,3 DS
5. Hx 4 EI
6. $Hx \supset Kx$ 1 UI
7. Kx 5,6 MP
8. $(x) \sim Kx$ 3 QN
9. $\sim Kx$ 8 UI
10. $Kx \cdot \sim Kx$ 7,9 Conj
11. $(\exists x)Kx$ 3-10 IP

(17) 4. $Qx \cdot Ax$ 2 EI
5. $\sim Bx \supset \sim Qx$ 3 UI
6. $Qx \supset Bx$ 5 Contra
7. Qx 4 Simp
8. Bx 6,7 MP
9. Ax 4 Simp
10. $Bx \cdot Ax$ 8,9 Conj
11. $(Bx \cdot Ax) \supset Dx$ 1 UI
12. Dx 10,11 MP
13. $Dx \cdot Qx$ 7,12 Conj
14. $(\exists x) (Dx \cdot Qx)$ 13 EG

(19) → 3.	$(x) (Sx \supset Px)$	AP/∴ $(x) (Sx \supset Rx)$
→ 4.	Sx	AP/∴ Rx
5.	$Sx \supset Px$	3 UI
6.	Px	4,5 MP
7.	$(Sx \cdot Px) \supset \sim Qx$	2 UI
8.	$Sx \cdot Px$	4,6 Conj
9.	$\sim Qx$	7,8 MP
10.	$Px \supset (Qx \vee Rx)$	1 UI
11.	$Qx \vee Rx$	6,10 MP
12.	Rx	9,11 DS
13.	$Sx \supset Rx$	4-12 CP
14.	$(x) (Sx \supset Rx)$	13 UG
15.	$(x) (Sx \supset Px) \supset (x) (Sx \supset Rx)$	3-14 CP

EXERCISE 10-1:

1. $\sim Mcb$
3. $Mb \supset \sim Mbj$
5. $\sim Sbk$
7. $(x)(Bxd \supset \sim Idx)$
9. $\sim Pbf \supset (x)(Px \supset \sim Pxf)$
11. $(\exists x)(Px \cdot Sxjh) \supset \sim (\exists x) (Px \cdot Sxab)$
13. $(x)(Wxb \supset Rhx)$
15. $(\exists x)(Mxa \cdot Lmx) \cdot (\exists x)(Mxa \cdot \sim Lmx)$
17. $(\exists x)(Px \cdot Txca) \cdot (\exists x)(Px \cdot \sim Txca)$
19. $\sim Hca \supset (\exists x)(Sxah \cdot \sim Tx)$

EXERCISE 10-2:

1. $(x) (y) Sxy$
 $(Saa \cdot Sab) \cdot (Sba \cdot Sbb)$
3. $(\exists x) (y) Sxy$
 $(Saa \cdot Sab) \vee (Sba \cdot Sbb)$
5. $\sim (x) (\exists y) Sxy$
 $\sim [(Saa \vee Sab) \cdot (Sba \vee Sbb)]$
7. $\sim (\exists x) (\exists y) Sxy$
 $\sim [(Saa \vee Sab) \vee (Sba \vee Sbb)]$
9. $\sim \sim (\exists x) (\exists y) Sxy$ (or $(\exists x) (\exists y) (Sxy)$)
 $\sim \sim [(Saa \vee Sab) \vee (Sba \vee Sbb)]$

11. $\sim (\exists x) Sxx$
 $\sim (Saa \vee Sbb)$

EXERCISE 10-3:

1. $(\exists x) (Sx \cdot (y) \sim Kxy)$
3. $(\exists x) \{Sx \cdot (y) [(Qy \cdot Txy) \supset Pxy]\}$
5. $(x) \{Gx \supset (y) [(Cy \cdot Txy) \supset Pxy]\}$
7. $Gm \cdot \sim (y) [(Ey \cdot Tmy) \supset Pmy]$
9. $(x) [(Cx \cdot Pmx) \supset Tjx]$
11. $\sim (\exists x) [Cx \cdot (y) (Py \supset Tyx)]$
13. $(x) [Qx \supset (y) (Ey \supset Ixy)]$
15. $\sim (\exists x) (Px \cdot Ixj)$
17. $(\exists x) \{Sx \cdot (y) [(Cy \cdot Txy) \supset (z) ((Cz \cdot Tjz) \supset \sim Iyz)]\}$ or
 $(\exists x) \{Sx \cdot (y) (z) [(Cy \cdot Cz) \cdot (Tjz \cdot Iyz)] \supset \sim Txy\}$
19. $\sim (\exists x) \{Px \cdot (y) [(Ey \cdot Tjy) \supset Pxy]\}$
21. $(\exists x) \{Sx \cdot (y) [(Cy \cdot \sim Dy) \supset Txy]\}$
23. $(x) (\{Sx \cdot (\exists y) [(Cy \cdot Dy) \cdot Txy]\} \supset Gx)$ or $(x) (y) \{[(Sx \cdot Cy) \cdot (Dy \cdot Txy)] \supset Gx\}$
25. $(x) \{(Cx \cdot Tmx) \supset (y) [(Cy \cdot Tjy) \supset Hxy]\}$
27. $(x) (\{Sx \cdot (y) [(Cy \cdot Txy) \supset Pxy]\} \supset Gx)$
29. $(x) [(Cx \cdot Tjx) \supset Tmx]$ or $(x) [Cx \supset (\sim Tmx \supset \sim Tjx)]$

EXERCISE 10-4:

1. $(\exists x) (\exists y) [(Px \cdot Cy) \cdot Ayx]$ ($Px =$ "x is a place"; $Cx =$ "x is a cheater"; $Axy =$ "x is at y")
3. $(x) [Px \supset (\exists y) (Cy \cdot Ayx)]$
5. $\sim (\exists x) \{Tx \cdot (\exists y) (\exists z) [(Py \cdot Wzy) \cdot Dzx]\}$ ($Px =$ "x is a person"; $Wxy =$ "x is the work of y"; $Tx =$ "x is a time"; $Dxy =$ "x is done at y")
7. That is, there are times when you can fool everyone: $(\exists x) [Tx \cdot (y) (Py \supset Fyx)]$
($Px =$ "x is a person"; $Tx =$ "x is a time"; $Fxy =$ "you can fool x at y")
9. That is, it is not the case that you can always fool everyone:
 $\sim (x) [Tx \supset (y) (Py \supset Fyx)]$
11. $\sim (\exists x) \{Tx \cdot (\exists y) [(Dy \cdot Byx) \cdot Iyx]\}$ ($Tx =$ "x is a time"; $Dx =$ "x is a dog"; $Bxy =$ "x barks at y"; $Ixy =$ "x bites at y")
13. $\sim (\exists x) [(Tx \vee Px) \cdot \sim (\exists y) (Sy \cdot Ayx)]$ ($Tx =$ "x is a time"; $Px =$ "x is a place"; $Sx =$ "x is a sin"; $Axy =$ "x is at y")
15. $(x) [Mx \supset (\exists y) (Sy \cdot Byx)]$ ($Mx =$ "x is a minute"; $Sx =$ "x is a sucker"; $Bxy =$ "x is born at y")

EXERCISE 10-5:

1. $(x) \{Dx \supset [Px \cdot (y) (Ty \supset \sim Lxy)]\}$ ($Dx =$ "x is a drama critic"; $Px =$ "x is a person"; $Tx =$ "x is a turn"; $Lxy =$ "x leaves y unstoned")

