A Unified Theory of Bond and Currency Markets

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Abstract

I show that an external habit model augmented with a heteroskedastic consumption growth process reproduces main domestic and international bond market puzzles, considered difficult to replicate simultaneously. Domestically, the model generates an upward sloping real yield curve and realistic violations of the expectation hypothesis. Depending on the parameters, the model can also generate a downward sloping real yield curve and predicts that the expectation hypothesis violations are stronger in countries with upward sloping real yield curves. Internationally, the model explains violations of the uncovered interest rate parity. Unlike a standard habit model, the model simultaneously features intertemporal smoothing to match domestic term structure and precautionary savings to reproduce international predictability. The model also replicates the imperfect correlation between consumption and bond prices/exchange rates through positive and negative consumption shocks affecting habit differently. Mechanisms of the model are empirically supported.

Keywords: fixed income, yield curve, expectation hypothesis, uncovered interest rate parity, habit, time-varying volatility

JEL codes: E43, G12, G15

1 Introduction

Bond markets present both domestic and international puzzles. In the United States the real yield curve is upward sloping and the expectation hypothesis is violated: the high slope of the yield curve predicts high returns on the long-term bonds over the life of short-term bonds. [1] At the same time, for instance, in the United Kingdom, the real yield curve is downward sloping and the expectation hypothesis is not violated: the high slope of the yield curve predicts low, instead of high, returns on the long term bonds over the life of short-term bonds. Internationally, the uncovered interest rate parity seems to be violated: a high differential between foreign and domestic interest rates strongly predicts high returns on borrowing in domestic bonds and investing in foreign bonds.

This paper shows that a simple consumption based term structure model is able to explain the phenomena above. Replicating both domestic and international bond dynamics within the same simple model is appealing, because most existing models explaining domestic bond markets produce counterfactual implications internationally, likewise most models addressing international bond markets produce counterfactual implications domestically. As modern domestic and international bond markets are closely integrated, this inability to explain them jointly constitutes one of the major puzzles in theoretical asset pricing.

The proposed model is a habit model. In habit models the utility of an agent is determined by the difference between the current and habitual consumptions. The habitual consumption (habit) is an aggregate of the past consumption history, which varies slowly over time. As the fluctuations in the difference between the current and habitual consumptions are percentually larger than fluctuations in the absolute consumption, in habit models the agent is more sensitive to consumption shocks than in models where the utility is determined by fluctuations in the absolute consumption alone (e.g., CRRA-utility based models). The key difference to standard habit models (Abel 1990; Cambpell and Cochrane, 1999) is that the model in this paper features a heteroskedastic consumption growth process. This can be interpreted as a time-varying amount of risk. At the same time, in contrast to the recent habit models (Campbell and Cochrane, 1999, Wachter,

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[1] For instance, if the difference between the yields on 5 years and 1 year Treasury bonds is high, the 1 year return on holding a 5 years bond has historically been relatively high in US.

2002, Verdelhan, 2010), the price of risk in the model is constant. I show that, unlike the standard habit specification, my specification allows to simultaneoulsy have intertempo-ral smoothing effect to match domestic term structure and precautionary savings effect to reproduce international bond market dynamics.

The main puzzles explained by the model are different slopes of the real yield curve, violations of the expectation hypothesis and violations of the uncovered interest rate parity. In the model, the average slope of the real yield curve is determined in the interaction between the intertemporal smoothing effect and the precautionary savings effect. If the intertemporal smoothing dominates: in times of low consumption the agent wants to consume more and not save, thus, the bond prices will be low. As a result, long-term bonds are a poor hedge against consumption shocks and should earn a premium against the short-term bonds: the real yield curve slopes upwards. If the precautionary savings effect dominates, in times of high consumption volatility the agent wants to save, thus, bond prices will be high. Consequently, long-term bonds are a good hedge against consumption volatility shocks and (assuming the agent dislikes volatility), should trade at a discount against the short-term bonds: the real yield curve slopes downwards.

The violation of the expectation hypothesis is an observation that holding returns on long-term bonds over the life of short-term bonds are high when the slope of the yield curve is high. Thus, there are two parts in the expectation hypothesis: a time-varying slope of the yield curve and time-varying returns on long-term bonds versus short-term bonds. Theoretically, the slope of the yield curve consists of two components: the expected change in the short rates in the future and the risk premium on holding a long term bond over the life of the short-term bond. The first term corresponds to the fact that if in the future short-term rates are expected to increase, the slope of the yield curve is higher. The second term corresponds to the fact that holding a long-term bond over a short period of time is risky because, until the payoff date, its price will fluctuate, and these fluctuations might be correlated with investors' marginal utility. Consequently, if investors require a compensation for holding a long-term bond over a short period of time because of these fluctuations, the yield on the long-term bond will be relatively high increasing the slope of the yield curve.

In the model, violations of the expectation hypothesis arise because an increase in the

consumption growth volatility increases the risk premium on holding a long term bond over the life of a short-term bond. This requires that the intertemporal smoothing is the dominant effect over precautionary savings. Indeed, if the intertemporal smoothing dominates, in times of low consumption the agent wants to consume instead of saving. This decreases the bond prices. Consequently, long-term bonds are a bad protection against conusmption shocks and thus the required risk premium for holding them is high when the magnitude (volatility) of these shocks is high. As explained above, this risk premium is the second component of the slope of the yield curve. Thus, the increase in this risk premium also increases the slope of the yield curve. Consequently, the slope of the yield curve positively predicts returns on long-term bonds over the life of short-term bonds.

Finally, in the model, violations of the uncovered interest parity are the result of the the time-varying consumption growth volatility. The strategy of borrowing in country 1 and lending in country 2 is a poor hedge against the consumption shocks in country 1. To see this, suppose that a bad consumption shock realizes in country 1. The consumption in country 1 becomes relatively scarce. Consequently, the agent who exchanges country 2's consumption for country 1's consumption will receive relatively little of country 1's consumption when she needs it the most (after negative consumption shocks). Because the strategy is a poor hedge against consumption shocks, the higher the magnitude (volatility) of these shocks, the higher premium the agent requires. Note that the higher consumption volatility in country 1 both decreases the interest rate in that country through the precautionary savings motive and increases the expected return on the strategy above.

The model in this paper addresses several problems of the main term-structure models: the classic habit model, a long-run risk model, and a rare-disaster model. In particular, the model is able to simulateneously replicate three major bond market puzzles: an upward sloping real yield curve, violations of the expectation hypothesis, and violations of the uncovered interest rate parity. The classic habit model by Wachter (2006) closely replicates US bond market dynamics: in particular, an upward sloping real yield curve and violations of the expectation hypothesis. However, a critique towards that model (Verdelhan, 2010, Bansal and Shaliastovich, 2013) is that the uncovered interest rate

parity still holds. Additionally, the effective coefficient of risk-aversion of 30 in the model is relatively high. Verdelhan (2010) proposes an alternative habit model which is able to explain the violations of the uncovered interest rate parity. However, in his model the real yield curve is downward sloping and the expectation hypothesis holds. Thus, it is not consistent with US bond markets. Additionally, in Verdelhan (2010) there is a strong correlation between the exchange rate changes and the consumption growth differentials, which is not the case in the data.

I also present a habit model. The main contribution over the previous habit-based term structure models is showing the importance of time-varying volatility in consumption growth for bond prices. Introducing a heteroskedastic consumption growth process into a habit model leads to the separation of the intertemporal smoothing and precautionary savings effects and thus allows to simultaneously match key domestic and international bond market puzzles. [2] Under my specification, the effective coefficient of risk-aversion is constant and is below 10 (which Mehra and Prescott, 1985, refer as an upper bound of realistic values and which is low compared to the previous habit literature), and the correlation between the exchange rate changes and the consumption growth differentials is weaker than in Verdelhan (2010).

A long-run risk model (Bansal and Shaliastovich, 2013) is able to simultaneously explain return predictability in domestic and international bond markets. Nevertheless, some of the model's implications are not completely clear. First, the long-run risk framework generally implies a downward sloping yield curve. Empirically, some countries (e.g., US) have an upward sloping real yield curve. Additionally, long-run risk models rely on the negative correlation between the consumption growth and inflation and/or the expected consumption growth and the expected inflation to generate an upward-sloping nominal yield curve. Such negative correlation has been observed in US during the twentieth century. However, Hasseltoft and Burkhardt (2012) and Fleckenstein, Longstaff, and Lustig (2013b) show that for the past decades this has not necessarily been the case. At the same time, the nominal yield curve still remained mostly upward sloping during this

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[2]Heyerdahl-Larsen (2012) and Stathopoulos (2012) start from a constant mean homoskedastic endowment processes and are able to generate some time variation in the mean and volatility of consumption growth through international trade, but this is not enough to sufficiently replicate bond returns predictability in either domestic or international markets (or both).

most recent period.

Second, in Bansal and Shaliastovich (2013) the real yield curve is downward sloping and the expectation hypothesis is strongly violated. However, anecdotal evidence suggests that countries with downward sloping real yield curves feature relatively weak violations of the expectation hypothesis (UK).[3]The model in this paper generates, depending on the parameters, both upward- and downward sloping real yield curves. Furthermore, the model predicts that the expectation hypothesis violations are stronger in countries with upward sloping yield curves.

At the first glance rare-disaster models (Farhi and Gabaix, 2010; Gabaix, 2012; Tsai, 2013) provide a good fit for both domestic and international bond market dynamics.[4] The main problem is that they heavily rely on the extremely low consumption growth outcomes not observed in many developed countries. Furthermore, in rare-disaster models, the real yield curve is either downward sloping or flat. In this paper extreme consumption outcomes are not needed to reproduce the observed asset pricing dynamics. In addition to the downward or flat real yield curves, the model in this paper can also have an upward sloping real yield curve.

The paper also makes two methodological contributions into the habit literature. First, I show that a heteroskedastic habit model allows to disentangle intertemporal smoothing from precautionary savings effects, unlike standard homoskedastic habit models (e.g., Abel 1990, 1999; Campbell and Cochrane, 1999; Wachter, 2002; Verdelhan, 2010). In that sense, the paper extends the heteroskedastic habit literature started by Bekaert (1996), who only analyze international bond markets, and Bekaert and Engstrom (2009), who only analyze basic asset pricing moments, such as the levels of the risk-free rate and the equity premium. The result has implications for the specification of habit models. Currently, there are two main specifications. The first and the most widespread specificiation is with i.i.d. consumption growth shocks and the habit's time-varying sensitivity to these shocks. This is approach taken, for instance, by Campbell and Cochrane (1999), Wachter

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[3]This critique generally applies to the most long-run risk models trying to explain the US bond market dynamics such as, for example, Hasseltoft (2012).

[4]Gabaix (2012) and Tsai (2013) do not explicitly analyze international puzzles but back-of-the-envelope calculations show that their models are able to address some of the international phenomena such as violations of the UIP.

(2002), and Verdelhan (2010) and can be interpreted as a time-varying price of risk and a fixed amount of risk approach. The second approach proposed by Bekaert and Engstrom (2009) features heteroskedastic consumption growth shocks and the habit's constant sensitivity to these shocks. This approach can be interpreted as a time-varying amount of risk and a fixed price of risk approach. Verdelhan (2010) argues that the first approach is not able to simultaneously explain US bond market dynamics and violations of the uncovered interest rate parity. I show that the second approach is able to address this task (due to the separation of the intertemporal smoothing from precautionary savings) and thus appears to be more general. Additionally, an advantage of the time-varying amount of risk is its computational simplicity: all solutions are available in closed form. At the same time, solving external habit models with a time-varying price of risk usually requires computationally heavy numerical procedures on a high precision grid (Wachter, 2005).

Second, I provide a potential explanation for why the consumption and asset prices are relatively weakly correlated in data (e.g., Backus and Smith, 1993, for the international finance case). I find empirical evidence suggesting that positive and negative consumption shocks affect habit differently. In particular, the intertemporal smoothing effect seems to be driven exclusively by negative consumption shocks. In habit models prices are determined by both habit and the consumption growth. If habit is imperfectly correlated with consumption (that is, for instance, negative consumption shocks affect habit differently than positive shocks), then the prices will be imperfectly correlated with consumption as well.

2   Bond and Currency Market Puzzles

This section provides qualitative and quantitative evidence on the real yield curve and its slope, the expectation hypothesis and the uncovered interest rate parity across countries.

2.1 Slope of the Real Yield Curve

Slope of the real yield curve is the difference between interest rates on long-term real bonds and short-term real bonds. Are these rates different? Although the trading history of inflation adjusted bonds in the US is relatively short (starting from 1997), Piazzesi and Schneider (2007) document that the real yield curve seems to be upward sloping. As time passes by, there accumulates more support for this conclusion. For instance, taking monthly US inflation adjusted rates from January 2004 to August 2013 from the extended appendix of Gurkaynak, Sack, and Wright (2009), the average difference between the 5 years yield and the 2 years yield is 0.41% (0.77% versus 0.36%) and is significant at the 1% significance level. A concern is that there are some liquidity issues in trading the inflation adjusted bonds in US (Fleckenstein et.al. 2013a). However, the slope is still economically large and statistically significant.

