IVRG WEEK 8
PROGRESS REPORT

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Here is a summary of what we discussed Monday, October 4 and Tuesday, October 5 plus a few updates.

- Tyler alerted us to an error in a previous claim I made: C hyperelliptic of genus g is *not* equivalent to the existence of an order 2 central element in $\mathrm{Aut}(C)$. The correct statement is:

  Proposition 0.1 *C is hyperelliptic of genus g if and only if there exists $f \in \mathrm{Aut}(C)$ with $f^2 = \mathrm{Id}$ and $f : C \to C$ has $2g + 2$ fixed points.*

  Dave will program a function for computing the number of fixed points. (Note added 10/11/2010: This is ready, see `FixXh` in `eichler.txt`.)

  Dave also found a result of Bujalance, Etayo and Martin [1] and claims that the following proposition follows from their main theorem (second page of their paper, p. 175):

  Would someone please check that the Proposition below really does follow directly from B-E-M's result plus the list from MSSV?

  Proposition 0.2 *Let C be hyperelliptic, and suppose that $\# \mathrm{Aut}(C) > 12(g-1)$. Then there are three possibilities for the genus and automorphism group of C:*
  (1) $g = 4$, $\mathrm{Aut}(C) = \langle 40, 8 \rangle$
  (2) $g = 5$, $\mathrm{Aut}(C) = \langle 120, 35 \rangle$
  (3) $g = 9$, $\mathrm{Aut}(C) = \langle 120, 35 \rangle$

  On Tuesday, Jacob and Dave discussed an approach for finding the genus 5 and 9 curves on the list above.
- Malik found two possible sets of equations for the curve with $g = 5$ and $\mathrm{Aut}(C) = \langle 160, 234 \rangle$. We are still not sure if these two sets of equations give rise to two different curves, or if there is a linear transformation on the variables which takes one set of equations to the other.
- Zach helped Dave identify a bug in DecomposeGAction. He was trying to compute equations for the genus 4 curve with $\mathrm{Aut}(C) = \langle 72, 42 \rangle$. This has been fixed—everyone should make sure they download and use version 2.
- Eddie found one set of equations for the genus 4 curve with automorphism group $\mathrm{Aut}(C) = \langle 120, 34 \rangle$. Dave revealed that this curve is classically known as Bring's curve, and that the group $\langle 120, 34 \rangle$ is S_5, and showed the following very nice system of equations for it:

$$\begin{aligned} a+b+c+d+e &= 0 \\ a^2+b^2+c^2+d^2+e^2 &= 0 \\ a^3+b^3+c^3+d^3+e^3 &= 0 \end{aligned}$$

  Remember that for a genus 4 curve, we are expecting one quadric and one cubic in $\mathbb{P}^3$. What we have done here is used one linear equation to form a $\mathbb{P}^3$ inside a $\mathbb{P}^4$.

Date: October 5, 2010.

QUESTION 0.3 When can this trick of adding extra variables lead to nicer equations for curves with automorphisms?

- Jacob worked on the genus 5 curve with automorphism group $\mathrm{Aut}(C) = \langle 96, 195\rangle$. Kuribayashi and Kimura list two groups of order 96 in [2, Prop. 3 item 5, p. 92]. He discovered that $G_2(96)$ is not $\langle 96, 195\rangle$. But when he entered the generators given for $G_1(96)$, he found that they actually generate a group of order 51840. Dave rechecked his notes from an earlier calculation and discovered that he had run into the same problem.

  Over the weekend, Dave programmed something called the *Eichler trace formula*, and used this plus `Magma` and another computer program called `GAP` to get matrix generators for $\langle 96, 195\rangle$. They are:

```
A:=elt<GL5K | 0,w,0,0,0, w^2,0,0,0,0, 0,0,0,0,1, 0,0,0,-1,0, 0,0,1,0,0>;
B:=elt<GL5K | 0,-1,0,0,0, 1,0,0,0,0, 0,0,0,-1,0, 0,0,1,0,0, 0,0,0,0,-1>;
```

  where `w` is a third root of unity, as usual.
- Tyler is working on the genus 5 curve with $\mathrm{Aut}(C) = \langle 192, 181\rangle$.
- Lev is working on the genus 5 curve with $\mathrm{Aut}(C) = \langle 64, 32\rangle$.

REFERENCES

[1] E. BUJALANCE, J. J. ETAYO, AND E. MARTÍNEZ, *Automorphism groups of hyperelliptic Riemann surfaces*, Kodai Math. J. **10** (1987), no. 2, 174–181, DOI 10.2996/kmj/1138037412. MR897252 (88i:30072) ←1

[2] AKIKAZU KURIBAYASHI AND HIDEYUKI KIMURA, *Automorphism groups of compact Riemann surfaces of genus five*, J. Algebra **134** (1990), no. 1, 80–103, DOI 10.1016/0021-8693(90)90212-7. MR1068416 (91j:30033) ←2

SOFTWARE PACKAGES REFERENCED

[3] THE GAP GROUP, `GAP`*: Groups, Algorithms, and Programming, a system for computational discrete algebra* (2008), available at http://www.gap-system.org. Version 4.4.11. ←

[4] SCHOOL OF MATHEMATICS AND STATISTICS COMPUTATIONAL ALGEBRA RESEARCH GROUP UNIVERSITY OF SYDNEY, `MAGMA` *computational algebra system* (2008), available at http://magma.maths.usyd.edu.au/magma/. Version 2.15-1. ←