3. $(x) \{Cx \supset [Px \cdot ((y) Pxy \cdot (y) \sim Vxy)]\}$ ($Cx =$ “ x is a cynic”; $Px =$ “ x is a person”; $Pxy =$ “ x knows the price of y ”; $Vxy =$ “ x knows the value of y ”). Or perhaps it is meant as a definition: $(x) \{Cx \equiv [Px \cdot ((y) Pxy \cdot (y) \sim Vxy)]\}$?
5. $(x) \{[Px \cdot (\exists y)(Sy \cdot Ayx)] \supset (\exists z) (Fz \cdot Azx)\}$ ($Px =$ “ x is a place”; $Sx =$ “smoke is at x ”; $Axy =$ “ x is at y ”; $Fx =$ “ x is fire”).
7. That is, if a person has sin then they are not permitted to cast the first stone:
 $(x) [(Px \cdot (\exists y) (Sy \cdot Hxy)) \supset \sim Cx]$ ($Px =$ “ x is a person”; $Cx =$ “ x is permitted to cast the first stone”; $Sx =$ “ x is sin”; $Hxy =$ “ x has y ”)
9. $(x) \{[Px \cdot (\exists y) (Sy \cdot Jxy)] \supset \sim (\exists z) (Wz \cdot Fxz)\}$ ($Px =$ “ x is a person”; $Sx =$ “ x is a scar”; $Jxy =$ “ x jests at y ”; $Wx =$ “ x is a wound”; $Fxy =$ “ x has felt y ”)
11. $(x) \{[Px \cdot (\exists y) (Sy \cdot Cxy)] \supset Ox\}$ ($Px =$ “ x is a person”; $Sx =$ “ x is a psychiatrist”; $Cxy =$ “ x consults y ”; $Ox =$ “ x ought to have his head examined”)
13. $(x) [(Px \cdot Hxx) \supset Hgx]$ ($Px =$ “ x is a person”; $Hxy =$ “ x helps y ”; $g =$ “God”)
15. $(x) [Hx \cdot (\exists y) (Ay \cdot Cxy)]$ ($Hx =$ “ x is a major horror of history”; $Ax =$ “ x is an altruistic motive”; $Cxy =$ “ x is committed in the name of y ”)
17. $(x) [(Lx \cdot Cxx) \supset (\exists z) (Tzx \cdot Fz)]$ ($Lx =$ “ x is a lawyer”; $Cxy =$ “ x pleads y ’s case”; $Txy =$ “ x is a client of y ”; $Fx =$ “ x is a fool”)
19. $(x) \{[Px \cdot (\exists y) (Py \cdot Sxy)] \supset (\exists z) (Pz \cdot Sxz)\}$ ($Px =$ “ x is a person”; $Sxy =$ “ x sheds y ’s blood”)

EXERCISE 10-6:

1. $(x) [(Cx \cdot Bx) \supset Dx]$ ($Cx =$ “ x is a company”; $Bx =$ “ x goes bankrupt”; $Dx =$ “ x deserves to go out of business”)
3. $(x) \{[Fx \cdot (\exists y) (Py \cdot \sim Cxy)] \supset (\exists z) (Lz \cdot Wz)\}$ ($Fx =$ “ x is a frontrunner”; $Px =$ “ x is a potential voter”; $Cxy =$ “ x caters to y ”; $Lx =$ “ x is a longshot”; $Wx =$ “ x is going to win”)
5. $(x) [(Px \cdot Bxg) \supset (y) (Cyg \supset Oxy)]$ ($Px =$ “ x is a person”; $Bxy =$ “ x believes in y ”; $Cxy =$ “ x is a commandment of y ”; $Oxy =$ “ x obeys y ”; $g =$ God)
7. $(x) \{[Px \cdot (\exists y) (Dy \cdot Bxy)] \supset (\exists z) (Sz \cdot Oxz)\}$ ($Px =$ “ x is a person”; $Dx =$ “ x is a scientific discovery”; $Bxy =$ “ x has benefited from y ”; $Sx =$ “ x is a scientist”; $Oxy =$ “ x owes money to y ”)
9. $(x) [Px \supset (\exists y) (Sy \cdot Bxy)] \supset (\exists x)[Px \cdot (\exists y) (Lyx \cdot \sim Pxy)]$
 $(Px =$ “ x is a person”; $Sx =$ “ x is a scientific discovery”; $Bxy =$ “ x has benefited from y ”; $Lxy =$ “ x is a bill of y ’s”; $Pxy =$ “ x has paid y ”)

11. $\sim (\exists x) \{Sx \cdot (\exists y)[(Py \cdot Ay) \cdot Hxy]\}$ ($Px =$ “ x is a person”; $Sx =$ “ x is a psychiatrist”;
“ $Ax =$ “ x acts on astrological advice”; $Hxy =$ “ x can help y ”)
13. $(x)(y) \{[(Px \cdot My) \cdot Oxy] \supset (\exists z)(Tz \cdot Pxyz)\}$ ($Mx =$ “ x is money”; $Px =$ “ x is a
person”; $Oxyz =$ “ x owes y to z ”; $Tx =$ “ x is a time”; $Pxyz =$ “ x should pay y to z
at u ”; $n =$ Isaac Newton)

EXERCISE 10-7:

1. Everyone believes in God.
3. Some people don't believe in God.
5. No one believes in God.
7. No one who is disenfranchised is his own master.
9. Everyone has some redeeming feature (or other).
11. There is some redeeming feature everybody has.
13. No one has any redeeming feature.
15. Anyone who has every redeeming feature believes in God.
17. Those who vote are masters of those who don't.

EXERCISE 10-8:

- (1) 1. $(Faa \cdot Fab) \vee (Fba \cdot Fbb)$
T T T T F F F
 $\therefore (Faa \cdot Fba) \cdot (Fab \cdot Fbb)$
T F F F T F F
- (3) 1. $(Faa \vee Fab) \vee (Fba \vee Fbb)$
F T T T F F F
 2. $(Gaa \vee Gab) \vee (Gba \vee Gbb)$
T T F T T T F
 $\therefore [(Faa \cdot Gaa) \vee (Fab \cdot Gab)] \vee [(Fba \cdot Gba) \vee (Fbb \cdot Gbb)]$
F F T F T F F F F F T F F F F

(5) 1. $(Faa \cdot Fab) \vee (Fba \cdot Fbb)$
F F T T T T
 $\therefore Faa$
F

(7) 1. $[(Faaa \vee Faab) \cdot (Faba \vee Fabb)] \cdot [(Fbaa \vee Fbab) \cdot (Fbba \vee Fbbb)]$
F T T T T T F T F T T T T F
 $\therefore [(Faaa \cdot Faab) \cdot (Faba \cdot Fabb)] \vee [(Fbaa \cdot Fbab) \cdot (Fbba \cdot Fbbb)]$
F F T F T F F F F F T F T F F

EXERCISE 10-9:

- (1) Inference to line 5 is invalid since x is used to replace an instance of y in line 4 when y is not free in line 4, **UG** can only be done on a free variable.
- (3) Inference to line 4 violates second restriction on **UG**.
- (5) 1. Inference to line 4 violates the requirement that quantifiers be dropped only if they quantify whole lines.
 2. Inference to line 7 is invalid because line 5 does not have the form required for **MP**.
 3. Inference to line 11 violates the third restriction on **UG**.
 4. Inference to line 13 violates the requirement that quantifiers added by **EG** or **UG** must quantify the whole line to which they are added.

EXERCISE 10-10:

- (1) $\left\{ \begin{array}{ll} 3. Ma & \text{AP/} \therefore Ob \\ 4. (y) [(Ma \vee Py) \supset (Oa \vee Ob)] & 2 \text{ UI} \\ 5. (Ma \vee Pb) \supset (Oa \vee Ob) & 4 \text{ UI} \\ 6. Ma \vee Pb & 3 \text{ Add} \\ 7. Oa \vee Ob & 5,6 \text{ MP} \\ 8. Ob & 1,7 \text{ DS} \\ \hline 9. Ma \supset Ob & 3-8 \text{ CP} \end{array} \right.$
- (3) $\left\{ \begin{array}{ll} 3. Aw \cdot Bw & 2 \text{ EI} \\ 4. (Aw \cdot Bw) \supset (\exists y) Cy & 1 \text{ UI} \\ 5. (\exists y) Cy & 3,4 \text{ MP} \\ 6. Cy & 5 \text{ EI} \\ 7. (\exists x) Cx & 6 \text{ EG} \end{array} \right.$
- (5) $\left\{ \begin{array}{ll} 2. \sim (y) (Mya \supset Oay) & \text{AP/} \therefore (y) (Mya \supset Oay) \\ 3. (\exists y) \sim (Mya \supset Oay) & 2 \text{ QN} \\ 4. \sim (Mwa \supset Oaw) & 3 \text{ EI} \\ 5. (y) (Mwa \supset Oay) & 1 \text{ UI} \\ 6. Mwa \supset Oaw & 5 \text{ UI} \\ \hline 7. (Mwa \supset Oaw) \cdot \sim (Mwa \supset Oaw) & 4,6 \text{ Conj} \\ 8. (y) (Mya \supset Oay) & 2-7 \text{ IP} \end{array} \right.$
- (7) $\left\{ \begin{array}{ll} 2. (y) (z) Azyw & 1 \text{ EI} \\ 3. (z) Azyw & 2 \text{ UI} \\ 4. Axyw & 3 \text{ UI} \\ 5. (\exists z) Axyz & 4 \text{ EG} \\ 6. (y) (\exists z) Axyz & 5 \text{ UG} \\ 7. (x) (y) (\exists z) Axyz & 6 \text{ UG} \end{array} \right.$
- (9) $\left\{ \begin{array}{ll} 2. (\exists y) \sim (\exists x) \sim (Bxy \cdot Byx) & 1 \text{ QN} \\ 3. \sim (\exists x) \sim (Bxy \cdot Byx) & 2 \text{ EI} \\ 4. (x) (Bxy \cdot Byx) & 3 \text{ QN} \\ 5. Byy \cdot Byy & 4 \text{ UI} \\ 6. Byy & 5 \text{ Simp} \\ 7. (\exists x) Bxx & 6 \text{ EG} \end{array} \right.$
- (11) $\left\{ \begin{array}{ll} 3. (x) (Fx \supset Gwx) & 2 \text{ EI} \\ 4. Fz \supset Gwz & 3 \text{ UI} \\ 5. (y) \sim Gwy & 1 \text{ UI} \\ 6. \sim Gwz & 5 \text{ UI} \\ 7. \sim Fz & 4,6 \text{ MT} \\ 8. (\exists x) \sim Fx & 7 \text{ EG} \\ 9. \sim (x) Fx & 8 \text{ QN} \end{array} \right.$