Although the trading of inflation adjusted bonds in the US has started only in 1997, several studies suggest that the average slope of the US real yield has been positive for a long time. These studies extract the real yield cruve from the nominal yield curve, inflation forecasts, and estimates of the inflation risk premium. For example, Chernov and Mueller (2012) show that the average difference between the 5 years and the 1 years real yields in US has been around 0.30% from 1971 to 2002. Ang and Ulrich (2012) estimate that the average difference between the 10 years and the 5 years US real yields has been around 0.45% from 1982 to 2008.

Internationally, there is also evidence of downward sloping real yield curves. For UK, Evans (1998) and Piazzesi and Schneider (2007) document a downward sloping yield curve: the difference between the 5 years yield and the 1 year yield is -0.30% (0.32% versus 0.62%).

Unfortunately, the further international evidence on the slope of the real yield curve is very limited, because inflation adjusted bonds are issued in relatively few countries and issues are mostly irregular and not very liquid (see Appendix A in Fleckenstein, 2013, for a good overview of international inflation adjusted bonds markets). Among a very few studies concentrating on the international real yield curves, Ejsing et.al. (2007) document that the real yield curve for France and Germany has been either flat or upward sloping.

Overall, it is puzzling why the real yield curve is upward sloping in some countries (US) and downward sloping in others (UK).

2.2 Expectation Hypothesis

The expectations hypothesis (EH) states that all variation in long-term risk-free rates is due to the variation in the future short-term risk-free rates. Informally, this means that if the slope of the yield curve is high, the short-term rates in the future are expected to rise and so will the yields on long-term bonds. Consequently, according to this theory, holding long-term bonds for the lifetime of short-term bonds when the slope of the yield curve is relatively high should result in relatively low returns.

Econometrically, the expectation hypothesis can be tested in several ways. In this paper, I follow the approach of Campbell and Shiller (1991)[5]. They run the following regression on the yields:

$$y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t, \tag{1}$$

where y is the logarithmic yield, n is the number of periods to maturity, and t is the time index. The expectation hypothesis implies that coefficients β_i should be equal to 1. However, Campbell and Shiller (1991) find that the coefficients are negative and decreasing with bonds maturities. This pattern has hold over time. Using the nominal US government bonds from June 1960 to June 2013, the coefficient in regression is gradually decreasing from -0.75 for the two years bonds to -1.57 for the five years bond (using the 1 year bond as the reference). Thus, the expectation hypothesis is violated. Economically, this means that long-term bonds deliver high, instead of low, returns when the slope of the yield curve is high.

Interestingly, the expectation hypothesis holds quite well for UK bonds. For instance, for regression (17) Bansal and Shaliastovich (2013) report positive regression coefficients which approach 1 at longer horizons.

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[5]Another popular approach is as in Fama and Bliss (1987).

2.3 Uncovered Interest Rate Parity

Uncovered interest rate parity (UIP) states that the difference in the risk-free rates between two countries should be equal to the expected change in the exchange rate between the currencies of the countries. Economically, this means that an investor should not be able to make profits from borrowing in country 1, exchanging the currency, lending in country 2, and then exchanging the currency back the next time period.

Econometrically, the UIP can be tested in the following way. Let e_t be the logarithm of the real exchange rate (the number of country 1 real consumption units given for a country 2 real consumption unit) and define $\Delta e_{t+1} = e_{t+1} - e_t$. Finally, let $y_{1,t}$ and $y_{1,t}^*$ be the real interest rates in countries 1 and 2, respectively. Then, $r_{t+1}^{FX} = -y_{1,t} + y_{1,t}^* + \Delta e_{t+1}$ is the return from the strategy of borrowing in country 1 and lending in country 2 mentioned above. The UIP implies that in the regression:

$$r_{t+1}^{FX} = \alpha_0 + \alpha_{FX}(y_{1,t} - y_{1,t}^*) + \epsilon_t, \tag{2}$$

the coefficient α_{FX} should be equal to 0. However, numerous studies starting from Hansen and Hodrick (1980) have shown that the coefficient is negative and often less than -1. For instance, Backus, Foresi, and Telmer (2001) document that for the US dollar-British pound pair the coefficient is -1.84, for the US dollar-Japanese yen pair -1.71, and for the US dollar-German mark pair -0.74. Thus, the UIP is violated.

The reported coefficients are stable at the prediction horizons from 1 month to 1 year. Some studies suggest that the UIP holds better at horizons longer than one year (Alexius, 2001, and Chinn and Meredith, 2004). However, this evidence is not conclusive (Bekaert, Wei, and Xing, 2007). Economically, this means that borrowing in low-interest rate countries and lending in high-interest rate countries generates profits. This is a so called carry trade strategy.

3 Model

The model belongs to the class of external habit models. The key difference to the standard external habit models (Abel, 1990, 1999; Campbell and Cochrane, 1999) is that

consumption growth shocks are heteroskedastic (which corresponds to a time-varying amount of risk in the economy) but the sensitivity of the habitual consumption to these shocks is constant over time (which corresponds to a fixed price of risk).

I only model the real side of the economy. This is because including money and inflation does not add much economic intuition. Calibrations show that including a consumption non-neutral inflation process into the model somewhat improves the empirical fit.

Modeling only the real side of the economy implicitly assumes that violations of the expectation hypothesis and uncovered interest rate parity are at least partially real phenomena. This assumption is empirically justified. Bansal and Shaliastovich (2007) in their Table II report that in US the inflation-adjusted expectation hypothesis violations are weaker than nominal expectations hypothesis violations but are still economically strong, statistically significant, and follow the same time pattern. Pflueger and Viceira (2011) document strong violations of the expectation hypothesis in inflation-indexed bonds. Hollifield and Yaron (2003) estimate that the uncovered interest rate parity violations should be attributed almost exclusively to real risks.

3.1 Preferences

A representative agent maximizes the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(\frac{C_t}{H_t})^{1-\gamma}}{1-\gamma}, \tag{3}$$

where C_t is the consumption at time t and H_t is the exogenous habit level (habitual consumption). The motivation for using the habit preferences in the model is that the agent becomes sensitive to relatively mild fundamentals' fluctuations.

Unlike the most of the recent literature (Campbell and Cochrane, 1999; Wachter, 2002; Bekaert and Enstrom, 2009; Verdelhan, 2010), I use the ratio habit utility instead of a difference habit utility. This is done for two reasons. First, in the ratio habit the coefficient of risk-aversion is constant, γ, which allows to concentrate on the core mechanism of this paper: the heteroskedasticity of the consumption process. Second, in the ratio habit framework, the habit has a straightforward interpretation of being a weighted average of past consumption shocks. The difference habit framework allows this interpretation only

after the approximation around the steady state. Other papers which employ the ratio habit utility are, for instance, Abel (1990, 1999) and Chan and Kogan (2002). Generally, as shown in the calibration section, all results in the paper can be replicated with the difference habit.[6]

3.2   Fundamentals Dynamics

The logarithmic consumption growth, $g_{t+1} = \ln \frac{C_{t+1}}{C_t}$, is a constant mean heteroskedastic process:

$$g_{t+1} = \bar{g} + \epsilon_{t+1}, \tag{4}$$

where $\bar{g}$ is a constant and ϵ_{t+1} is a mean 0 heteroskedastic shock. In this paper, ϵ_{t+1} is modeled as a mixture of two demeaned gamma-distributed shocks, $\omega_{p,t+1}$ and $\omega_{n,t+1}$:

$$\begin{aligned}
\epsilon_{t+1} &= \sigma_{cp}\omega_{p,t+1} - \sigma_{cn}\omega_{n,t+1}, \\
\omega_{p,t+1} &\sim \Gamma(\bar{p}, 1) - \bar{p}, \\
\omega_{n,t+1} &\sim \Gamma(n_t, 1) - n_t,
\end{aligned} \tag{5}$$

where $\bar{p}$ and n_t are shape parameters of the gamma distributions. $\omega_{p,t+1}$ roughly corresponds to the right tail and $\omega_{n,t+1}$ to the left tail of the consumption growth distribution. Economically, the consumption shock in the model has two components: one coming from a good regime ($\omega_{p,t+1}$) and another coming from a bad regime ($\omega_{n,t+1}$).

Gamma distributions are used instead of more standard Gaussian distributions for purely quantitative, not qualitative, reasons. Indeed, all main theoretical results in the paper can be straightforwardly replicated with heteroskedastic Gaussian consumption shocks, $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma_t)$. However, as it will be discussed in the calibration section, quantitatively matching predictability patterns observed in the data is much easier with gamma distributed shocks. This is because gamma shocks have fatter tails than Gaussian shocks. Thus, they increase the sensitivity of the agent to the shocks, which makes matching asset prices easier. It should be also pointed out that gamma shocks are not simply a

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[6]In fact, the calibration results with the difference habit are even stronger than with the ratio habit, as the countercyclical risk-aversion amplifies the asset pricing dynamics.

fancy technical tool but are empirically well justified: Bekaert and Engstrom (2009) show that gamma shocks describe the consumption dynamics better than Gaussian shocks.

At the first glance, the model might look like a rare disaster model but it turns out not to be the case. In equation (5) the right tail has a constant shape while the shape of the left tail varies over time. This indeed reminds a time-varying probability/size of a disaster dynamics as, e.g., in Gabaix (2010) or Tsai (2013). However, in the calibration the probability of an extreme consumption growth outcome in the model is very low: much lower than in actual US data and by magnitudes lower than in rare disaster models. In the model, the time-varying shape parameter of the left tail is used to model the time-varying volatility of the consumption growth: the volatility of the $\omega_{n,t+1}$ shock is indeed n_t.[7] Thus, instead of a rare disaster model a more appropriate title for the model is a habit model with time-varying volatility of the consumption growth. Consistently with this logic, in the remainder of the paper, I will refer to n_t and $\bar{p}$ as to the volatility parameters.

The shape parameter of the $\omega_{n,t+1}$ shock follows a lag 1 autoregressive process:

$$n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1}. \tag{6}$$

The role of the state variable n_t, corresponding to the time-varying volatility of the consumption, is to determine the strength of the precautionary savings effect.

In equations (5) and (6) the negative shock to the consumption growth and the shock to the volatility are the same. This corresponds to the observation that the volatility is higher during recessions.

Finally, in line with the earlier work on the external habit formation, for the brevity of exposition, I choose to model the logarithm of the consumption-habit ratio, $s_t = \ln\frac{C_t}{H_t}$ instead of the habit itself. Similarly to the previous literature, the logarithm of the consumption-habit ratio follows a lag 1 autoregressive process:

$$s_{t+1} = \bar{s} + \rho_s(s_t - \bar{s}) + \sigma_{sp}\omega_{p,t+1} + \sigma_{sn}\omega_{n,t+1}. \tag{7}$$

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[7]It is also possible to make the right tail of the consumption growth distribution time-varying. This is not implemented for parsimony. Calibrations show that a model with a time-varying right tail instead of a time-varying left tail performs quantitatively approximately as well as a model with time-varying left tail, although the predictability patterns are weaker and fundamentals more volatile.

Under this specification, the habit has an economically appealing interpretation of being a weighted average of past consumption shocks. To see this, note that $\ln H_t = g_t - s_t = \bar{g} + \sigma_{cp}\omega_{p,t} - \sigma_{cn}\omega_{n,t} - \bar{s} - \rho_s(s_t - \bar{s}) - \sigma_{sp}\omega_{p,t} - \sigma_{sn}\omega_{n,t}$. Plugging the process s_t from (7), the habit is $\ln H_t = constant + \sum_{i=0}^{\infty}(\alpha_i^p\omega_{p,t-i} + \alpha_i^n\omega_{n,t-i})$, where α_i^p and α_i^n are constants. The difference habit utility allows this interpretation only as an approximation around the steady state.

Similarly to Campbell and Cochrane (1999), the consumption growth and the consumption-habit ratio in equation (7) are driven by the same shocks.[8] However, equation (7) allows positive and negative shocks to affect the habit in different way than they affect consumption: generally σ_{sp} and σ_{sn} are not linked to σ_{cp} and σ_{cn}. Quantitatively, this assumption is only important for replicating the weak correlation between consumption growth and asset prices: prices are affected by the habit which in this specification is not perfectly correlated with the consumption. All other moments can be approximately reproduced from the model where the shock to the consumption growth and the shock to the consumption-habit ratio are perfectly correlated (that is, $\sigma_{sp} = \alpha\sigma_{cp}$ and $\sigma_{sn} = -\alpha\sigma_{cn}$ for a constant $\alpha > 0$). However, in Section 5 I show that the assumption of positive and negative consumption shocks affecting habit differently enjoys some empirical support and thus it is justifiable to include it into the model. [9]

Note that the consumption-habit ratio, s_t, in equation (7) plays a different role than the consumption-habit ratio in Campbell and Cochrane (1999). In my model, the role of s_t is purely the intertemporal smoothing (determining how valuable is consumption today compared to consumption in the future), the risk-aversion is constant. In Cambpell and Cochrane (1999) the consumption-habit ratio, in addition to intertemporal smoothing,

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[8]Campbell and Cochrane (1999), Wachter (2002), Bekaert and Engstrom (2009), and Verdelhan (2010) use the difference habit utility. For that reason, they are modeling the inverse surplus ratio instead of the consumption-habit ratio. The inverse surplus ratio and the consumption-habit ratio are technically different specifications to model the economically same thing. Thus, to avoid the confusion with terminology, in the paper I will refer to both of them as to the consumption-habit ratio.