- | | | | |
|------|----|--------------------------------------|--------|
| (13) | 3. | $(y) Awy$ | 1 EI |
| | 4. | $(\exists y) (Awy \supset \sim Bww)$ | 2 UI |
| | 5. | $Awz \supset \sim Bww$ | 4 EI |
| | 6. | Awz | 3 UI |
| | 7. | $\sim Bww$ | 5,6 MP |
| | 8. | $(\exists y) \sim Byy$ | 7 EG |
| | 9. | $\sim (y) Byy$ | 8 QN |

EXERCISE 10-11:

- | | | | |
|-----|-----|--|----------|
| (1) | 3. | $Az \cdot (y) (Qy \supset Lzy)$ | 1 EI |
| | 4. | $Az \supset (y) (Ey \supset \sim Lzy)$ | 2 UI |
| | 5. | Az | 3 Simp |
| | 6. | $(y) (Ey \supset \sim Lzy)$ | 4,5 MP |
| | 7. | $Ex \supset \sim Lzx$ | 6 UI |
| | 8. | $(y) (Qy \supset Lzy)$ | 3 Simp |
| | 9. | $Qx \supset Lzx$ | 8 UI |
| | 10. | $\sim \sim Lzx \supset \sim Ex$ | 7 Contra |
| | 11. | $Lzx \supset \sim Ex$ | 10 DN |
| | 12. | $Qx \supset \sim Ex$ | 9,11 HS |
| | 13. | $(x) (Qx \supset \sim Ex)$ | 12 UG |

- | | | | |
|-----|----|---------------------------------|---------------------------------|
| (3) | 2. | $(x) Fx$ | AP/ $\therefore (\exists y) Gy$ |
| | 3. | Fx | 2 UI |
| | 4. | $(\exists y) (\sim Fx \vee Gy)$ | 1 UI |
| | 5. | $\sim Fx \vee Gy$ | 4 EI |
| | 6. | $\sim \sim Fx$ | 3 DN |
| | 7. | Gy | 5,6 DS |
| | 8. | $(\exists y) Gy$ | 7 EG |
| | 9. | $(x) Fx \supset (\exists y) Gy$ | 2-8 CP |

- | | | | |
|-----|-----|---------------------------------|--------|
| (5) | 3. | $Hx \cdot (y) (Ky \supset Lxy)$ | 2 EI |
| | 4. | Hx | 3 Simp |
| | 5. | $(\exists x) Hx$ | 4 EG |
| | 6. | $(\exists y) Ky$ | 1,4 MP |
| | 7. | Ky | 6 EI |
| | 8. | $(y) (Ky \supset Lxy)$ | 3 Simp |
| | 9. | $Ky \supset Lxy$ | 8 UI |
| | 10. | Lxy | 7,9 MP |
| | 11. | $(\exists y) Lxy$ | 10 EG |
| | 12. | $(\exists x) (\exists y) Lxy$ | 11 EG |

(7)	3.	$\sim (\exists y) (Gy \vee Lx)$	AP/∴	$(\exists z) (Ez \cdot Nzx)$
	4.	$(y) \sim (Gy \vee Lx)$	3	QN
	5.	$\sim (Gy \vee Lx)$	4	UI
	6.	$\sim Gy \cdot \sim Lx$	5	DeM
	7.	$\sim Lx$	6	Simp
	8.	$(y) [(\sim Lx \vee Mx) \supset Nyx]$	2	UI
	9.	$(\sim Lx \vee Mx) \supset Nyx$	8	UI
	10.	$\sim Lx \vee Mx$	7	Add
	11.	Nyx	9,10	MP
	12.	$Ey \vee Gy$	1	UI
	13.	$\sim Gy$	6	Simp
	14.	Ey	12,13	DS
	15.	$Ey \cdot Nyx$	11,14	Conj
	16.	$(\exists z) (Ez \cdot Nzx)$	15	EG
	17.	$\sim (\exists y) (Gy \vee Lx) \supset$ $(\exists z) (Ez \cdot Nzx)$	3-16	CP
	18.	$(x) [\sim (\exists y) (Gy \vee Lx) \supset$ $(\exists z) (Ez \cdot Nzx)]$	17	UG

(9)	4.	$(x) \sim (Axa \cdot \sim Bxb)$	1	QN
	5.	$(x) \sim (Cxc \cdot Cbx)$	2	QN
	6.	$\sim (Aea \cdot \sim Beb)$	4	UI
	7.	$\sim Aea \vee \sim \sim Beb$	6	DeM
	8.	$\sim Aea \vee Beb$	7	DN
	9.	$Aea \supset Beb$	8	Impl
	10.	$Beb \supset Cbf$	3	UI
	11.	$\sim (Cfc \cdot Cbf)$	5	UI
	12.	$\sim Cfc \vee \sim Cbf$	11	DeM
	13.	$\sim Cbf \vee \sim Cfc$	12	Comm
	14.	$Cbf \supset \sim Cfc$	13	Impl
	15.	$Aea \supset Cbf$	9,10	HS
	16.	$Aea \supset \sim Cfc$	14,15	HS
	17.	$\sim Aea \vee \sim Cfc$	16	Impl
	18.	$\sim (Aea \cdot Cfc)$	17	DeM

(11)	2. $\sim (\exists z) (Bz \cdot Cxz)$	AP/∴ $\sim (\exists y) (Ay \cdot Cxy)$
	3. $(z) \sim (Bz \cdot Cxz)$	2 QN
	4. $\sim (Bz \cdot Cxz)$	3 UI
	5. $\sim Bz \vee \sim Cxz$	4 DeM
	6. $Bz \supset \sim Cxz$	5 Impl
	7. Az	AP/∴ $\sim Cxz$
	8. $Az \supset Bz$	1 UI
	9. Bz	7,8 MP
	10. $\sim Cxz$	6,9 MP
	11. $Az \supset \sim Cxz$	7-10 CP
	12. $\sim Az \vee \sim Cxz$	11 Impl
	13. $\sim (Az \cdot Cxz)$	12 DeM
	14. $(y) \sim (Ay \cdot Cxy)$	13 UG
	15. $\sim (\exists y) (Ay \cdot Cxy)$	14 QN
	16. $\sim (\exists z) (Bz \cdot Cxz) \supset$ $\quad \sim (\exists y) (Ay \cdot Cxy)$	2-15 CP
	17. $(\exists y) (Ay \cdot Cxy) \supset$ $\quad (\exists z) (Bz \cdot Cxz)$	16 Contra
	18. $(x) [(\exists y) (Ay \cdot Cxy) \supset$ $\quad (\exists z) (Bz \cdot Cxz)]$	17 UG

(13)	3. $\sim (x) [(Fx \cdot Px) \supset (\exists y) Gy]$	AP/∴ $(x) [(Fx \cdot Px) \supset (\exists y) Gy]$
	4. $(\exists x) \sim [(Fx \cdot Px) \supset (\exists y) Gy]$	3 QN
	5. $\sim [(Fx \cdot Px) \supset (\exists y) Gy]$	4 EI
	6. $\sim [\sim (Fx \cdot Px) \vee (\exists y) Gy]$	5 Impl
	7. $\sim \sim (Fx \cdot Px) \cdot \sim (\exists y) Gy$	6 DeM
	8. $\sim \sim (Fx \cdot Px)$	7 Simp
	9. $Fx \cdot Px$	8 DN
	10. Fx	9 Simp
	11. $(\exists x) Fx$	10 EG
	12. $(x) [Px \supset (\exists y) Qxy]$	1,11 MP
	13. $Px \supset (\exists y) Qxy$	12 UI
	14. Px	9 Simp
	15. $(\exists y) Qxy$	13,14 MP
	16. Qxy	15 EI
	17. $(y) (Qxy \supset Gx)$	2 UI
	18. $Qxy \supset Gx$	17 UI
	19. Gx	16,18 MP
	20. $\sim (\exists y) Gy$	7 Simp
	21. $(y) \sim Gy$	20 QN
	22. $\sim Gx$	21 UI
	23. $Gx \cdot \sim Gx$	19,22 Conj
	24. $(x) [(Fx \cdot Px) \supset (\exists y) Gy]$	3-23 IP

EXERCISE 10-12:

(1) \rightarrow	1. $(x)(y) Fxy$	AP $\therefore (y)(x) Fxy$
	2. $(y) Fxy$	1 UI
	3. Fxy	2 UI
	4. $(x) Fxy$	3 UG
	5. $(y)(x) Fxy$	4 UG
	<hr/>	
	6. $(x)(y) Fxy \supset (y)(x) Fxy$	1-5 CP
\rightarrow	7. $(y)(x) Fxy$	AP $\therefore (x)(y) Fxy$
	8. $(x) Fxy$	7 UI
	9. Fxy	8 UI
	10. $(y) Fxy$	9 UG
	11. $(x)(y) Fxy$	10 UG
	<hr/>	
	12. $(y)(x) Fxy \supset (x)(y) Fxy$	7-11 CP
	13. $6 \cdot 12$	6,12 Conj
	14. $(x)(y) Fxy \equiv (y)(x) Fxy$	13 Equiv
(3) \rightarrow	1. $(\exists x) Gx$	AP $\therefore (\exists y) Gy$
	2. Gx	1 EI
	3. $(\exists y) Gy$	2 EG
	<hr/>	
	4. $(\exists x) Gx \supset (\exists y) Gy$	1-3 CP
\rightarrow	5. $(\exists y) Gy$	AP $\therefore (\exists x) Gx$
	6. Gy	5 EI
	7. $(\exists x) Gx$	6 EG
	<hr/>	
	8. $(\exists y) Gy \supset (\exists x) Gx$	5-7 CP
	9. $4 \cdot 8$	4,8 Conj
	10. $(\exists x) Gx \equiv (\exists y) Gy$	9 Equiv

(5)	1. $(x)(Fx \cdot Gx)$	AP/∴ $(x)Fx \cdot (x)Gx$
	2. $Fx \cdot Gx$	1 UI
	3. Fx	2 Simp
	4. $(x)Fx$	3 UG
	5. Gx	2 Simp
	6. $(x)Gx$	5 UG
	7. $(x)Fx \cdot (x)Gx$	4,6 Conj
	8. $(x)(Fx \cdot Gx) \supset$ $[(x)Fx \cdot (x)Gx]$	1-7 CP
	9. $(x)Fx \cdot (x)Gx$	AP/∴ $(x)(Fx \cdot Gx)$
	10. $(x)Fx$	9 Simp
	11. Fx	10 UI
	12. $(x)Gx$	9 Simp
	13. Gx	12 UI
	14. $Fx \cdot Gx$	11,13 Conj
	15. $(x)(Fx \cdot Gx)$	14 UG
	16. $[(x)Fx \cdot (x)Gx] \supset (x)(Fx \cdot Gx)$	9-15 CP
	17. 8 · 16	8,16 Conj
	18. $(x)(Fx \cdot Gx) \equiv [(x)Fx \cdot (x)Gx]$	17 Equiv

(7)	1. $(x)(Fx \supset Gx)$	AP/∴ $(\exists x)Fx \supset (\exists x)Gx$
	2. $(\exists x)Fx$	AP/∴ $(\exists x)Gx$
	3. Fx	2 EI
	4. $Fx \supset Gx$	1 UI
	5. Gx	3,4 MP
	6. $(\exists x)Gx$	5 EG
	7. $(\exists x)Fx \supset (\exists x)Gx$	2-6 CP
	8. $(x)(Fx \supset Gx) \supset$ $[(\exists x)Fx \supset (\exists x)Gx]$	1-7 CP

(9)	1. $(\exists x) (Fx \vee Gx)$	AP $\therefore (\exists x) Fx \vee (\exists x) Gx$
	2. $Fx \vee Gx$	1 EI
	3. $\sim (\exists x) Fx$	AP $\therefore (\exists x) Gx$
	4. $(x) \sim Fx$	3 QN
	5. $\sim Fx$	4 UI
	6. Gx	2,5 DS
	7. $(\exists x) Gx$	6 EG
	8. $\sim (\exists x) Fx \supset (\exists x) Gx$	3-7 CP
	9. $\sim \sim (\exists x) Fx \vee (\exists x) Gx$	8 Impl,
	10. $(\exists x) Fx \vee (\exists x) Gx$	9 DN
	11. $(\exists x) (Fx \vee Gx) \supset$ $[(\exists x) Fx \vee (\exists x) Gx]$	1-10 CP

Equivalence can actually be proven:

	12. $\sim (\exists x) (Fx \vee Gx)$	AP $\therefore \sim [(\exists x) Fx \vee (\exists x) Gx]$
	13. $(x) \sim (Fx \vee Gx)$	12 QN
	14. $\sim (Fy \vee Gy)$	13 UI
	15. $\sim Fy \cdot \sim Gy$	14 DeM
	16. $\sim Fy$	15 Simp
	17. $(x) \sim Fx$	16 UG
	18. $\sim (\exists x) Fx$	17 QN
	19. $\sim Gy$	15 Simp
	20. $(x) \sim Gx$	19 UG
	21. $\sim (\exists x) Gx$	20 QN
	22. $\sim (\exists x) Fx \cdot \sim (\exists x) Gx$	18,21 Conj
	23. $\sim [(\exists x) Fx \vee (\exists x) Gx]$	22 DeM
	24. $\sim (\exists x) (Fx \vee Gx) \supset$ $\sim [(\exists x) Fx \vee (\exists x) Gx]$	12-23 CP
	25. $[(\exists x) Fx \vee (\exists x) Gx] \supset$ $(\exists x) (Fx \vee Gx)$	24 Contra
	26. 11 \cdot 25	11,25 Conj
	27. $(\exists x) (Fx \vee Gx) \equiv$ $[(\exists x) Fx \vee (\exists x) Gx]$	26 Equiv

(11) →	1. $(x)(Fx \vee P)$	AP ∴ $(x) Fx \vee P$
	2. $Fx \vee P$	1 UI
	3. $\sim P$	AP ∴ $(x) Fx$
	4. Fx	2,3 DS
	5. $(x) Fx$	4 UG
	6. $\sim P \supset (x) Fx$	3-5 CP
	7. $\sim \sim P \vee (x) Fx$	6 Impl
	8. $P \vee (x) Fx$	7 DN
	9. $(x) Fx \vee P$	8 Comm
	10. $(x)(Fx \vee P) \supset [(x) Fx \vee P]$	1-9 CP
→	11. $(x) Fx \vee P$	AP ∴ $(x)(Fx \vee P)$
→	12. $\sim P$	AP ∴ Fx
	13. $(x) Fx$	11,12 DS
	14. Fx	13 UI
	15. $\sim P \supset Fx$	12-14 CP
	16. $\sim \sim P \vee Fx$	15 Impl
	17. $P \vee Fx$	16 DN
	18. $Fx \vee P$	17 Comm
	19. $(x)(Fx \vee P)$	18 UG
	20. $[(x) Fx \vee P] \supset (x)(Fx \vee P)$	11-19 CP
	21. $10 \cdot 20$	10,20 Conj
	22. $(x)(Fx \vee P) \equiv [(x) Fx \vee P]$	21 Equiv
(13) →	1. $(x)(Fx \supset P)$	AP ∴ $(\exists x) Fx \supset P$
→	2. $(\exists x) Fx$	AP ∴ P
	3. Fx	2 EI
	4. $Fx \supset P$	1 UI
	5. P	3,4 MP
	6. $(\exists x) Fx \supset P$	2-5 CP
	7. $(x)(Fx \supset P) \supset [(\exists x) Fx \supset P]$	1-6 CP
→	8. $(\exists x) Fx \supset P$	AP ∴ $(x)(Fx \supset P)$
→	9. Fy	AP ∴ P
	10. $(\exists x) Fx$	9 EG
	11. P	8,10 MP
	12. $Fy \supset P$	9-11 CP
	13. $(x)(Fx \supset P)$	12 UG
	14. $[(\exists x) Fx \supset P] \supset (x)(Fx \supset P)$	8-13 CP
	15. $7 \cdot 14$	7,14 Conj
	16. $(x)(Fx \supset P) \equiv [(\exists x) Fx \supset P]$	15 Equiv