[9]A different sensitivity to negative and positive shocks can be related to reference point-utility preferences (Koszegi and Rabin, 2009), where bad consumption ouctomes hurt more than good consumption outcomes benefit. There is empirical evidence in favor of such preferences (Crawford and Meng, 2009, Pope and Schweitzer, 2011, Sydnor, 2010, among others) and thus they are not simply a convenient technical tool.

also determines the risk-aversion (which in their model is time-varying).

Economically, positive shocks should increase the consumption habit ratio and negative consumption shocks should decrease the consumption-habit ratio (corresponding to the situation where positive consumption shocks increase utility and negative consumption shocks decrease it). Thus, σ_{qp} is expected to be negative and σ_{qn} is expected to be positive.

The key difference between the model in this paper and the standard habit model by Cambpell and Cochrane (1999) is that in this paper the consumption growth shocks are heteroskedastic, but sensitivity of the agent's consumption-to-habit ratio to these shocks is constant, in contrast in Campbell and Cochrane (1999) the consumption growth shocks are homoskedastic, but sensitivity of the consumption-to-habit ratio to these shocks is time-varying. This turns out to be very important because this paper's specification allows to disentangle the intertemporal smoothing and precautionary savings effects from each other. Indeed, the intertemporal smoothing process is governed mainly by the consumption-habit ratio process (7), while the precautionary savings are driven by the volatility process (6). In the standard habit model (Campbell and Cochrane, 1999; Wachter, 2002; Verdelhan, 2010), there is no time-varying consumption volatility. Both the intertemporal smoothing and precautionary savings effects are driven by the consumption-habit ratio process. Depending on the specification of that process, either the intertemporal smoothing or the precautionary savings effect is dominant all the time. This severely restricts term structure dynamics and, in particular, precludes explaining domestic and international bond markets dynamics simultaneously: in Wachter (2002) the intertemporal smoothing dominates and the uncovered interest rate parity holds, while in Verdelhan (2010) the precautionary savings dominate and the real yield curve is downward sloping and the expectation hypothesis holds. The model in this paper disentangles the intertemporal smoothing and precautionary savings effects and allows them to operate to some extent independently. As shown in the next section, this allows to simultaneously match domestic and international bond market dynamics.

4   Asset Pricing Implications

This section explains how the model is able to generate bond market predictability patterns often considered puzzling. To clarify economic intuition, I concentrate on the examples with one and two period bonds and then discuss how the results are generalized for longer term bonds. All asset prices in the model are closed-form and thus easy to interpret. For brevity, in this section I only discuss the formulas necessary for intuition. All other equations are relegated to the appendix.

In line with the previous external habit literature, the agent only maximizes her utility with respect to consumption and takes the habit as given. Consequently, the stochastic discount factor, M_{t+1}, is equal to the ratio of marginal utilities of consumption at $t+1$ and t:

$$M_{t+1} = \beta \frac{C_t}{C_{t+1}} \frac{(\frac{C_{t+1}}{H_{t+1}})^{1-\gamma}}{(\frac{C_t}{H_t})^{1-\gamma}} = \beta e^{-g_{t+1} + (1-\gamma)(s_{t+1} - s_t)}. \tag{8}$$

Risk premia on the assets are determined by the covariance of their returns with the innovations to the stochastic discount factor. The innovations to the (logarithmic) stochastic discount factor are:

$$\begin{aligned}
m_{t+1} - E_t m_{t+1} &= a_p \omega_{p,t+1} + a_n \omega_{n,t+1}, \\
a_p &= (1-\gamma)\sigma_{sp} - \sigma_{cp}, \\
a_n &= (1-\gamma)\sigma_{sn} + \sigma_{cn}.
\end{aligned} \tag{9}$$

For simplicity, I only consider the case where positive consumption growth shocks decrease the stochastic discount factor (corresponding to good times) and negative consumption growth shocks increase the stochastic discount factor (corresponding to bad times). In (9), this corresponds to $a_p < 0$ and $a_n > 0$.

4.1   Yield Curve

All bonds are real risk-free zero-coupon bonds. It is covenient to work with the continuously compounded yields defined as:

$$y_{n,t} = -\frac{1}{n} \ln P_{n,t}, \tag{10}$$

where t is the time index, n is the number of periods to the payoff, and P is the bond price.

Let us first analyze the one period risk-free rate. Using the formulas for expectations of gamma distributed variables from the appendix, (8) yields:

$$y_{1,t} = -\ln\beta + \bar{g} - (1-\gamma)(1-\rho_s)\bar{s} + f(a_p)\bar{p} + \underbrace{(1-\gamma)(1-\rho_s)}_{<0}\, s_t + \underbrace{f(a_n)}_{<0}\, n_t, \quad (11)$$

where:

$$f(x) = x + \ln(1-x). \tag{12}$$

Note that in (12) function $f(x)$ is always negative.

The risk-free rate in equation (11) consists of two components: the intertemporal smoothing and precautionary savings. First, the risk-free rate loads negatively on the consumption-habit ratio s_t.[10] This is the intertemporal smoothing effect: in times of low consumption (low s_t) the agent wants to consume now and not in the future and thus needs to be compensated for transfering consumption into the future. Second, the risk-free rate loads negatively on the consumption growth volatility n_t. This is the precautionary savings effect: in times of high consumption growth volatility (high n_t) the agent wants to save to hedge the uncertainty.

To understand how the average slope of the yield curve is determined, let us consider an example with only one and two period zero-coupon bonds. The return on holding a two period bond over one period is $R_{2,t\to t+1} = \frac{P_{1,t+1}}{P_{2,t}}$. Taking logs:

$$r_{2,t\to t+1} = -y_{1,t+1} + 2y_{2,t}. \tag{13}$$

Rearranging and taking unconditional expectations of (13) results in:

$$E(y_{2,t} - y_{1,t}) = \frac{1}{2}\underbrace{E(y_{1,t+1} - y_{1,t})}_{\text{expected change in the short rate}} + \frac{1}{2}\underbrace{E(r_{2,t\to t+1} - y_{1,t})}_{\text{risk-premium for holding a 2 period bond over 1 period}},$$

$$E(y_{2,t} - y_{1,t}) = \frac{1}{2}E(r_{2,t\to t+1} - y_{1,t}). \tag{14}$$

The left hand side of (14) is the average slope of the yield curve in our two bonds example. In the first line it consists of two parts. The first part corresponds to the expected return

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[10]Throughout the paper I assume that the risk-aversion coefficient, γ, is greater than 1.

on holding a two period bond once the one period bond expires: from time $t + 1$ to time $t + 2$. This correponds to the expected change in short rates. The second part is the expected excess return on holding a two period bond over the life time of the one period bond. Note that this return is not necessarily 0, because the two period bond is only riskless at the two period horizon, at the one period horizon its price will fluctuate and these fluctuations might be correlated with the agent's marginal utility (stochastic discount factor). From the second line of (14), this risk-premium determines the average slope of the yield curve. This is because on average the 1 period yields today and the next period will be equal and thus the first term cancels out.

By definition, the risk-premium on holding a two period bond over 1 period, $E(r_{2,t \to t+1} - y_{1,t})$, is determined by the covariance of $r_{2,t \to t+1}$ with the stochastic discount factor: $E(R_{2,t \to t+1} - R^f_{t \to t+1}) = -\frac{cov(M_{t+1} R_{2,t \to t+1})}{EM_{t+1}}$[11]. In logs:

$$E(r_{2,t \to t+1} - y_{1,t}) \approx -cov(m_{t+1}, r_{2,t \to t+1}) = cov(m_{t+1}, y_{1,t+1}), \qquad (15)$$

where the first part is an approximation, because there is also a Jensen inequality term due to the concavity of the logarithm function.

The logic behind (15) is that if a future short yield $(y_{1,t+1})$ is positively correlated with the stochastic discount factor holding a two period bond is risky. This is because if the next period the stochastic discount factor will be high (a bad situation for the agent), the short term yield is likely to be high as well, implying a low bond price. Consequently, the agent needs to be compensated for holding a two period bond. From (14), this implies a positive average yield curve slope.

In the model, the average slope of the yield curve is determined in the interaction of the intertemporal smoothing and precautionary savings effects. To see this, note that plugging (15) into (14) yields:

$$E(y_{2,t} - y_{1,t}) = \frac{1}{2} E(r_{2,t \to t+1} - y_{1,t}) \approx \frac{1}{2} cov(m_{t+1}, y_{1,t+1})$$

$$= \underbrace{-S_1 \sigma_{sn} a_n Var(n_t)}_{>0} + \underbrace{N_1 \sigma_{nn} a_n Var(n_t)}_{<0} ,$$

intertemporal smoothing, $\propto cov_t(m_{t+1}, s_{t+1})$ precautionary savings, $\propto cov_t(m_{t+1}, n_{t+1})$

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[11]This follows from substituting the definition of the covariance into the definiton of the stochastic discount factor: $E(M_{t+1} R_{2,t \to t+1}) = 1$.

$$\tag{16}$$

where the second line is computed by substituting in (9) and (11). The first term in (16) is the intertemporal smoothing term, which comes from the covariance of the stochastic discount factor with consumption-habit ratio. If a negative consumption shock is realized, the consumption-habit ratio drops, increasing the short-term yield (equation (11)) and consequently decreasing bond prices. Thus, a two period bond is a bad hedge against consumption shocks and should trade at premium. Indeed, the first term in (16) is positive.

The second term in (16) is the precautionary savings term, which comes from the covariance of the stochastic discount factor with consumption growth volatility. If a negative consumption shock is realized, the consumption growth volatility rises, decreasing the short-term yield (equation (11)) and consequently increasing bond prices. Thus, a two period bond is a good hedge against consumption shocks and should trade at discount. Indeed, the second term in (16) is negative.

Overall, the slope of the yield curve will be determined by which of the two effects is stronger:

1) Long-tem bonds are a poor hedge against the consumption shocks because they decrease the consumption-habit ratio decreasing bond prices. Thus, the real yield curve should slope up.

2) Long-term bonds are a good hedge against the consumption shocks because they increase the consumption growth volatility increasing bond prices. Thus, the real yield curve should slope down.

For longer horizons, the model can reproduce a very rich set of real yield curves: upward-sloping (as, e.g., for US in Gurkaynak, Sack, and Wright 2009), downward-sloping (as, e.g., for UK in Piazzesi and Schneider, 2007), hump-shaped (as, e.g., for US in Ang, Bekaert, and Wei, 2008), or U-shaped. This is because in the model the relative strength of the intertemporal smoothing and precautionary savings effects are different at different horizons: at some horizons the intertemporal smoothing effect might be dominant increasing the slope of the yield curve, while at other horizons the precautionary savings effect is dominant decreasing the slope of the yield curve.

4.2 Expectation Hypothesis

To understand the implications of the model for the expectation hypothesis, let us again consider the two bonds example. Going back to equation (13), rearranging and taking conditional expectations:

$$\underbrace{y_{2,t} - y_{1,t}}_{\text{slope of the yield curve}} = \frac{1}{2} \underbrace{E_t(y_{1,t+1} - y_{1,t})}_{\text{expected change in the short rate}} + \frac{1}{2} \underbrace{E_t(r_{2,t \to t+1} - y_{1,t})}_{\text{risk-premium for holding a 2 period bond over 1 period}} . \tag{17}$$

The left hand side of (17) is the time t slope of the real yield curve, which consists of two parts. The first part corresponds to the expected relative return on holding a two period bond once the one period bond expires: it is equal to the expected change in the short rate. That is, if the expected change in the short rate next period, $y_{1,t+1} - y_{1,t}$ is high, then the slope of the real yield curve is going to be high as well. This is consistent with the expectation hypothesis: the high yield curve slope should predict high future short rates and thus low returns on long-term bonds.

The second part of the time t slope of the real yield curve in equation (17) is the risk-premium for holding a two period bond over the first period (that is over the life of the one period bond). This risk-premium is not necessarily zero or even constant, because the price of a two period zero-coupon bond next period is not sure: it will depend on the short rate next period. Furthermore, this dependance might vary through time. As the short rate next period might be correlated with the marginal utility (stochastic discount factor) of the agent, there will be a premium (or a discount) for holding a two period bond over one period.

The model explains violations of the expectation hypothesis via the time-varying volatility of the consumption shocks. Suppose that the time t consumption growth volatility is relatively high. The first component of the real yield curve in (17), the expected short rate next period, is then relatively high as well. To see this, note that due to the mean-reversion in the volatility (equation (6)), the volatility next period is expected to be lower. Consequently, the precautionary savings effect is expected to be weaker and the expected short interest rate is high (equation (11)).

If the intertemporal smoothing is the dominant effect, then the second component of the

real yield curve in (17), the risk-premium on holding a two period bond over one period, is also high in times of high consumption growth volatility. To see this, let us analyze the conditional version of equation (16):

$$E_t(r_{2,t\rightarrow t+1} - y_{1,t}) = \overbrace{\underbrace{-S_1\sigma_{sn}a_nVar_t(n_{t+1})}_{>0}}^{\text{intertemporal smoothing, } \propto cov_t(m_{t+1},s_{t+1})} + \overbrace{\underbrace{N_1\sigma_{nn}a_nVar_t(n_{t+1})}_{<0}}^{\text{precautionary savings, } \propto cov_t(m_{t+1},n_{t+1})} .$$

(18)

From (6), $Var_t(n_{t+1})$ is proportional to n_t. If the intertemporal smoothing is the dominant effect, in (18) the first term is more important and the risk premium on holding a two period bond is high when the consumption growth volatility is high. The intuition is that, as the two period bond is a bad hedge against consumption shocks, the agent should be compensated more for holding it when the magnitude (volatility) of these shocks is large. Thus, as both components of the slope of the real yield curve are high in times of high consumption growth volatility, the slope is itself high in these times.

To summarize, assuming that the intertemporal smoothing is the dominant effect in the model, the high consumption growth volatility drives up both the slope of the yield curve and the holding returns on long-term bonds. Thus, the high slope of the yield curve predicts high returns on the long-term bonds. The expectation hypothesis is violated.

Assuming that the precautionary savings is the dominant effect, the model can also replicate the non-violated expectation hypothesis, as observed, for example, in UK. To see this, note from (18) that, if the precautionary savings effect is the dominant effect, the risk premium on holding a two period bond over one period is low when the consumption growth volatility is high. The intuition is that, as the two period bond is a good hedge against consumption shocks, the agent should be compensated less for holding it when the magnitude (volatility) of these shocks is large. Thus, when the consumption growth volatility is high, the first term in (17) will be high and the second term in (17) will be low. Thus, depending on which of these two terms is more important, in times of the high consumption growth volatility the slope of the real yield curve will be either high or low and thus will be positively (the expectation hypothesis is violated) or negatively (the expectation hypothesis holds) correlated with the expected return on holding a two period bond over one period.

Comparing this and the previous subsection, note the link between the slope of the

real yield curve and the expectation hypothesis. If the intertemporal smoothing effect is dominant, the yield curve is sloping up and the expectation hypothesis is likely to be violated. At the same time, if the precautionary savings effect is dominant, the yield curve is sloping down and the expectation hypothesis is more likely to hold. This prediction is in line with the anecdotal two country evidence (US and UK) that the expectation hypothesis holds better in countries with the downward sloping real yield curve.

In the model, the degree of the expectation hypothesis violations might vary at different time horizons (as is the case, for instance, in US data). This is because, depending on the model's parameters, at each time horizon, the relative strength of the intertemporal smoothing and precautionary savings effects might be different.

4.3 Uncovered Interest Rate Parity

To understand the implications of the theory for the uncovered interest rate parity, let us analyze the return, r_{t+1}^{FX}, from borrowing in country 1 and lending the same amount in country 2. I assume that there are two countries with one country specific consumption good each.[12] Following Backus et.al. (2001), Verdelhan (2010), and Bansal and Shaliastovich (2013), I assume that financial markets are complete and there are no arbitrage opportunities. Under these assumptions, the exchange rate change is equal to the difference in stochastic discount factors: $\Delta e_{t+1} = m_{t+1}^* - m_{t+1}$. Thus:

$$E_t r_{t+1}^{FX} = -y_{1,t} + y_{1,t}^* + E_t(m_{t+1}^* - m_{t+1}) = \underbrace{N_1}_{>0}(n_t - n_t^*). \tag{19}$$

The intuition behind the return on the cross-country borrowing-and-lending strategy is as follows. First, the return is independent of the consumption-habit ratio s_t. This is because the role of s_t is intertemporal smoothing. Intertemporally, nothing changes for the agent: she borrows in country 1 and lends the same amount in country 2, her time t consumption is unaffected.

Second, the return is higher the higher is the volatility in country 1, n_t. This is because the

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[12]This assumption is in line with an empirical evidence by Burstein, Eichenbaum, and Rebelo (2006) that most of the real exchange rate fluctuations are driven by non-tradable goods. Recall that, as shown by Hollifield and Yaron (2003), the real exchange rate fluctuations are the most relevant for the uncovered interest rate parity violations.

strategy is a poor hedge against negative shocks, $\omega_{n,t+1}$, to the stochastic discount factor and, thus, according to (9) should earn positive returns when the magnitude (volatility) of these shocks (n_t) is higher. To see why the strategy is a poor hedge against negative shocks, suppose a large negative shock in country 1 realizes. Then the country 1's consumption is scarce compared to the country 2's consumption. Consequently, the agent who will be exchanging the country 2's consumption units for the country 1's consumption units will get little consumption when she needs it the most (in the case of large negative consumption shocks in country 1).

Note that the argument in the previous paragraph is a volatility, not a disaster, argument. The strategy above is also a poor hedge against positive shocks, $\omega_{p,t+1}$. To see this, suppose a large positive shock in country 1 realizes. Then the country 1's consumption is abundant compared to country 2's consumption. Consequently, the agent who will be exchanging the country 2's consumption units for the country 1's consumption units will get a lot of consumption when she needs it the least (in the case of large positive consumption shocks in country 1). Thus, the argment for the positive shock is the same as for the negative shock. As already mentioned before, the time-varying volatility of only the negative shock is a choice made mainly for the brevity of the exposition: the logic of the model is replicable with time-varying volatility of positive shocks.

The model's explanation for the violations of the uncovered interest rate parity is based on the fact that consumption growth volatility affects both the interest rate and the return on the cross-country borrowing-and-lending stragegy at the same time. Indeed, from (11) the interest rate is decreasing in volatility n_t due to the precautionary savings. From (19) $E_t r_{t+1}^{FX}$ is in increasing in the volatility n_t because the trading strategy is a poor hedge against consumption shocks and thus is riskier when the magnitude (volatility) of these shocks is higher. Thus, when the agent borrows from a low interest rate country and invests into the high interest rate country, she has a large exposure to consumption shocks in the low-interest rate country.[13] Thus, on average, she should earn a premium

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[13]In (11) the interest rate also depends on the consumption-habit ratio s_t. As s_t and n_t are negatively correlated, when n_t is high, the risk-free rate might be relatively high, instead of low, due to the low s_t. This would make the uncovered interest rate parity hold. However, quantitatively, in the economically sensible parameter region, the impact of the s_t on the uncovered interest rate parity seems to be always smaller than the impact of the volatility driven dependence discsussed in the main text.

from such a strategy.

4.4 Equity

Similarly to the previous habit literature starting from Campbell and Cochrane (1999), the equity is modeled as the claim to the each period's consumption. The price-dividend ratio is solved in closed form in the appendix by using the formulas for expectations of gamma distributed variables. The solution and intuition for equity are largely from Campbell and Cochrane (1999) and Bekaert and Engstrom (2009) and thus an interested reader is referred to these papers for a detailed discussion.

5   Empirical Evidence and Simulation

I start by demonstrating empirical evidence in favor of main mechanisms driving the model. Next, I show that under reasonable parameters the model is able to reproduce bond and equity market dynamics in US and internationally, including different slopes of the yield curve, violations of the expectation hypothesis and the uncovered interest rate parity. I also quantitatively demonstrate the link between the slope of the real yield curve and the violations of the expectation hypothesis established in the previous section.

5.1   Empirical evidence

5.1.1   Domestic bond markets

In modeling the domestic term-structure, there are 3 key mechanisms in the model:

1) intertemporal smoothing: interest rates are low when the consumption-habit ratio is high and interest rates are high when the consumption-to-habit ratio is low

2) precautionary savings: interest rates are low when the consumption growth volatility is high and interest rates are high when the consumption growth volatility is low

3) different sensitivity of the consumption-habit ratio to positive and negative consumption shocks

The risk-free rate equation (11) is the key equation to empirically evaluate the mechanisms above. In particular, it states that the real risk-free rate should be decreasing in the consumption-habit ratio and decreasing in the conditional consumption growth volatility. These predictions can be tested by regressing a proxy for the real risk-free rate on proxies for the consumption-habit ratio and the conditional consumption growth volatility. Furthermore, the different sensitivity of the consumption-habit ratio to positive and negative shocks can be studied by constructing proxies for the consumption-habit ratio where positive and negative shocks contribute differently and investigating if these proxies are more successful in explaining asset prices than proxies where all consumption shocks affect the consumption-habit ratio in the same way.

In order to test the mechanisms in data, I need a proxy for the real interest rate as well as for the consumption-habit ratio and the consumption growth volatility. I approximate the real interest rate by taking the nominal 3 month Treasury bill rate from the St.Louis Fed website and reducing the expected inflation for the next quarter from the Survey of Professional Forecasters. I operate at the quarterly frequency, because ignoring the inflation risk premium is better justified for short time intervals as the inflation is strongly predictable.The time period is 1969Q1-2012Q4, because the inflation forecasts are not available before that.

I approximate the consumption-habit ratio using its evolution process given in (7). First, assume that the shock to the consumption-habit ratio is the the same as to the consumption growth (mathematically, $\sigma_{sp} = \alpha \sigma_{cp}$ and $\sigma_{sn} = -\alpha \sigma_{cn}$). Then, by iterating equation (7) backwards, the consumption-habit ratio can be expressed as:

$$s_t = const + \alpha \sum_{i=0}^{\infty} \rho_s^i (\sigma_{cp} \omega_{p,t-i} - \sigma_{cn} \omega_{n,t-i}). \tag{20}$$

Note that, under the model's assumptions, shocks $(\sigma_{cp} \omega_{p,t-i} - \sigma_{cn} \omega_{n,t-i})$ in (20) are simply consumption growth shocks and are thus directly observable from data. This allows to approximate the consumption-habit ratio using the past consumption growth data. In particular, Wachter (2002) argues that around 40 quarters of past consumption is needed to approximate habit, and this is the value I use in this paper. ρ_s is set to 0.97 which is the average quarterly habit persistence in the previous habit literature (Campbell and Cochrane, 1999; Wachter, 2002, 2006; Verdelhan, 2010) and this paper.

The general specification in equation (7) allows positive and negative components of consumption growth shocks to affect the consumption-habit ratio in different ways than they affect consumption growth (mathematically, $\sigma_{sp} = \alpha_1 \sigma_{cp}$ and $\sigma_{sn} = -\alpha_2 \sigma_{cn}$ with $\alpha_1 \neq \alpha_2$). By iterating equation (7) backwards, the consumption-habit ratio can be expressed as:

$$s_t = const + \alpha_1 \sum_{i=0}^{\infty} \rho_s^i \sigma_{cp} \omega_{p,t-i} + \alpha_2 \sum_{i=0}^{\infty} \rho_s^i (-\sigma_{cn} \omega_{n,t-i}). \tag{21}$$

Filtering $\sigma_{cp} \omega_{p,t-i}$ and $-\sigma_{cn} \omega_{p,t-i}$ for equation (26) from data is non-obvious. Thus, as a first approximation, to simplify the computational burden, I set:

$$\begin{aligned}
\sum_{i=0}^{\infty} \rho_s^i \sigma_{cp} \omega_{p,t-i} &:= \sum_{i=0}^{\infty} \rho_s^i (g_{t-i} - \bar{g}) \mathbb{1}_{(g_{t-i} - \bar{g}) \geq 0}, \\
\sum_{i=0}^{\infty} \rho_s^i (-\sigma_{cn} \omega_{n,t-i}) &:= \sum_{i=0}^{\infty} \rho_s^i (g_{t-i} - \bar{g}) \mathbb{1}_{(g_{t-i} - \bar{g}) < 0},
\end{aligned} \tag{22}$$

where $\mathbb{1}$ is an indicator function. The interpretation of equation (22) is that positive components of the consumption growth shocks are simply estimated as the positive consumption growth shocks, and negative components of the consumption growth shocks are estimated as the negative consumption growth shocks. This is an approximation because in the model every consumption growth shock is always a mixture of a positive and a negative component (see equation (5)). As components follow a demeaned gamma distribution, $\omega_{p,t}$ might well be negative and $-\omega_{n,t}$ positive. In empirically evaluating (22), I again follow the previous literature and use 40 quarters of data and $\rho_s{=}0.97$.

Finally, I estimate the conditional volatility of consumption growth using two different ways. First, I assume that the consumption growth follows a constant mean heteroskedastic process, where the error term is following a GARCH(1,1) process. That is I estimate the following process for the quarterly consumption growth data via maximizing the likelihood:

$$\begin{aligned}
g_t &= \bar{g} + \sigma_t \epsilon_t, \\
\sigma_t^2 &= \bar{\sigma} + \rho_\sigma \sigma_{t-1}^2 + \phi (g_{t-1} - \bar{g})^2, \\
\epsilon_t &\sim \mathcal{N}(0,1).
\end{aligned} \tag{23}$$

I employ σ_t in (23) as the first measure of the conditional volatility of the consumption growth.

Note that (23) is an approximation because as equation (5) states the shocks in the model are gamma and not normally distributed. To address this issue, I also compute the conditional volatility using the gamma shocks. In particular, I estimate the following BEGE-GARCH (Bekaert, Engstrom, and Ermolov, 2014) process for the quarterly consumption growth data (again via maximizing the likelihood):

$$
\begin{aligned}
g_t &= \bar{g} + \sigma_{cp}\omega_{p,t} - \sigma_{cn}\omega_{n,t}, \\
\omega_{p,t} &\sim \Gamma(\bar{p}, 1), \\
\omega_{n,t} &\sim \Gamma(n_{t-1}, 1), \\
n_t &= \bar{n} + \rho_n n_t + \phi_n (g_{t-1} - \bar{g})^2,
\end{aligned}
\tag{24}
$$

I employ $\sqrt{\sigma_{cp}^2 \bar{p} + \sigma_{cn}^2 n_{t-1}}$ as the second measure of the conditional volatility of the consumption growth. Note that (24) is still an approximation to the theoretical process in the paper, because for computational reasons I do not filter $\omega_{n,t}$ and $\omega_{p,t}$ shocks and, thus, unlike in the theoretical model, the shock to the volatility (n_t) does not equal the $\omega_{n,t}$mshock to the consumption growth.