(15) 1.	$(\exists x) (P \vee Fx)$	AP $\therefore P \vee (\exists x) Fx$
2.	$\sim P$	AP $\therefore (\exists x) Fx$
3.	$P \vee Fx$	1 EI
4.	Fx	2,3 DS
5.	$(\exists x) Fx$	4 EG
6.	$\sim P \supset (\exists x) Fx$	2-5 CP
7.	$\sim \sim P \vee (\exists x) Fx$	6 Impl
8.	$P \vee (\exists x) Fx$	7 DN
9.	$(\exists x) (P \vee Fx) \supset [P \vee (\exists x) Fx]$	1-8 CP
10.	$P \vee (\exists x) Fx$	AP $\therefore (\exists x) (P \vee Fx)$
11.	$\sim P$	AP $\therefore Fy$
12.	$(\exists x) Fx$	10,11 DS
13.	Fy	12 EI
14.	$\sim P \supset Fy$	11-13 CP
15.	$\sim \sim P \vee Fy$	14 Impl
16.	$P \vee Fy$	15 DN
17.	$(\exists x) (P \vee Fx)$	16 EG
18.	$[P \vee (\exists x) Fx] \supset (\exists x) (P \vee Fx)$	10-17 CP
19.	$9 \cdot 18$	9,18 Conj
20.	$(\exists x) (P \vee Fx) \equiv [P \vee (\exists x) Fx]$	19 Equiv

EXERCISE 10-13:

(1) 2.	$\sim (\exists x) Fx$	AP $\therefore (\exists x) Fx$
3.	$(x) \sim Fx$	2 QN
4.	$Fx \cdot Gx$	1 EI
5.	$\sim Fx$	3 UI
6.	Fx	4 Simp
7.	$Fx \cdot \sim Fx$	5,6 Conj
8.	$(\exists x) Fx$	2-7 IP
(3) 2.	$(y) (Hy \cdot Ky)$	AP $\therefore \sim (y) (Hy \cdot Ky)$
3.	$Hy \cdot Ky$	2 UI
4.	$Hy \supset \sim Ky$	1 UI
5.	Hy	3 Simp
6.	$\sim Ky$	4,5 MP
7.	Ky	3 Simp
8.	$Ky \cdot \sim Ky$	6,7 Conj
9.	$\sim (y) (Hy \cdot Ky)$	2-8 IP
10.	$(\exists y) \sim (Hy \cdot Ky)$	9 QN

(5) →	4. $\sim (Tb \cdot Rb)$	AP/∴ $Tb \cdot Rb$
	5. Rb	3 UI
	6. $\sim Tb \vee \sim Rb$	4 DeM
	7. $\sim \sim Rb$	5 DN
	8. $\sim Tb$	6,7 DS
	9. $(Rb \cdot Ab) \supset Tb$	1 UI
	10. $Rb \cdot Ab$	2,5 Conj
	11. Tb	9,10 MP
	12. $Tb \cdot \sim Tb$	8,11 Conj
	<hr/>	
	13. $Tb \cdot Rb$	4-12 IP

(7) →	3. $\sim (\exists x) \sim Fx$	AP/∴ $(\exists x) \sim Fx$
	4. $(x) \sim \sim Fx$	3 QN
	5. $\sim Gx$	2 EI
	6. $Fx \supset Gx$	1 UI
	7. $\sim Fx$	5,6 MT
	8. $\sim \sim Fx$	4 UI
	9. $\sim Fx \cdot \sim \sim Fx$	7,8 Conj
	<hr/>	
	10. $(\exists x) \sim Fx$	3-9 IP

(9) →	3. $\sim (x) (\sim Sx \supset \sim Bx)$	AP/∴ $(x) (\sim Sx \supset \sim Bx)$
	4. $(\exists x) \sim (\sim Sx \supset \sim Bx)$	3 QN
	5. $\sim (\sim Sx \supset \sim Bx)$	4 EI
	6. $\sim (\sim \sim Sx \vee \sim Bx)$	5 Impl
	7. $\sim (Sx \vee \sim Bx)$	6 DN
	8. $\sim Sx \cdot \sim \sim Bx$	7 DeM
	9. $\sim Sx$	8 Simp
	10. $Mx \supset Sx$	1 UI
	11. $\sim Mx$	9,10 MT
	12. $\sim Bx \vee Mx$	2 UI
	13. $\sim Bx$	11,12 DS
	14. $\sim \sim Bx$	8 Simp
	15. $\sim Bx \cdot \sim \sim Bx$	13,14 Conj
	<hr/>	
	16. $(x) (\sim Sx \supset \sim Bx)$	3-15 IP

EXERCISE 10-14:

- | | | |
|-------|--|--|
| (1) → | 3. $\sim (\exists x) Gx$ | AP/∴ $(\exists x) Gx$ |
| | 4. $(x) \sim Gx$ | 3 QN |
| | 5. $(\exists y) Fxy$ | 2 EI |
| | 6. Fxy | 5 EI |
| | 7. $(y) (Fxy \supset Gx)$ | 1 UI |
| | 8. $Fxy \supset Gy$ | 7 UI |
| | 9. Gy | 6,8 MP |
| | 10. $\sim Gy$ | 4 UI |
| | 11. <u>$Gy \cdot \sim Gy$</u> | 9,10 Conj |
| | 12. $(\exists x) Gx$ | 3-11 IP |
| | | |
| (3) → | 2. $\sim (\exists x) (Fx \cdot Hxa)$ | AP/∴ $(\exists x) (Fx \cdot Hxa)$ |
| | 3. $Fx \cdot (y) Hxy$ | 1 EI |
| | 4. $(x) \sim (Fx \cdot Hxa)$ | 2 QN |
| | 5. $\sim (Fx \cdot Hxa)$ | 4 UI |
| | 6. Fx | 3 Simp |
| | 7. $(y) Hxy$ | 3 Simp |
| | 8. Hxa | 7 UI |
| | 9. $Fx \cdot Hxa$ | 6,8 Conj |
| | 11. <u>$(Fx \cdot Hxa) \cdot \sim (Fx \cdot Hxa)$</u> | 5,9 Conj |
| | 12. $(\exists x) (Fx \cdot Hxa)$ | 2-11 IP |
| | | |
| (5) → | 2. $\sim (y) [By \supset (\exists x) (Ax \cdot Cxy)]$ | AP/∴ $(y) [By \supset (\exists x) (Ax \cdot Cxy)]$ |
| | 3. $(\exists y) \sim [By \supset (\exists x) (Ax \cdot Cxy)]$ | 2 QN |
| | 4. $\sim [By \supset (\exists x) (Ax \cdot Cxy)]$ | 3 EI |
| | 5. $\sim [\sim By \vee (\exists x) (Ax \cdot Cxy)]$ | 4 Impl |
| | 6. $\sim \sim By \cdot \sim (\exists x) (Ax \cdot Cxy)$ | 5 DeM |
| | 7. $Ax \cdot (y) (By \supset Cxy)$ | 1 EI |
| | 8. Ax | 7 Simp |
| | 9. $(y) (By \supset Cxy)$ | 7 Simp |
| | 10. $By \supset Cxy$ | 9 UI |
| | 11. $\sim \sim By$ | 6 Simp |
| | 12. By | 11 DN |
| | 13. Cxy | 10,12 MP |
| | 14. $\sim (\exists x) (Ax \cdot Cxy)$ | 6 Simp |
| | 15. $(x) \sim (Ax \cdot Cxy)$ | 14 QN |
| | 16. $\sim (Ax \cdot Cxy)$ | 15 UI |
| | 17. $\sim Ax \vee \sim Cxy$ | 16 DeM |
| | 18. $\sim \sim Ax$ | 8 DN |
| | 19. $\sim Cxy$ | 17,18 DS |
| | 20. <u>$Cxy \cdot \sim Cxy$</u> | 13,19 Conj |
| | 21. $(y) [By \supset (\exists x) (Ax \cdot Cxy)]$ | 2-20 IP |

(7) →	2. $\sim (\exists y) (x) (Fx \supset Gy)$	AP/∴ $(\exists y) (x) (Fx \supset Gy)$
	3. $(y) \sim (x) (Fx \supset Gy)$	2 QN
	4. $(y) (\exists x) \sim (Fx \supset Gy)$	3 QN
→	5. $(\exists x) Gx$	AP/∴ $\sim (\exists x) Gx$
	6. Gy	5 EI
	7. $(\exists x) \sim (Fx \supset Gy)$	4 UI
	8. $\sim (Fx \supset Gy)$	7 EI
	9. $\sim (\sim Fx \vee Gy)$	8 Impl
	10. $\sim \sim Fx \cdot \sim Gy$	9 DeM
	11. $\sim Gy$	10 Simp
	12. $Gy \cdot \sim Gy$	6,11 Conj
	13. $\sim (\exists x) Gx$	5-12 IP
	14. $\sim (\exists x) Fx$	1,13 MT
	15. $(x) \sim Fx$	14 QN
	16. $(\exists x) \sim (Fx \supset Gy)$	4 UI
	17. $\sim (Fz \supset Gy)$	16 EI
	18. $\sim (\sim Fz \vee Gy)$	17 Impl
	19. $\sim \sim Fz \cdot \sim Gy$	18 DeM
	20. $\sim \sim Fz$	19 Simp
	21. $\sim Fz$	15 UI
	22. $\sim Fz \cdot \sim \sim Fz$	20,21 Conj
	23. $(\exists y) (x) (Fx \supset Gy)$	2-22 IP

EXERCISE 11-1:

1. The problem is that it would allow for invalid inferences, basically moving from the claim that there is at least one thing with a certain property to a claim that states or assumes that everything has that property. For example, it would allow the following inference which is clearly invalid.