Table 1 illustrates that the model in this paper seems to be roughly consistent with US data while previous habit models seem to be not. In particular, specification 1 shows that taken alone, a consumption-habit ratio seems to be positively (albeit statistically insignificantly) associated with risk-free rates. This casts doubt on the traditional habit-based explanation of an upward sloping real yield curve and the violations of the expectation hypothesis (Wachter, 2006), as this explanation requires a negative relationship between the consumption-habit ratio and short-term risk-free rates (in order for long-term bonds to be risky securities).

Specification 2 in Panel A of Table 1 shows that impact of positive and negative consumption shocks on the consumption to habit ratio is indeed statistically significantly different (the coefficient of the *Consumption-habit⁻* is statistically significantly negative). In particular, while for positive consumption shocks the relationship between the consumption-habit ratio and risk-free rates is still positive (the coefficient of the *Consumption-habit* is positive), the relationship between the negative shocks and the consumption-habit ratio becomes negative (0.5715-0.8452=-0.2737). This implies that empirically the intertemporal smoothing effect is pronounced for negative consumption shocks and is absent for positive consumption shocks. Indeed, in data a large in magnitude negative shock to the

consumption-habit ratio leads to higher risk-free interest rates implying that bonds are a bad hedge for consumption shocks. Interestingly, a large in magnitude positive shock also implies higher risk-free rate. The results of specifications 1 and 2 in Panel A suggest that in order to justify some kind of the intertemporal smoothing in data under the habit framework, one should allow negative and positive consumption shocks to affect the consumption-habit ratio differently.

Specifications 3 and 4 in Panel A of Table 1 show the evidence of the intertemporal smoothing and precautionary savings effects in data. Specification 3 shows that if positive and negative shocks are not allowed to affect the consumption-habit ratio differently, there is no evidence of either intertemporal smoothing or precautionary savings effects. Contrary, both the higher consumption-habit ratio and the higher conditional volatility of the consumption growth are associated with the higher risk-free rates (coefficients of the *Consumption-habit* and *Conditional volatility* are positive). However, specification 4 indicates that if positive and negative shocks do affect the consumption-habit ratio differently (and they do affect the consumption-habit ratio differently as the *Consumption-habit⁻* coefficient is statistically significantly negative), there appears to be the intertemporal smoothing effect for negative shocks (consistently with results in specifications 1 and 2) and the precautionary savings effect (a negative coefficient for the *Conditional volatility*). Thus, in line with specifications 1 and 2, specifications 3 and 4 suggest that once positive and negative consumption shocks are allowed to affect the consumption-habit ratio differently, there is evidence of both intertemporal smoothing and precautionary savings effects. Results using gamma distributed shocks (in Panel B of Table 1) are in line with results obtained with Gaussian shocks.

Overall, the empirical evidence suggests that the positive and negative consumption shocks affect the consumption-habit ratio differently, and, after accounting for this, there are intertemporal smoothing and precautionary savings effects.[14] The intertemporal smoothing seems to be driven almost exclusively by negative shocks. Consistently with this observation, in the calibration of the model negative consumption shocks will have greater impact on the consumption-to-habit ratio than positive shocks.

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[14]Importantly, this also adresses the Hartzmark (2014)'s empirical critique towards the absence of the link between economic (consumption) growth and interest rates.

Table 1 about here.

5.1.2 International bond markets

Data supports the model's volatility based explanation for the uncovered interest rate parity violations. From Panel A of Table 2 it can be seen that, in line with the model's mechanism, the consumption growth volatilities for the low interest countries are higher than for high interest countries.[15] Panel B of Table 2 shows that the diffferences are often statistically significant. Interestingly, the differences in consumption growth volatilities are smaller in the most recent sample. This is largely in line with the observation that G10 carry trade returns have been economically relatively weak and statistically insignificant in 1990:s and 2000:s (see, for example, Table 8 in Chabi-Yo and Song, 2013, for the most recent evidence).

Table 2 about here.

5.2 Calibration

I calibrate the model to match the bond and equity markets dynamics in US. Internationally, I match the uncovered interest rate parity coefficient observed between US dollar and some common currencies. I calibrate the model at the annual frequency. This is to avoid the time aggregation issues.

In the calibration, I aim to match the following set of moments:

a) US per capita consumption growth moments: mean, standard deviation, skewness, excess kurtosis, probabilities of extreme outcomes (<mean-2×standard deviation, <mean-4×standard deviation). I obtain these moments from NIPA tables 1.1.6 (the real consumption growth) and 7.1 (the population growth). The time period is 1929-2012.

b) US real interest rates: annual real interest rates for 1,2,3,4, and 5 years and the volatility of the 1 year interest rate. This data is from the updated appendix of Gurkaynak et.al. 2009 and covers the period from 2004 to 2012. The TIPS data is only available at

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[15]Ready, Roussanov, and Ward (2014) report the same empirical evidence in the context of a different model.

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annual maturities with the shortest maturity being 2 years. For this reason, I linearly extrapolate the values for the 1 year real bonds from the 2,3,4, and 5 years bonds. The time period is relatively short because the history of inflation adjusted bonds trading in US is short.

c) US equity: equity risk-premium, Sharpe-ratio, price dividend ratio. The risk-premium and Sharpe-ratio are from Kenneth French's data library. The price-dividend ratio is from Boudoukh et.al. (2007). The time period is 1929-2012. [16]

d) US expectation hypothesis coefficients for 2,3,4, and 5 years bonds using the 1 year bond as a reference short-term security. The expectation hypothesis coefficients are not directly available for real bonds. Thus, I compute them by running regression (17) for the nominal bonds. The data on the yields is from the updated appendix of Gurkaynak et.al. (2006). I do not aim to match these coefficients exactly, because I am operating with real, not nominal, yields. I aim to match the negativity of the coefficients and that the coefficients are more negative for the bonds with longer maturities.

e) The uncovered interest rate parity coefficient between US and other major economies. The uncovered interest rate parity coefficients are not directly available in real terms because there is not long enough history of real government bonds trading for the most countries. The uncovered interest rate parity coefficients are from Backus et.al. (2001).

The calibration is the mix of minimizing the weighted squared distance between the model implied moments and their empirical counter-parts and the hand-calibration. I partially rely on the hand calibration because it is not feasible to get consistent standard errors for all the moments of interest: some moments are only available in nominal terms and time periods for different moments are rather different. For the identification purposes, the discount factor β is fixed to 0.975. This is the average of the discount factors from Campbell and Cochrane (1999) and Wachter (2002). The level of the consumption-habit

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[16]Calibrating the price dividend ratio is somewhat complicated. In the model, the price-dividend ratio corresponds to the equity payout ratio: that is the ratio of the equity price to the total payout (not only dividends but also share repurchases and equity issuance). Boudoukh et.al. (2007) show that in recent years this statistic has gone very small and even negative. This means that the capital have been flowing from households to firms. The model in this paper is not designed to have such negative dividends. Thus, instead of the payout ratio I follow most of the asset pricing literature, such as Campbell and Cochrane (1999) or Bansal and Yaron (2004), and match the price-dividend ratio from Boudoukh et.al. (2007).

ratio is set to 1. Unlike in the Campbell and Cochrane (1999)-type models, the level of the consumption-habit ratio is irrelevant for pricing, because, as it can be seen from equations (8) and (9), the stochastic discount factor is only dependent on the ratio (not levels) of the consumption-habit ratios across different time periods.[17] Thus, $\bar{s}$ can have an arbitrary positive value.

The parameters which will fit consumption and asset pricing dynamics reasonably well are summarized in Table 3. The local relative risk-aversion (which is a rough counterpart to the CRRA risk-aversion in the model) is relatively low compared to other models which try to explain bond markets predictability such as Wachter (2002) and Bansal and Shaliastovich (2013).The persistence of the habit might look low, but this is an annual persistence, not monthly or quarterly persistence used in most other habit models. The persistence of the habit is roughly in line with the annualized value from Bekaert and Engstrom (2009). In line with economic intuition, positive consumption shocks increase and the negative consumption shocks decrease the consumption-habit ratio. The left tail of the consumption growth distribution is much stronger non-Gaussian than the right-tail. This is consistent with US consumption dynamics analyzed in Bekaert and Engstrom (2009). Figure 1 visualizes the consumption growth distribution in the model.

Table 3 about here.

Figure 1 about here.

5.3 Results

The model fits both consumption and asset pricing dynamics reasonably well. Table 4 shows that the model implied consumption dynamics is more Gaussian than US consumption in 1929-2012. The probability of the consumption disasters in the model is very low compared to the rare disaster models: around 2% for the two standard deviations disaster and almost 0 for the four standard deviastions disaster. For instance, in the rare disaster literature, Gabaix (2012) requires the probability of the four standard deviation

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[17]In the Campbell and Cochrane (1999)-type models, the level of the consumption-habit ratio is important, because it affects the habit's sensitivity to consumption shocks. In this paper this sensitivity is constant.

consumption growth disaster to be 3%. Additionally, in his model the disasters are very severe, corresponding to around -30% annual consumption growth. This has never been observed in US (although has been observed internationally). Tsai (2013) requires the probability of the four standard deviations consumption growth disaster to be 1.8% percent but in his model the consumption disasters can be significantly larger than in Gabaix (2013). [18] Thus, as already discussed, the model is not a rare disasters-type model.

Table 4 about here.

Table 5 demonstrates the model's ability to match the key asset pricing characteristics. In particular, the model adresses a common critique towards the habit model regarding its inability to simultaneously explain violations of the expectation hypothesis and uncovered interest parity (Verdelhan, 2010; Bansal and Shaliastovich, 2013).

Table 5 about here.

Consistently with the data, the expectation hypothesis violations in the model are stronger at the longer horizons (β-coefficients in Table 5 decreasing over time). This is just the consequence of the deep model parameters. In the calibration, the sensitivity of the yield curve's slope to the consumption volatility is mainly determined by the habit and volatility persistence components, ρ_s and ρ_n. At the same time, the sensitivity of the long-term bonds holding returns to the consumption volatility is mainly determined by the habit sensitivity parameters, σ_{sp} and σ_{sn}. In order for β-coefficients in Table 5 to decrease over time, the parameters should be chosen so that holding returns on long-term bonds are more sensitive to the consumption volatility than the slope of the yield curve. This is doable because the slope and the holding returns are largely affected by different model parameters.

The expectation hypothesis coefficients in the model are not decreasing as fast over time as they do in the data. This might be partially attributed to the fact that the model is the real model: non-neutral inflation would apmlify the effects. It should also be pointed out

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[18]In Nakamura et.al. (2013) the probability of the four standard deviation disaster is 1.12%, which is in line with the twentieth century US data, but authors are not concerned about predictability patterns, they only match basic asset pricing moments.

that in the long-run risk models such as Hasseltoft (2012) and Bansal and Shaliastovich (2013) the time pattern in the expectation hypothesis coefficients is also somewhat weaker than in the data.[19]

Note that the volatility of the real interest rate in the model is realistically low. As Cambpell and Cochrane (1999) point out, habit models usually generate too high interest rate volatility. In this paper, this is not a problem because the intertemporal smoothing and precautionary savings effects always drive the risk-free rate in opposite directions reducing its fluctuations. For instance, if a large negative shock does realize, the consumption-to-habit ratio goes down increasing the interest rate and the consumption growth volatility goes up decreasing the interest rate.

The equity premium in the model is a little bit low compared to the data. At the same time, the equity premium in the model is roughly consistent with the US equity premium over the longer time period: for instance, for the period from 1889 to 1994 Campbell et.al. (1997) in their Table 8.1 report the equity premium of 4.18%. The relatively low equity premium is a general problem of the ratio-habit model (see, e.g., Chan and Kogan, 2002). In the next section I show that using the difference habit instead of ratio habit addresses the low equity premium in my model.

An obvious problem with the model is that the exchange rate volatility is too high. This corresponds to the observation by Brandt et.al. (2006) that it is difficult to reconcile high domestic Sharpe-ratios with the low exchange rate volatility. The reason is that to fit the high domestic Sharpe-ratios the variance of the stochastic discount factors should be high (Mehra and Prescott, 1985). However, the high volatility of the stochastic discount factor implies high volatility of the exchange rates: $Var\Delta e_{t+1} = Var(m_{t+1}) + Var(m_{t+1}^*) - 2\rho(m_{t+1}, m_{t+1}^*)\sqrt{Var(m_{t+1})Var(m_{t+1}^*)}$. In the habit framework, Verdelhan (2010) solves this problem by introducing between the countries trading costs. This mechanism can also be applied to this paper at the cost of losing the intuition provided by closed form solutions.

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[19]Hasseltoft (2012) uses a slightly different method to test the expectation hypothesis and the time pattern in the model is slightly weaker than in data. In Bansal and Shaliastovich (2013) the appropriate comparision to this paper is the cross-country calibration in Table 7.