1.	$(\exists y) (Fy \supset Gy)$	p
2.	$(x) Fx$	p
3.	$Fy \supset Gy$	1 EI
4.	Fy	2 UI
5.	Gy	3,4 MT
6.	$(x) Gx$	5 UG (invalid)

The inference to line 6 basically assumes that line 3 is true of everything when it is not.

3. Such a requirement would prohibit valid inferences, and the inference given is a valid one.

5. It is invalid. If for each person there is someone who is her or his parent, it does not follow that for each person there is someone who is his or her own parent.

7. (No answer is provided because the questions's importance lies in the way students explain and defend their answer.)

EXERCISE 11-2:

- (1)
- | | | |
|----|-----------------------------|----------------------|
| 1. | $(x)(y)(Fxy \supset Gx)$ | p |
| 2. | $(\exists x)(\exists y)Fxy$ | p/∴ $(\exists x)Gx$ |
| 3. | $(\exists y)Fay$ | 2 EI , flag a |
| 4. | Fab | 3 EI , flag b |
| 5. | $(y)(Fay \supset Ga)$ | 1 UI |
| 6. | $Fab \supset Ga$ | 5 UI |
| 7. | Ga | 4,6 MP |
| 8. | $(\exists x)Gx$ | 7 EG |
-
- (3)
- | | | |
|----|--------------------------------|---------------------------------|
| 1. | $(\exists x)[Fx \cdot (y)Hxy]$ | p/∴ $(\exists x)(Fx \cdot Hxa)$ |
| 2. | $Fb \cdot (y)Hby$ | 1 EI , flag b |
| 3. | Fb | 2 Simp |
| 4. | $(y)Hby$ | 2 Simp |
| 5. | Hba | 4 UI |
| 6. | $Fb \cdot Hba$ | 3,6 Conj |
| 7. | $(\exists x)(Fx \cdot Hxa)$ | 6 EG |
-
- (5)
- | | | |
|-----|---|---|
| 1. | $(\exists x)[Ax \cdot (y)(By \supset Cxy)]$ | p/∴ $(y)[By \supset (\exists x)(Ax \cdot Cxy)]$ |
| 2. | flag a | |
| 3. | $Ab \cdot (y)(By \supset Cby)$ | 1 EI , flag b |
| 4. | Ab | 3 Simp |
| 5. | $(y)(By \supset Cby)$ | 3 Simp |
| 6. | $Ba \supset Cba$ | 5 UI |
| 7. | Ba | AP/∴ $(\exists x)(Ax \cdot Cxa)$ |
| 8. | Cba | 6,7 MP |
| 9. | $Ab \cdot Cba$ | 4,8 Conj |
| 10. | $(\exists x)(Ax \cdot Cxa)$ | 9 EG |
| 11. | $Ba \supset (\exists x)(Ax \cdot Cxa)$ | 7-10 CP |
| 12. | $(y)[By \supset (\exists x)(Ax \cdot Cxy)]$ | 11 UG |

(7) Trying direct proof results in violating the third restriction on flagged constants:

- | | | |
|----|---------------------------------------|--|
| 1. | $(\exists x)Fx \supset (\exists x)Gx$ | p/∴ $(\exists y)(x)(Fx \supset Gy)$ |
| 2. | flag a | |
| 3. | Fa | AP/∴ Gb |
| 4. | $(\exists x)Fx$ | 3 EG |
| 5. | $(\exists x)Gx$ | 1,4 MP |
| 6. | Gb | 5 EI , flag b |
| 7. | $Fa \supset Gb$ | 3-6 CP |
| 8. | $(x)(Fx \supset Gb)$ | 7 UG (Invalid, b occurs outside the subproof) |
| 9. | $(\exists y)(x)(Fx \supset Gy)$ | 8 EG |

So one needs to use an indirect method:

1. $(\exists x)Fx \supset (\exists x)Gx$	$p / \therefore (\exists y)(x)(Fx \supset Gy)$
2. $\sim (\exists y)(x)(Fx \supset Gy)$	AP / $\therefore (\exists y)(x)(Fx \supset Gy)$
3. $(y) \sim (x)(Fx \supset Gy)$	2 QN
4. $\sim (x)(Fx \supset Ga)$	3 UI
5. $(\exists x) \sim (Fx \supset Ga)$	4 QN
6. $\sim (Fb \supset Ga)$	5 EI, flag b
7. $\sim (\sim Fb \vee Ga)$	6 Impl
8. $\sim \sim Fb \cdot \sim Ga$	7 DeM
9. $\sim \sim Fb$	8 Simp
10. Fb	9 DN
11. $(\exists x) Fx$	10 EG
12. $(\exists x) Gx$	1,11 MP
13. Gc	12 EI, flag c
14. $\sim (x)(Fx \supset Gc)$	3 UI
15. $(\exists x) \sim (Fx \supset Gc)$	14 QN
16. $\sim (Fd \supset Gc)$	15 EI, flag d
17. $\sim (\sim Fd \vee Gc)$	16 Impl
18. $\sim \sim Fd \cdot \sim Gc$	17 DeM
19. $\sim Gc$	18 Simp
20. <u>$Gc \cdot \sim Gc$</u>	13,19 Conj
21. $(\exists y)(x)(Fx \supset Gy)$	2-20 IP

EXERCISE 12-1:

(1) 1.	$(x)(Fx \supset Gx)$	p
✓2.	$(\exists x) \sim Gx$	$p / \therefore (\exists x) \sim Fx$
✓3.	$\sim (\exists x) \sim Fx$	Denial of conclusion
4.	$(x) \sim \sim Fx$	3 FC2
5.	$\sim Ga$	2 FC4
✓6.	$Fa \supset Ga$	1 FC5
✓7.	$\sim \sim Fa$	4 FC5
8.	Fa	7 FC2
9.	$\begin{array}{c} \swarrow \quad \searrow \\ \sim Fa \quad Ga \end{array}$	6 FC3
10.	Valid. All paths close.	

(3)	1.	$(x)(Ax \supset Bx)$	p
	2.	$(x)(\sim Ax \supset Cx)$	p / $\therefore (x)(\sim Bx \supset \sim Cx)$
	✓3.	$\sim (x)(\sim Bx \supset \sim Cx)$	Denial of conclusion
	✓4.	$(\exists x) \sim (\sim Bx \supset \sim Cx)$	3 FC2
	✓5.	$\sim (\sim Ba \supset \sim Ca)$	4 FC4
	6.	$\sim Ba$	5 FC3
		✓ $\sim \sim Ca$	
	7.	Ca	6 FC2
	✓8.	$Aa \supset Ba$	1 FC5
	✓9.	$\sim Aa \supset Ca$	2 FC5
	10.	$\sim Aa$ Ba	8 FC3
	11.	$\sim \sim Aa$ Ca	9 FC3
	✓12.	Aa	12 FC2
	13.		
	14.		






Invalid. Open path. Let $V(Aa) = \mathbf{F}$, $V(Ba) = \mathbf{F}$, and $V(Ca) = \mathbf{T}$. The expansions give us $Aa \supset Ba$ for the first premise, $\sim Aa \supset Ca$ for the second premise, and $\sim Ba \supset \sim Ca$ for the conclusion; thus true premises, false conclusion.