The model is to some extent able to match the non-perfect correlation between the exchange rate changes and consumption growth differentials. This weak correlation first documented in Backus and Smith (1993) is theoretically often considered puzzling. Indeed, Backus and Smith (1993) show that under the no-arbitrage condition in complete markets with the CRRA utility, the exchange rate changes should be proportional to the consumption growth differentials between two countries: $\Delta e_{t+1} = -\gamma(g^*_{t+1} - g_{t+1})$. Thus, the exchange rate changes and the consumption growth differentials should be perfectly correlated.

The habit model is usually able to match the low correlation between the exchange rate changes and the consumptiong growth differentials better than CRRA-based models, because the exchange rates are determined not only by the consumption growth but also by the habit. However, as Verdelhan (2010) points out the correlation in habit models is generally still too strong (around 0.75 in his model). In this paper the correlation between the exchange rate changes and the consumption growth differentials is weaker than in the previous habit models. This is because the shocks to the consumption growth and the inverse surplus ratio are imperfectly correlated: $\omega_{n,t+1}$ and $\omega_{p,t+1}$ affect the consumption and habit with different weights.

Table 6 confirms the theoretically predicted ability of the model to produce the downward sloping yield curve along with "non-violated" expectation hypothesis. Note that the main difference between the parameters in Tables 3 and 6 is the higher risk-aversion and volatility in the later specification. This makes sense, because, as discussed in the previous section, in order for the yield curve to slope downward and the expectation hypothesis to hold the precautionary savings effect should dominate. The result is also indirectly supported empirically as historically the consumption growth volatility in UK has been higher than in US.

Table 6 about here.

5.4   Role of Model Ingredients

In this section, I analyze two of the model's features which are different from the mainstream literature: using gamma shocks instead of Gaussian shocks and using the ratio

habit instead of the difference habit.

5.4.1 Gamma Shocks

The role of gamma shocks in this paper is largely quantitative, not qualitative. Indeed, the logic of the model is easily replicable with Gaussian shocks with time-varying volatility.

However, calibrating the model with Gaussian shocks is much more challenging. In particular, because gamma shocks have fatter tails than Gaussian shocks, volatility of Gaussian shocks should be relatively high to generate the same amount of predictability in asset prices. As it can be seen from Figure 1 and Table 4, the consumption growth in the model is less volatile than US consumption growth in 1929-2012. With Gaussian consumption shocks, the unconditional volatility of the consumption growth would be somewhat higher than in 1929-2012 US in order to generate roughly the same amount of predictability as reported in Tables 5 and 6. Additionally, the mean effective coefficient of risk-aversion would be around 20. It is important to emphasize that gamma shocks are not just a convenient theoretical tool: they are empirically well justified. For instance, Bekaert and Engstrom (2009) show that gamma shocks match the consumption growth dynamics (e.g., conditional consumption growth distributions) significantly better than Gaussian shocks.

5.4.2 The Ratio Habit versus The Difference Habit

I use the ratio habit utility instead of a more popular difference habit utility as it allows to concentrate on the core mechanism of the model and is economically better motivated. I show that all the results of the model can be replicated and even improved using the difference habit utility.

Again there is a representative agent. The agent maximizes the expected utility as in Campbell and Cochrane (1999):

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1 - \gamma}, \tag{25}$$

where C_t is the consumption at time t and H_t is an exogenously modeled level of habit

(habitual consumption), satisfying $H_t < C_t$. For computational convenience, instead of modeling H_t, I follow most of the external habit literature starting from Cambpell and Cochrane (1999) and model the inverse surplus ratio $Q_t = \frac{C_t}{C_t - H_t}$. The local coefficient of relative risk-aversion, which usually roughly corresponds to the CRRA risk-aversion coefficient, is time-varying and equal to γQ_t. The exact risk-aversion over wealth is a complicated function of the agent's value function and is not analyzed here. In line with the earlier work on the external habit formation, the logarithm of the inverse surplus ratio also follows a lag 1 autoregressive process:

$$q_{t+1} = \bar{q} + \rho_q(q_t - \bar{q}) + \sigma_{qp}\omega_{p,t+1} + \sigma_{qn}\omega_{n,t+1}. \tag{26}$$

The rest of the model is the same as in the case of the ratio habit utility:

$$
\begin{aligned}
g_{t+1} &= \bar{g} + \sigma_{cp}\omega_{p,t+1} - \sigma_{cn}\omega_{n,t+1}, \\
\omega_{p,t+1} &\sim \Gamma(\bar{p}, 1) - \bar{p}, \\
\omega_{n,t+1} &\sim \Gamma(n_t, 1) - n_t, \\
n_{t+1} &= \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1}.
\end{aligned}
\tag{27}
$$

Again ω_{p,t_1} and $\omega_{n,t+1}$ shocks affect the consumption growth and the inverse-surplus ratio with different weights. Again this is only important to generate a non-perfect correlation between the consumption growth and asset prices (exchange rates): all other moments can be calibrated to realistic values with the consumption and the inverse surplus ratio shocks being perfectly correlated.

This version of the model is solved exactly in the same way as the ratio habit version. All solutions are closed form and the intuitions are similar to the intuitions for the ratio habit model.

Table 7 shows that the difference habit version of the model is able to match many asset pricing moments, in particular, equity premium, better than the ratio habit model version. This is largely due to the time-varying risk-aversion, which is the key distinction between the ratio habit and the difference habit models.

Figure 2 shows that the unconditional consumption growth distribution in the difference habit model is much less volatile than in 1929-2012 US and is in fact very similar to the postwar US consumption growth distribution. However, the average local coefficient of

risk-aversion in the difference habit version is somewhat higher than in the ratio habit version. At the same time, the average local coefficient of risk-aversion in the difference habit version is significantly lower than in the earlier habit models of Wachter (2002) and Verdelhan (2010). This is because gamma distributions have fatter tails and thus the agent is more sensitve to the fluctuations in fundamentals than in the case of Gaussian distributions used by these papers.

Table 7 about here.

Figure 2 about here.

5.5   Comparision to Other Models

Table 8 shows that the model in this paper outperforms major existing models along several dimensions. First, only the model in this paper and the rare disaster model are using an empirically realistic risk aversion coefficient (based on the literature review in Mehra and Prescott, 1985). However, the rare disasters model assumes an abnormally high (compared to US data) probability of very low consumption growth outcomes. The model in this paper does not need this assumption.

Second, the model in this paper is the only model which is able to simultaneously replicate three major bond market puzzles: an upward-sloping real yield curve, violations of the expectation hypothesis, and violations of the uncovered interest rate parity. The long-run risk model, the rare-disaster model, and the habit model by Verdelhan (2010) do not replicate an upward-sloping real yield curve. Additionally, in the model by Verdelhan (2010), the expectation hypothesis holds. The habit model by Wachter (2002), although is able to replicate an upward sloping real yield curve and violations of the expectation hypothesis, is not generating violations of the uncovered interest rate parity.

Table 8 about here.

6   Conclusion

This paper shows that an external habit model augmented with a heteroskedastic consumption process goes a long way in explaining many domestic and international bond market puzzles. The key innovation is disentangling the intertemporal smoothing and precautionary savings effects from each other: the time-varying combination of these two effects produces a rich and economically intuitive term structure. The model adresses the popular critique towards the habit model regarding its inability to simultaneously match violations of expectation hypothesis and uncovered interest rate parity (Verdelhan, 2010; Bansal and Shaliastovich, 2013). This is because, unlike in a standard habit model, in this paper there are simultaneously intertemporal smoothing (through the variations in the consumption-habit ratio) to match domestic term structure and precautionary savings (through the time-varying consumption volatility) to reproduce international predictability. The model also has some advantages over other popular term structure models such as long-run risk and rare-disaster models. For instance, the model is able to produce both downward and upward sloping real yield curves and does not rely on extreme negative consumption growth outcomes.

It should be emphasized that the role of the habit in the model is very different than in the mainstream habit literature (Campbell and Cochrane, 1999; Wachter, 2006; Verdelhan, 2010, among others) where it drives the price of risk. In my model, the price of risk is constant and habit is simply a level factor which makes the agent sensitive to relatively mild fluctuations in fundamentals and thus makes the levels of standard asset pricing moments (for instance, equity premium) realistic: qualitatively all the logic of the model comes from the time-varying volatility and goes through with the CRRA utility.

Results have methodological implications for the habit literature. First, due to its ability to generate a richer term structure, a habit model with a time-varying amount of risk and a fixed price of risk might be a better choice than a more standard habit model with a fixed amount of risk and a time-varying price of risk. Second, I show that a habit model where positive and negative shocks are allowed to affect habit differently is more consistent with US data than a traditional habit model where the impact of both shocks is the same. Interestingly, in data the intertemporal smoothing seems to be driven exclusively by negative consumption shocks.

Investigating model's predictions empirically is a fruitful area for future research. For example, jointly estimating model's parameters and filtering the fundamental shocks from consumption and asset pricing data to evaluate the model's fit would be interesting. Of course, this is challenging as the shocks are gamma distributed.

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Appendix

The appendix contains the solutions for the consumption and asset pricing moments in the paper.

Properties of gamma distributions

Suppose $X \sim \Gamma(a, 1)$, where a is the shape parameter. Then all consumption and asset pricing formulas below can be obtained using these formulas:

$$E(X) = a,$$

$$Var(X) = a,$$

$$Skw(X) = \frac{2}{\sqrt{a}},$$

$$\text{Excess kurtosis(X)} = \frac{6}{a},$$

$$E(e^{bX}) = e^{-a\ln(1-b)}.$$

Consumption moments

$$Eg_{t+1} = \bar{g},$$

$$Var(g_{t+1}) = \sigma_{cp}^2 \bar{p} + \sigma_{cn}^2 \bar{n},$$

$$Skw(g_{t+1}) = \frac{2(\sigma_{cp}^3 \bar{p} - \sigma_{cn}^3 \bar{n})}{Var(g_{t+1})^{\frac{3}{2}}},$$

$$Kur(g_{t+1}) = \frac{6(\sigma_{cp}^4 \bar{p} + \sigma_{cn}^4 \bar{n})}{Var(g_{t+1})^2} - 3.$$

Domestic bond markets

Second moments of the state variables:

$$Var(n_t) = \frac{\sigma_{nn}^2 \bar{n}}{1 - \rho_n^2},$$

$$Var(s_t) = \frac{\sigma_{sp}^2 \bar{p} + \sigma_{sn}^2 \bar{n}}{1 - \rho_s^2},$$

$$Cov(s_t, n_t) = \frac{\sigma_{sn}\sigma_{nn}\bar{n}}{1 - \rho_s \rho_n}.$$

Price of a 0-coupon n-period bond at time t:

$$P_{n,t} = e^{C_n + S_n s_t + N_n n_t},$$

$$C_n = \bar{G}_n + \bar{S}_n \bar{s} + \bar{P}_n \bar{p} + \bar{N}_n \bar{n},$$

$$\bar{G}_1 = \ln \beta - \gamma \bar{g},$$

$$\bar{S}_1 = -(\gamma - 1)(1 - \rho_s),$$

$$\bar{P}_1 = -f(a_p),$$

$$\bar{N}_1 = 0,$$

$$S_1 = (\gamma - 1)(1 - \rho_s),$$

$$N_1 = -f(a_n),$$

$$\bar{G}_n = \ln \beta - \gamma \bar{g} + \bar{G}_{n-1},$$

$$\bar{S}_n = -(\gamma - 1)(1 - \rho_s) + \bar{S}_{n-1} + S_{n-1}(1 - \rho_s),$$

$$\bar{P}_n = \bar{P}_{n-1} - f(a_p + S_{n-1}\sigma_{sp}),$$

$$\bar{N}_n = \bar{N}_{n-1} + N_{n-1}(1 - \rho_n),$$

$$S_n = (\gamma - 1)(1 - \rho_s) + \rho_s S_{n-1},$$

$$N_n = \rho_n N_{n-1} - f(a_n + S_{n-1}\sigma_{sn} + N_{n-1}\sigma_{nn}).$$

Variance of the one period risk-free rate:

$$Var(y_1) = (1 - \gamma)^2 (1 - \rho_s)^2 Var(s_t) + f(a_n)^2 Var(n_t) + 2(1 - \gamma)(1 - \rho_s)f(a_n)Cov(s_t, n_t).$$

Expectations hypothesis coefficient from the regression $y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t$:

$$\beta_n = \frac{Cov(E_t(y_{n-1,t+1} - y_{n,t}), y_{n,t} - y_{1,t})}{Var(y_{n,t} - y_{1,t})}$$

$$= 1 - \frac{(\rho_n N_{n-1} - N_n + N_1)(-\frac{1}{n}N_n + N_1)Var(n_t) + (\rho_n N_{n-1} - N_n + N_1)(-\frac{1}{n}S_n + S_1)Cov(s_t, n_t)}{(-\frac{1}{n}S_n + S_1)^2 Var(s_t) + (-\frac{1}{n}N_n + N_1)^2 Var(n_t) + 2(-\frac{1}{n}S_n + S_1)(-\frac{1}{n}N_n + N_1)Cov(s_t, n_t)}.$$

International markets

Two countries are assumed to have the same parameters. The shocks are assumed to be uncorrelated across two countries.