Number (5) can be done more easily using the unrestricted order method:

(5)	✓1.	$(\exists x)[Fx \cdot (y)(Gy \supset Lxy)]$	p
	2.	$(x)[Fx \supset (y)(My \supset \sim Lxy)]$	p / $\therefore (x)(Gx \supset \sim Mx)$
	✓3.	$\sim (x)(Gx \supset \sim Mx)$	Denial of concl.
	✓4.	$(\exists x) \sim (Gx \supset \sim Mx)$	3 FC2
	✓5.	$Fa \cdot (y)(Gy \supset Lay)$	1 FC4
	✓6.	$\sim (Gb \supset \sim Mb)$	4 FC4
	✓7.	Gb	6 FC3
		✓ $\sim \sim Mb$	
	8.	Mb	7 FC2
	9.	Fa	5 FC3
		$(y)(Gy \supset Lay)$	
	✓10.	$Fa \supset (y)(My \supset \sim Lay)$	2 FC5
	11.	$\sim Fa$ $(y)(My \supset \sim Lay)$	10 FC3
	12.		
	✓13.	$Mb \supset \sim Lab$	11 FC5
	14.	$\sim Mb$ $\sim Lab$	13 FC3
	15.		
	✓16.	$Gb \supset Lab$	9 FC5
	17.	$\sim Gb$ Lab	16 FC3
	18.		

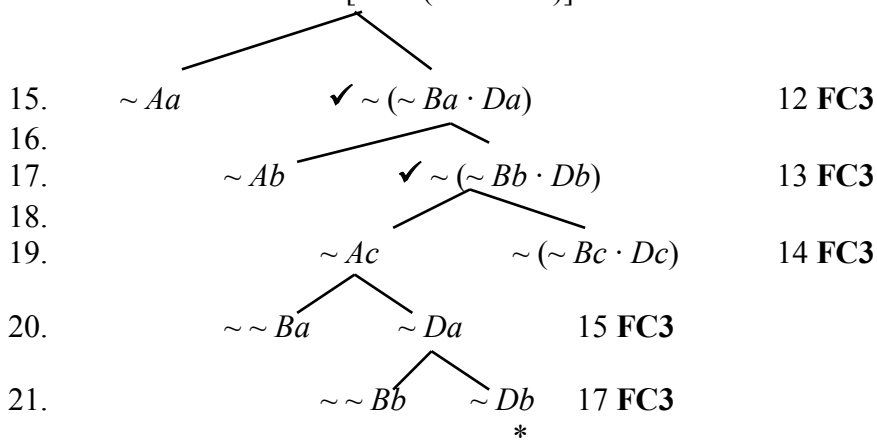
Valid. All paths close.

Number (5) done by the prescribed order method is below:

(5) ✓1.	$(\exists x)[Fx \cdot (y)(Gy \supset Lxy)]$	p
2.	$(x)[Fx \supset (y)(My \supset \sim Lxy)]$	p / $\therefore (x)(Gx \supset \sim Mx)$
✓3.	$\sim (x)(Gx \supset \sim Mx)$	Denial of concl.
✓4.	$(\exists x) \sim (Gx \supset \sim Mx)$	3 FC2
✓5.	$Fa \cdot (y)(Gy \supset Lay)$	1 FC4
6.	Fa	5 FC3
	$(y)(Gy \supset Lay)$	
✓7.	$\sim (Gb \supset \sim Mb)$	4 FC4
8.	Gb	7 FC3
	✓ $\sim \sim Mb$	
9.	Mb	8 FC2
✓10.	$Fa \supset (y)(My \supset \sim Lay)$	2 FC5
✓11.	$Fb \supset (y)(My \supset \sim Lby)$	2 FC5
✓12.	$Ga \supset Laa$	6 FC5
✓13.	$Gb \supset Lab$	6 FC5
		
14.	$\sim Fa$ $(y)(My \supset \sim Lay)$	10 FC3
15.		
		
16.	$\sim Gb$ Lab	13 FC3
17.		
		
18.	$\sim Fb$ $(y)(My \supset \sim Lby)$	11 FC3
19.		
	$\sim Ga$ Laa $\sim Ga$ Laa	12 FC3
20.	$Ma \supset \sim Laa$ $Ma \supset \sim Laa$ $Ma \supset \sim Laa$ $Ma \supset \sim Laa$	14 FC5
✓21.	$Mb \supset \sim Lab$ $Mb \supset \sim Lab$ $Mb \supset \sim Lab$ $Mb \supset \sim Lab$	14 FC5
22.	$Ma \supset \sim Lba$ $Ma \supset \sim Lba$ $Ma \supset \sim Lba$ $Ma \supset \sim Lba$	16 FC5
23.	$Mb \supset \sim Lbb$ $Mb \supset \sim Lbb$ $Ma \supset \sim Lbb$ $Ma \supset \sim Lbb$	16 FC5
		
24.	$\sim Mb$ $\sim Lab$ $\sim Mb$ $\sim Lab$ $\sim Mb$ $\sim Lab$ $\sim Mb$ $\sim Lab$	19 FC3
25.		

Valid. All paths close.

(7) ✓ 1.	$(\exists x)(Ax \cdot \sim Bx)$	p
✓ 2.	$(\exists x)(Ax \cdot \sim Cx)$	p
✓ 3.	$(\exists x)(\sim Bx \cdot Dx)$	p
		$\therefore (\exists x)[Ax \cdot (\sim Bx \cdot Dx)]$
✓ 4.	$\sim (\exists x)[Ax \cdot (\sim Bx \cdot Dx)]$	Denial of conclusion
5.	$(x) \sim [Ax \cdot (\sim Bx \cdot Dx)]$	4 FC2
✓ 6.	$Aa \cdot \sim Ba$	1 FC4
7.	Aa	6 FC3
	$\sim Ba$	
✓ 8.	$Ab \cdot \sim Cb$	2 FC4
9.	Ab	8 FC3
	$\sim Cb$	
✓ 10.	$\sim Bc \cdot Dc$	3 FC4
11.	$\sim Bc$	10 FC3
	Dc	
✓ 12.	$\sim [Aa \cdot (\sim Ba \cdot Da)]$	5 FC5
✓ 13.	$\sim [Ab \cdot (\sim Bb \cdot Db)]$	5 FC5
✓ 14.	$\sim [Ac \cdot (\sim Bc \cdot Dc)]$	5 FC5



Invalid. Open path.

In the branch marked with *, let $V(Aa) = \mathbf{T}$, $V(Ab) = \mathbf{T}$, $V(Ac) = \mathbf{F}$,

$V(Ba) = \mathbf{F}$, $V(Bc) = \mathbf{F}$,

$V(Cb) = \mathbf{F}$,

$V(Da) = \mathbf{F}$, $V(Db) = \mathbf{F}$, and $V(Dc) = \mathbf{T}$,

and let $V(Bb)$, $V(Ca)$, $V(Cc)$ be either \mathbf{T} or \mathbf{F} . The expansions give us

1. $(Aa \cdot \sim Ba) \vee [(Ab \cdot \sim Bb) \vee (Ac \cdot \sim Bc)]$

2. $(Ab \cdot \sim Cb) \vee [(Ab \cdot \sim Cb) \vee (Ab \cdot \sim Cb)]$

3. $(\sim Ba \cdot Da) \vee [(\sim Bb \cdot Db) \vee (\sim Bc \cdot Dc)]$

$\therefore [Aa \cdot (\sim Ba \cdot Da)] \vee \{[Ab \cdot (\sim Bb \cdot Db)] \vee [Ac \cdot (\sim Bc \cdot Dc)]\}$,

thus true premises, false conclusion.

(9) ✓1.	$(\exists x)(Ax \vee \sim Bx)$	
2.	$(x)[(Ax \cdot \sim Bx) \supset Cx]$	p
✓3.	$\sim (\exists x)Cx$	p / $\therefore (\exists x)Cx$
4.	$(x) \sim Cx$	Denial of conclusion
✓5.	$Aa \vee \sim Ba$	3 FC2
	$Aa \vee \sim Ba$	1 FC4
6.	$Aa \quad \quad \quad \sim Ba$	5 FC3
✓7.	$(Aa \cdot \sim Ba) \supset Ca \quad (Aa \cdot \sim Ba) \supset Ca$	2 FC5
8.	$\sim Ca \quad \quad \quad \sim Ca$	4 FC5
✓9.	$\sim (Aa \cdot \sim Ba) \quad Ca$	7 FC3
10.	$\sim (Aa \cdot \sim Ba)$	
✓11.	$\sim Aa \quad \quad \quad \sim \sim Ba$	9 FC3
12.		
13.	Ba	11 FC3
	*	

Invalid. Open path.
 In the branch marked with *, let $V(Aa) = \mathbf{F}$, $V(Ba) = \mathbf{F}$, and $V(Ca) = \mathbf{F}$. The expansions give us $Aa \vee \sim Ba$ for the first premise, $(Aa \cdot \sim Ba) \supset Ca$ for the second premise, and Ca for the conclusion; thus true premises, false conclusion.