The uncovered interest rate coefficient from the regression $r_{t+1}^{FX} = \alpha_0 + \alpha_{FX}(y_{1,t} - y_{1,t}^*) + \epsilon_t$:

$$\alpha_{UIP} = \frac{Cov(-y_{1,t} + y_{1,t}^* + E_t(m_{t+1}^* - m_{t+1}), y_{1,t} - y_{1,t}^*)}{Var(y_{1,t} - y_{1,t}^*)}$$

$$= \frac{-N_1^2 Var(n_t) - S_1 N_1 Cov(s_t, n_t)}{S_1^2 Var(s_t) + (N_1^2)Var(n_t) + 2S_1 N_1 Cov(s_t, n_t)}.$$

Variance of the changes in the real exchange rates:

$$Var(\Delta e_{t+1}) = Var(m_{t+1}^* - m_{t+1}) = 2((1 - \gamma)^2 (1 - \rho_s)^2 Var s_t + a_p^2 \bar{p} + a_n^2 \bar{n}).$$

Correlation between the exchange rate changes and the consumption growth differentials:

$$Corr(\Delta e_{t+1}, g_{t+1}^* - g_{t+1}) = Corr(m_{t+1}^* - m_{t+1}, g_{t+1}^* - g_{t+1}) = \frac{2(\sigma_{cp}a_p\bar{p} - \sigma_{cn}a_n\bar{n})}{\sqrt{2Var\Delta e_{t+1}Var(g_{t+1})}}.$$

Equity

Assuming, following Cambpell and Cochrane (1999) and Bekaert and Engstrom (2009), that dividends equal consumption, the price-dividend ratio is:

$$\frac{P_t}{D_t} = \sum_{i=1}^{\infty} e^{C_i^e + S_i^e s_t + N_i^e n_t},$$

$$C_i^e = G_i^e + \bar{S}_i^e \bar{s} + \bar{P}_i^e \bar{p} + \bar{N}_1^e \bar{n},$$

$$G_1^e = \ln\beta + (1-\gamma)\bar{g},$$

$$\bar{S}_1^e = -(\gamma-1)(1-\rho_s),$$

$$\bar{N}_1^e = 0,$$

$$\bar{P}_1^e = -f(a_p + \sigma_{cp}),$$

$$S_1^e = (\gamma-1)(1-\rho_s),$$

$$N_1^e = -f(a_n - \sigma_{cn}),$$

$$G_n^e = G_{n-1}^e + \ln\beta + (1-\gamma)\bar{g},$$

$$\bar{S}_n^e = \bar{S}_{n-1}^e + S_{n-1}^e(1-\rho_s) - (\gamma-1)(1-\rho_s),$$

$$\bar{P}_n^e = \bar{P}_{n-1}^e - f(a_p + \sigma_{cp} + S_{n-1}^e \sigma_{sp}),$$

$$\bar{N}_n^e = \bar{N}_{n-1}^e + N_{n-1}^e(1-\rho_n),$$

$$S_n^e = (\gamma-1)(1-\rho_s) + \rho_s S_{n-1},$$

$$N_n^e = N_{n-1}^e \rho_n - f(a_n - \sigma_{cn} + S_{n-1}^e \sigma_{sn} + N_{n-1}^e \sigma_{nn}).$$

Note that the expression above is non-linear in the state variables n_t and q_t, which is not very convenient to compute the expected returns. The first order Taylor approximation of the logarithm of the price dividend ratio around the steady state is:

$$pd_t \approx \ln\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}} + \frac{\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}} S_i^e}{\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}}}(s_t - \bar{s}) + \frac{\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}} N_i^e}{\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}}}(n_t - \bar{n})$$

$$\equiv K_1 + K_1^s s_t + K_1^n n_t.$$

Similarly:

$$\ln(1 + \frac{P_t}{D_t}) \approx \ln(1 + \sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}}) + \frac{\sum_{i=1}^{\infty} e^{C_i^e + s_i^e \bar{s} + N_1^e \bar{n}} S_i^e}{1 + \sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}}}(s_t - \bar{s}) + \frac{\sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}} N_i^e}{1 + \sum_{i=1}^{\infty} e^{C_i^e + S_i^e \bar{s} + N_1^e \bar{n}}}(n_t - \bar{n})$$

$$\equiv K_2 + K_2^s s_t + K_2^n n_t.$$

Using the linear approximations above, the equity premium is:

$$
\begin{aligned}
E_t(r_{mkt,t+1} - y_1, t) = & E_t(\ln(1 + \frac{P_{t+1}}{D_{t+1}}) - \ln \frac{P_t}{D_t} + g_{t+1} - y_{1,t}) \\
\approx & K_2 - K_1 + K_2^s(1 - \rho_s)\bar{s} + K_2^n(1 - \rho_n)\bar{n} + \ln \beta - (\gamma - 1)(1 - \rho_s)\bar{s} - f(a_p)\bar{p} + \\
& (K_2^s \rho_s - K_1^s + (\gamma - 1)(1 - \rho_s))s_t + (K_2^n \rho_n - K_1^n - f(a_n))n_t.
\end{aligned}
$$

Figure 1: Unconditional Distribution of Consumption Growth in US and in the Model. The data are annual. The US data is the logarithmized per capita total consumption growth. The time period is 1929-2012. The model implied distribution is obtained by sampling 100,000 annual observations from the model.
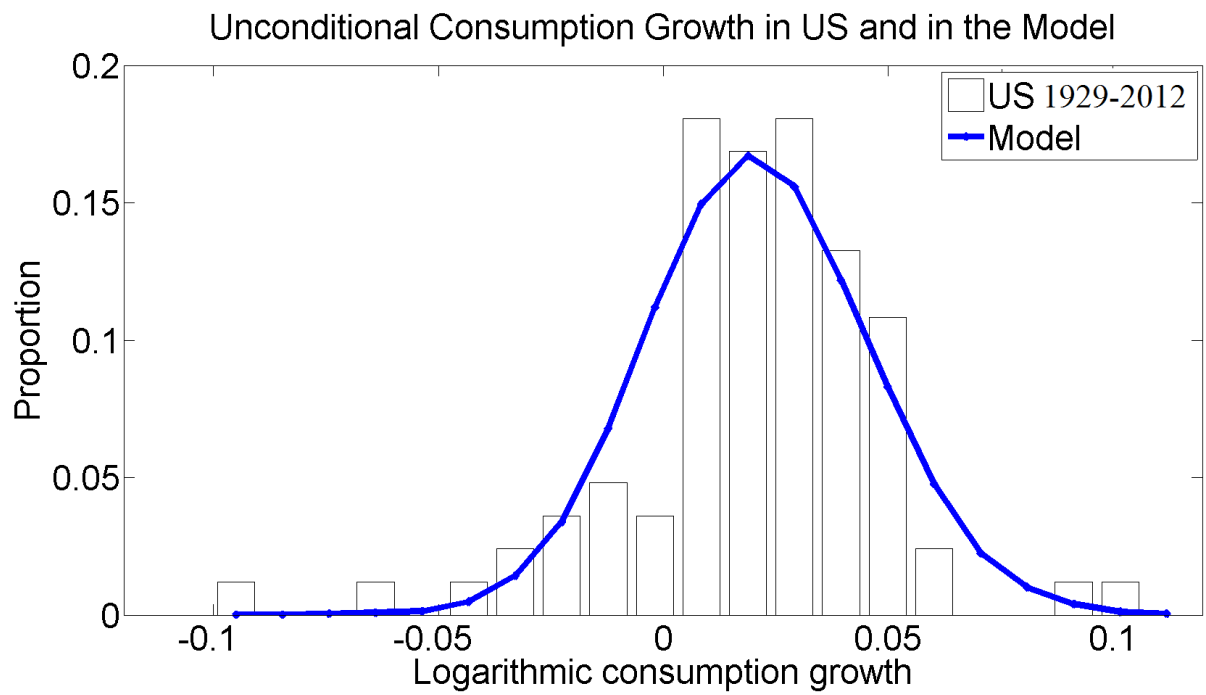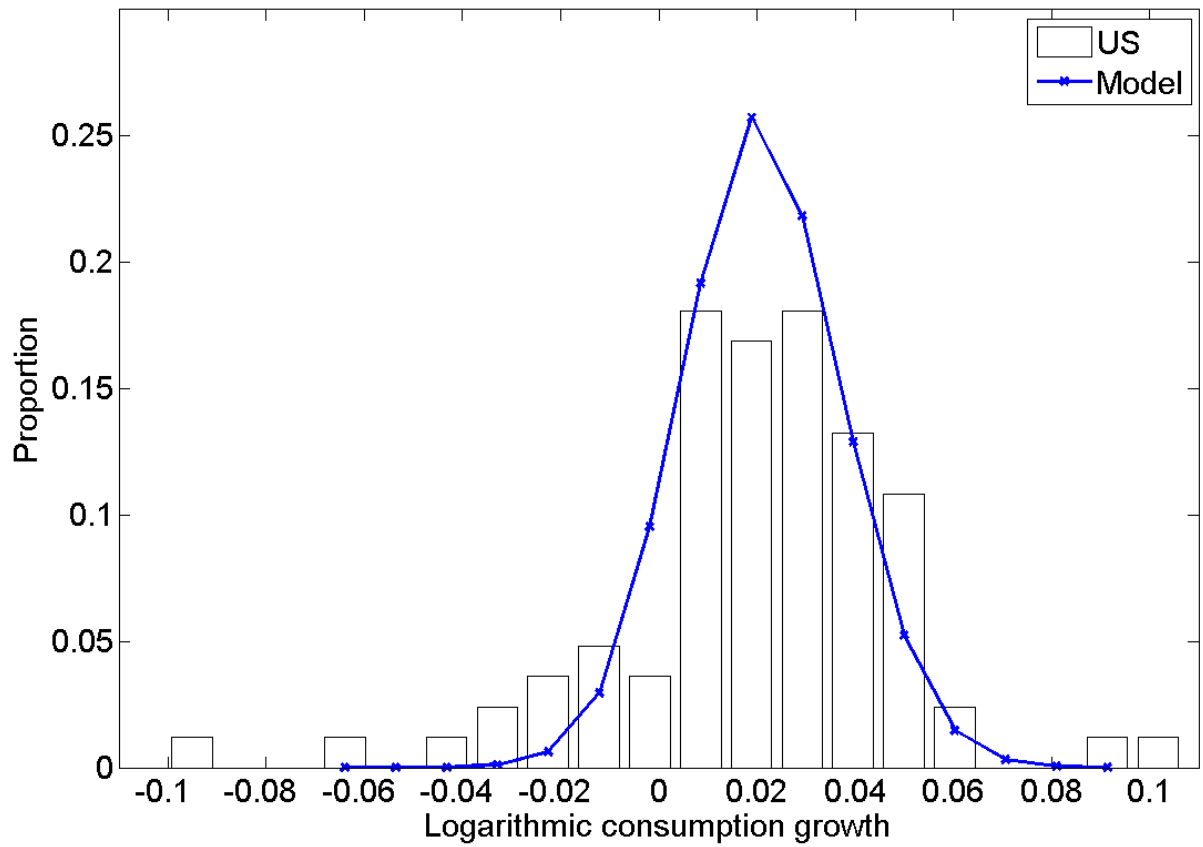
Figure 2: The Difference Habit: Unconditional Distribution of Consumption Growth in US and in the Model. The data are annual. The US data is the logarithmized per capita total consumption growth. The time period is 1929-2012. The model implied distribution is obtained by sampling 100,000 annual observations from the model.

Table 1: Determinants of the Risk-free Rate. Data are quarterly US observations from 1969Q1 to 2012Q4. The main regression is $r^{\$}_{t,t+1} - E_t\pi_{t+1} = \alpha_0 + \alpha_1 \cdot$ consumption-to-habit $+ \alpha_2 \cdot$ consumption-to-habit$^- + \alpha_3 \cdot$ conditional volatility $+ \epsilon_{t+1}$. $r^{\$}_{t,t+1}$ is a nominal 90 days Treasury bill interest rate. $E_t\pi_{t+1}$ is the expectation of the inflation for the next 3 months from the Survey of Professional Forecasters. *Consumption-habit* is a proxy for the total log consumption-to-habit ratio over past 40 quarters assuming shocks to habit and consumption growth are the same: $\sum_{i=1}^{40} \rho_s^i(g_{t-i} - \bar{g})$. *Consumption-habit$^-$* is a proxy for the log consumption-to-habit ratio computed using only negative consumption shocks: $\sum_{i=1}^{40} \rho_s^i(g_{t-i} - \bar{g})\mathbb{1}_{(g_{t-i}-\bar{g})<0}$. g_t is the time t consumption growth and $\bar{g}$ is the mean consumption growth. $\rho_s = 0.97$, which corresponds to the average quarterly persistence of habit in the literature. *Conditional volatility* is the conditional volatility of the consumption growth for the next quarter. In Panel A, the conditional volatility is computed assuming that quarterly consumption growth follows a constant mean heteroskedastic process where the error term is of the GARCH(1,1) structure. In Panel B, the conditional volatility is computed assuming that quarterly consumption growth follows a constant mean heteroskedastic process where the error term is a mixture of two gamma shocks. Standard errors in brackets are 60 lags Newey-West standard errors.

	Specification 1	Specification 2	Specification 3	Specification 4
Panel A: Gaussian shocks				
Constant	0.0421***	-0.0084	-0.0001	0.0000
	(0.0091)	(0.0146)	(0.0154)	(0.0184)
Consumption − habit	0.1720	0.5715***	0.2270	0.8010***
	(0.1674)	(0.2012)	(0.1812)	(0.2414)
Consumption − habit$^-$		-0.8452***		-1.4544***
		(0.2736)		(0.2411)
Conditional volatility			6.2180**	-6.6084*
			(2.6823)	(3.9976)
R^2	0.0673	0.3057	0.2098	0.3341
Panel B: gamma-shocks				
Constant	0.0440***	-0.0055	0.0218	-0.0037
	(0.0077)	(0.0143)	(0.0084)	(0.0165)
Consumption − habit	0.1702	0.5857***	0.2089	0.5994***
	(0.1693)	(0.2075)	(0.1654)	(0.2102)
Consumption − habit$^-$		-0.8302***		-0.8997***
		(0.2712)		(0.2200)
Conditional volatility			3.1454*	-0.8244
			(1.6165)	(1.4297)
R^2	0.0653	0.2964	0.1997	0.3090

Table 2: Aggregate Per Capita Consumption Growth Volatilities for G10 Carry Trade Countries. Data is annual. Portfolio consumption growths are GDP weighted consumption growths of portfolio countries. In Panel A, Volatilities are computed from the time series of consumption growths. Bootstrap standard errors are in brackets. In Panel B, in each bootstrap run a time series of the historical length is sampled for each portfolio, preserving the time relationship between the time series (that is if the observation for 1996 is sampled for the high-interest rate portfolio it is also sampled for the mid- and low-interest rate portfolios). Then, volatilities of these time series are computed. p-value is the proportion of bootstrap runs where the volatility of a row time series is larger than the volatility of a column time series. 10,000 bootstrap runs are performed for both Panels A and B. The asterisks, *, **, and ***, correspond to the statistical significance at 10, 5, and 1 percent levels, respectively.

Panel A: Historical consumption growth volatilities		
Low-interest rate countries	Mid-interest rate countries	High-interest rate countries
Japan, Switzerland	Canada, Germany, Sweden, UK, US	Australia, New Zealand, Norway
Whole time period: 1971-2012		
1.94%	1.42%	1.18%
(0.22%)	(0.14%)	(0.13%)
Modern time period: 1988-2012		
1.42%	1.24%	1.21%
(0.22%)	(0.21%)	(0.22%)
Panel B: Bootstrap p-values for the differences in historical consumption growth volatilities		
Whole time period: 1971-2012		
	Low-interest rate countries	Mid-interest rate countries	High-interest rate countries
Low-interest rate countries	-	0.9867**	0.9976***
Mid-interest rate countries	0.0133**	-	0.8881
High-interest rate countries	0.0024***	0.1119	-
Modern time period: 1988-2012		
	Low-interest rate countries	Mid-interest rate countries	High-interest rate countries
Low-interest rate countries	-	0.8217	0.7960
Mid-interest rate countries	0.1783	-	0.5387
High-interest rate countries	0.2040	0.4613	-

Table 3: Parameters of the Model. Preferences and consumption dynamics parameters for the baseline specification of the model. The parameters are calibrated to match the basic consumption and asset pricing dynamics. The parameters are for the annual calibration frequency. The implied local relative risk-aversion, defined as γe^{q_t} approximately corresponds to the CRRA-utility risk-aversion. The distribution of the implied local relative risk-aversion is computed by sampling 100,000 annual observations from the model.

Parameter	Interpretation	Value
Preferences		
β	discount factor	0.9750
γ	risk-aversion	6.6865
$\bar{s}$	average consumption-habit ratio	1.0000
ρ_s	persistence of the consumption-habit ratio	0.7860
σ_{sp}	sensitivity of the consumption-habit ratio to positive shocks	0.0001
σ_{sn}	sensitivity of the consumption-habit ratio to negative shocks	-0.1604
Consumption dynamics		
$\bar{g}$	average consumption growth	0.0210
$\bar{p}$	shape parameter of positive shocks	202.2554
σ_{cp}	impact of the positive shocks on the consumption growth	0.0017
$\bar{n}$	average shape parameter of negative shocks	0.0185
σ_{cn}	impact of the negative consumption shocks on the consumption growth	0.0519
ρ_n	volatility persistence	0.8758
σ_{nn}	scale of the volatility shock	0.0770

Table 4: Per Capita Consumption Growth Dynamics in The Model and Data. The US data is the logarithmized annual per capita total consumption growth. The time period is 1929-2012. The model statistics for disaster probabilities is computed by sampling 100,000 annual observations from the model. $\bar{g}$ and σ_g are unconditional mean and standard deviation of the consumption growth, respectively.

	Mean	Standard deviation	Skewness	Excess kurtosis	$P(< \bar{g} - 2\sigma_g)$	$P(< \bar{g} - 4\sigma_g)$
Model	2.10%	2.52%	-0.20	2.02	1.93%	0.11%
US	2.00%	2.98%	-0.83	3.52	4.96%	1.20%

Table 5: Asset Pricing Moments. All moments are annual moments. For the international statistics, it is assumed that two countries have exactly the same model parameters as reported in Table 3, and shocks are independent across two countries. All model generated moments are in real terms. All data moments are in real terms except the expectation hypothesis coefficients and the uncovered interest rate parity coefficients. Real yields data are US 2004-2012 data. Expectation hypothesis data are US 1961-2012 data. Equity data are US 1926-2012 data. The uncovered interest rate parity coefficient is the average UIP coefficient for the US dollar and most common foreign currencies from Backus et.al. (2001). The correlation between the consumption growth rates differentials and the real exchange rate changes are from Benigno and Thoenissen (2008). The volatility of the real exchange rate changes is from Croce and Colacito (2011).

Moment	Description	Model	Data
	Real zero-coupon yields		
y_1	1 year	0.20%	0.23%
y_2	2 years	0.48%	0.37%
y_3	3 years	0.75%	0.49%
y_4	4 years	1.01%	0.63%
y_5	5 years	1.26%	0.78%
$\sigma(y_1)$	volatility of the 1 year yield	1.65%	1.58%
	Expectation hypothesis coefficients from the regression $y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t$		
β_2	n=2 years	-1.18	-0.71
β_3	n=3 years	-1.21	-1.04
β_4	n=4 years	-1.27	-1.29
β_5	n=5 years	-1.31	-1.48
	Equity		
$r_{mkt} - y_1$	equity premium	4.45%	5.67%
Sharpe-ratio		0.36	0.29
pd	logarithm of the price-dividend ratio	3.66	3.40
$Corr(pd_{t-1}, pd_t)$	autocorrelation of the price-dividend ratio	0.81	0.85
	International		
α_{FX}	from regression $r_{t+1}^{FX} = \alpha_0 + \alpha_{FX}(y_{1,t} - y_{1,t}^*) + \epsilon_t$	-1.92	[-0.74,-1.84]
$\sigma(\Delta e_{t+1})$	volatility of the real exchange rate	20.12%[20]	11.21%
$Corr(\Delta e_{t+1}, g_{t+1}^* - g_{t+1})$	correlation between the exchange rate changes and the consumption growth differentials	-0.49	[-0.55,0.53]

[20]20.12% is for the case of uncorrelated consumption growth shocks across two countries. For the case where the correlation between consumption growth shocks in two countries is 0.32, which corresponds to the correlation between US and UK consumption growths, the exchange rate volatility is 16.46%.

Table 6: Alternative Model Calibration: Downward Sloping Yield Curve And "Non-violated" Expectation Hypothesis. All moments are annual real moments. $\bar{g}$ and σ_g are unconditional mean and standard deviation of the consumption growth, respectively. y_n is the yield for a horizon n risk-free zero-coupon bond. β_n is the coefficient from the $y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t$ regression. $r_{mkt} - y_1$ is the equity risk-premium. pd is the natural logarithm of the price-dividend ratio.

PANEL A: PARAMETERES						
β	γ	$\bar{s}$	ρ_s	σ_{sp}	σ_{sn}	
0.9700	14.9134	1.0000	0.8464	0.0001	-0.0449	
Consumption dynamics						
$\bar{g}$	$\bar{p}$	σ_{cp}	$\bar{n}$	σ_{cn}	ρ_n	σ_{nn}
0.0210	119.5582	0.0023	0.1018	0.0420	0.6669	0.2755
PANEL B: CONSUMPTION AND ASSET PRICING MOMENTS						
Consumption growth						
Mean	Standard deviation	Skewness	Excess kurtosis	$P(< \bar{g} - 2\sigma_g)$	$P(< \bar{g} - 4\sigma_g)$	
2.10%	2.84%	2.97.	-0.53	2.26%	0.27%	
Real yields						
y_1	y_2	y_3	y_4	y_5	$\sigma(y_1)$	
0.68%	0.43%	0.26%	0.18%	0.17%	2.40%	
Expectation hypothesis coefficients						
β_2	β_3	β_4	β_5			
2.55	2.27	1.91	1.45			
Equity						
$r_{mkt} - y_1$	Sharpe-ratio	pd	$Corr(pd_{t-1}, pd_t)$			
4.08%	0.23	3.62	0.86			

Table 7: The Difference Habit Calibration. All moments are annual real moments. $\bar{g}$ and σ_g are unconditional mean and standard deviation of the consumption growth, respectively. y_n is the yield for a horizon n risk-free zero-coupon bond. β_n is the coefficient from the $y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t$ regression. α_{FX} is the coefficient from the $r_{t+1}^{FX} = \alpha_0 + \alpha_{FX}(y_{1,t} - y_{1,t}^*) + \epsilon_t$ regression. $r_{mkt} - y_1$ is the equity risk-premium. pd is the natural logarithm of the price-dividend ratio.

PANEL A: PARAMETERES						
β	γ	$\bar{q}$	ρ_q	σ_{qp}	σ_{qn}	
0.9964	3.6740	1.0000	0.8049	-0.0002	0.1956	
Consumption dynamics						
$\bar{g}$	$\bar{p}$	σ_{cp}	$\bar{n}$	σ_{cn}	ρ_n	σ_{nn}
0.0210	197.4329	0.0011	0.1095	0.0114	0.8581	0.1878
Implied local relative risk-aversion						
Mean	Median	90th-percentile	95th-percentile	99th-percentile		
10.0522	9.6007	10.9753	12.1335	16.1032		

PANEL B: CONSUMPTION AND ASSET PRICING MOMENTS					
Consumption growth					
Mean	Standard deviation	Skewness	Excess kurtosis	$P(< \bar{g} - 2\sigma_g)$	$P(< \bar{g} - 4\sigma_g)$
2.10%	1.62%	0.06	0.19	2.05%	0.02%
Real yields					
y_1	y_2	y_3	y_4	y_5	$\sigma(y_1)$
0.55%	0.78%	1.00%	1.21%	1.40%	1.41%
Expectation hypothesis coefficients					
β_2	β_3	β_4	β_5		
-0.87	-0.88	-0.90	-0.94		
Equity					
$r_{mkt} - y_1$	Sharpe-ratio	pd	$Corr(pd_{t-1}, pd_t)$		
5.53%	0.37	3.21	0.80		
International					
α_{FX}	$\sigma(\Delta e_{t+1})$	$Corr(\Delta e_{t+1}, g_{t+1}^* - g_{t+1})$			
-1.56	38.50%[21]	-0.46			

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[21]38.50% is for the case of uncorrelated consumption growth shocks across two countries. For the case where the correlation between consumption growth shocks in two countries is 0.32, which corresponds to the correlation between US and UK consumption growths, the exchange rate volatility is 31.75%.

Table 8: Comparision to Other Models. All values are at annual frequency unless mentioned otherwise. Samples, and thus calibration objectives, are slightly different across the studies. These differences do not affect qualitative implications of the models (e.g., does the expectation hold or not). μ is the average consumption growth. σ is the standard deviation of the consumption growth. y_5 and y_1 are 1 and 5 years yields, respectively. β_2 is the regression coefficient from the $y_{1,t+1} - y_{1,t} = \beta_0 + \beta_2(y_{2,t} - y_{1,t}) + \epsilon_t$ regression run with annual data. UIP coefficient is the α_{FX} coefficient from the regression $r_{t+1}^{FX} = \alpha_0 + \alpha_{FX}(y_{1,t} - y_{1,t}^*) + \epsilon_t$. A colored cell points out an inconsistency between the model and the data.

	This paper	Habit: Wachter (2002)	Habit: Verdelhan (2010)	Long-run risk: Bansal and Shaliastovich (2013)	Rare disasters: Gabaix, 2012	Data: US
Risk-aversion	6.69	≈ 30	≈ 30	20.90	4	< 10[1]
Disaster probability ($g_t < \mu - 4\sigma$)	0.11%	0.00%	0.00%	0.03%	3.63%	1.20%
Real yield curve slope, $\overline{y_5 - y_1}$	1.06%	0.89%	-0.46%	$<0.00\%$	0.00%	0.55%
Expectation hypothesis: β_2	-1.18	-0.97[2]	>0.00	-0.45[2,3]	-1.92[2,3]	-0.71[3]
UIP coefficient	-1.92	>0.00	-1.99	-2.06[3]	<-1.00	-2.29[4]

[1] - based on the literature review in Mehra and Prescott (1985)

[2] - the short-term bond is a 3 month treasury bill

[3] - nominal value

[4] - for the US-UK pair