EXERCISE 13-1:

- | | | |
|-----|-------------------------------|------------------|
| (1) | 3. $Fx \cdot Gx$ | 2 EI |
| | 4. $(x) [Fx \supset (x = a)]$ | 1 Simp |
| | 5. $Fx \supset (x = a)$ | 4 UI |
| | 6. Fx | 3 Simp |
| | 7. $x = a$ | 5,6 MP |
| | 8. Gx | 3 Simp |
| | 9. Ga | 7,8 ID |
| | | |
| (3) | 5. Hc | 3 Simp |
| | 6. Kc | 1,5 MP |
| | 7. Md | 3 Simp |
| | 8. Nd | 2,7 MP |
| | 9. Kd | 4,6 ID |
| | 10. Nc | 4,8 ID |
| | 11. $Kd \cdot Nc$ | 9,10 Conj |

(5)	3. $\sim\sim Pa$	AP/∴ $\sim Pa$
	4. Pa	3 DN
	5. $\{Px \cdot (y) [Py \supset (y = x)]\} \cdot Qx$	1 EI
	6. $Px \cdot (y) [Py \supset (y = x)]$	5 Simp
	7. $(y) [Py \supset (y = x)]$	6 Simp
	8. $Pa \supset (a = x)$	7 UI
	9. $a = x$	4,8 MP
	10. Qx	5 Simp
	11. Qa	9,10 ID
	12. $Qa \cdot \sim Qa$	2,11 Conj
	13. $\sim Pa$	3-12 IP
(7)	3. $\sim(\sim Px \vee \sim Ex)$	2 EI
	4. $\sim\sim Px \cdot \sim\sim Ex$	3 DeM
	5. $Px \cdot Ex$	4 DN (twice)
	6. $Pz \cdot \{(y) [Py \supset (y = z)] \cdot Qz\}$	1 EI
	7. $(y) [Py \supset (y = z)] \cdot Qz$	6 Simp
	8. $(y) [Py \supset (y = z)]$	7 Simp
	9. $Px \supset (x = z)$	8 UI
	10. Px	5 Simp
	11. $x = z$	9,10 MP
	12. Qz	7 Simp
	13. Qx	11,12 ID
	14. Ex	5 Simp
	15. $Ex \cdot Qx$	13,14 Conj
	16. $(\exists x) (Ex \cdot Qx)$	15 EG
(9)	4. $Gb \supset Fb$	1 UI
	5. Fb	3,4 MP
	6. $(y) [(Fb \vee Fy) \supset (b = y)]$	2 UI
	7. $(Fb \vee Fc) \supset (b = c)$	6 UI
	8. $Fb \vee Fc$	5 Add
	9. $b = c$	7,8 MP
	10. $(Fb \vee Fa) \supset (b = a)$	6 UI
	11. $Fb \vee Fa$	5 Add
	12. $b = a$	10,11 MP
	13. $a = c$	9,12 ID

EXERCISE 13-2:

1. $Fg \cdot (x) [Fx \supset (x = g)]$
3. $(\exists x) (Fx \cdot Gx)$
5. $(\exists x)(\exists y)(\exists z)(\{[(Fx \cdot Sx) \cdot [(Fy \cdot Sy) \cdot (Fz \cdot Sz)]]\} \cdot \{[(x \neq y) \cdot (y \neq z)] \cdot (x \neq z)\}) \cdot (w)\{(Fw \cdot Sw) \supset ([(w = x) \vee (w = y)] \vee (w = z))\})$

7. $(x) \{ [Px \cdot (x \neq h)] \supset Hhx \}$
9. $(x) \{ [Px \cdot (x \neq a)] \supset Lxa \}$, where $Lxy =$ “x is less compassionate than y.”

EXERCISE 13-3:

1. $(x) \{ Sx \supset (\exists y) \{ [Sy \cdot (x \neq y)] \cdot Ixy \} \}$ ($Sx =$ “x is a student”; $Ixy =$ “x is more intelligent than y”)
3. $(x) [Gx \supset (y) (\sim Gy \supset Hxy)]$ ($Gx =$ “x is a good example”; $Hxy =$ “x is harder to put up with than y”) [Note: the sentence is really making a generalization about good examples, and not claiming that there is one hardest thing to put up with and that it is a good example.]
5. $Cf \cdot (x) \{ [Cx \cdot (x \neq f)] \supset Bfx \}$ ($Cx =$ “x is a chess player”; $f =$ “Bobby Fisher”; $Bxy =$ “x is a better chess player than y”)
7. $Bka \cdot (x) [(Cx \cdot Bxa) \supset (x = k)]$ ($Cx =$ “x is a current chess player”; $a =$ “Anatoli Karpov”; $k =$ “Gary Kasparov”) [Note: the sentence could also be understood as just making a conditional claim, in which case the first conjunct is dropped.]
9. $(\exists x)(\exists y) \{ \{ [(Cx \cdot Cy) \cdot (x \neq y)] \cdot (z)(Cz \supset [(z = x) \vee (z = y)]) \} \cdot (Dxy \cdot Dyx) \}$
($Cx =$ “x is a presidential candidate”; $Dxy =$ “x dislikes y”)
11. $Dms \cdot (x) [Dxs \supset (x = m)]$ ($m =$ “Mary”; $s =$ “Sonia”; $Dxy =$ “x is a daughter of y”) [Note: this says that Mary is the only daughter of Sonia, if the sentence is just saying that Mary is Sonia’s daughter, then the second conjunct is dropped. Alternately, if one wants to symbolize “Sonia is Mary’s mother”, use $Mxy =$ “x is a mother of y” and get: $Msm \cdot (x) [Mxm \supset (x = s)].$]

EXERCISE 13-4:

- A.1. Nonsymmetrical, nontransitive, nonreflexive
3. Nonsymmetrical, transitive, reflexive
5. Asymmetrical, transitive, irreflexive
7. Nonsymmetrical, transitive, irreflexive

B.	1. $(x)(y)(Fxy \supset \sim Fyx)$	$p/\therefore (x) \sim Fxx$
	2. $\sim (x) \sim Fxx$	AP $\therefore (x) \sim Fxx$
	3. $(\exists x) Fxx$	2 QN
	4. Fxx	3 EI
	5. $(y)(Fxy \supset \sim Fyx)$	1 UI
	6. $Fxx \supset \sim Fxx$	5 UI
	7. $\sim Fxx$	4,6 MP
	8. $Fxx \cdot \sim Fxx$	4,7 Conj
	9. $(x) \sim Fxx$	2-8 IP

EXERCISE 13-5:

- Some say that the levels-of-language theory solves this paradox. For it forbids a statement from talking about any statement in the same level of language, thus forbidding it to talk about itself. (A) does, and thus denying the meaningfulness of (A).
-
- Some say that the theory of types solves this paradox. For the interestingness of the lowest uninteresting number is said to be a higher type than that of, say, the interesting number seven (similar to the rareness of the property of being rare).

EXERCISE 13-6:

- Those that require an identity clause (using obvious abbreviations):
 - $(\exists x) [\{(Hx \cdot Wx) \cdot (y) [(Hy \cdot Wy) \supset (x = y)]\} \cdot Lx]$
 - $Cm \cdot (x) \{[Cx \cdot (x \neq m)] \supset Gmx\}$ ($Gxy =$ "x is a greater composer than y")
 - $Cm \cdot (x) \{[Px \cdot (x \neq m)] \supset Bmx\}$ ($Px =$ "x is a person"; $Bxy =$ "x is a better composer than y")
 - $(x) \{[Sx \cdot (x \neq s)] \supset Hsx\}$ ($Sx =$ "x is a student")

Those that don't require an identity clause:

- $(x) [Hx \supset (Ix \cdot Ax)]$
 - $\sim (\exists x) (Sx \cdot Hxs)$
 - $(x) (Wx \supset Lx)$
- The "gold" example on page 307 is said by some to illustrate that a truth-functional implication is not sufficient to capture the idea of potentiality, or of causality, that lies behind the gold example.
- (1) False. The problem is that if the conditional is understood as the material conditional then the sentence would be true merely if we never ate the food in question. Further, in that case, the sentence "Food f does not have the power to nourish" would be translated as "If we eat f , then we won't be nourished," and it too would be true because of its false antecedent. But we don't think that both sentences can be true. Also, the original sentence implies that a food is nourishing, and not otherwise, even if no one ever eats it. Thus the material

conditional is inadequate for capturing the idea of potentiality asserted in the sentence.

- (2) True. In both cases, having to use the material conditional results in inadequate translations. See discussion on pp.301-303.
- (3) False. The problem is similar to the problem in (1). Suppose we never make cigarettes illegal. Then $C \supset S$ has a false antecedent and so is true automatically. Its contrary “If we were to make cigarettes illegal, then fewer people would smoke”, symbolized as $C \supset \sim S$, would also be true.