Time-varying Risk of Nominal Bonds: How Important Are Macroeconomic Shocks?

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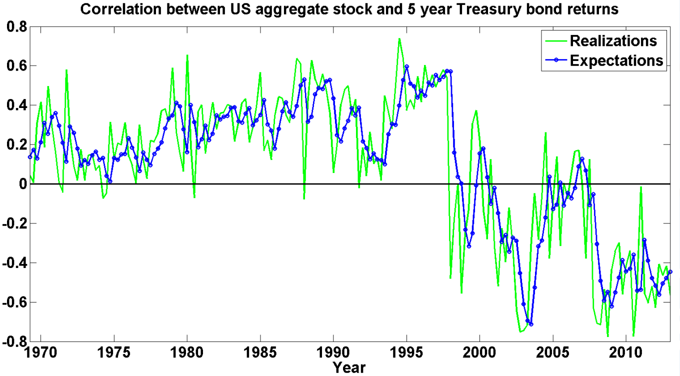
February 7, 2015

Motivation: Time-varying stock and bond return correlation



- Unconditional correlation is 0.02
- Computed quarterly from daily data
- Expectations=dynamic conditional correlation of Colacito et.al. (2009)

Stock and bond return correlation - important but difficult to explain moment

- Important:
  - First order effect on portfolio variance
  - Stocks and bonds large and closely integrated markets: should be modeled jointly
- Difficult to explain:
  - Theoretically: starting from Shiller and Beltratti (1992)
  - Empirically: e.g., even in dynamic factor models (Baele et.al., 2010)

Question

- Are macroeconomic shocks (consumption growth and inflation) related to time-varying stock and bond return correlation?
  - Can they generate correlations of observed magnitudes?
  - Historically, how much do they matter at different points in time?

Contribution: Methodology

- Tractable stuctural model for analyzing macroeconomic risk of nominal assets

	Campbell et.al., 2014	Burkhardt and Hasseltoft, 2012; Song, 2014	Me
Type	Habit	Long-run risk	Habit
Non-Gaussian macro dynamics	No	Yes	Yes
Exact closed form solutions	No	No	Yes
Realistic term structure	Yes	No	Yes
Macroeconomic shocks from consumption and inflation data	No	No	Yes
Do macroeconomic shocks matter for the risk of nominal assets?	Not much	A lot	Half of the sample

Contribution: Empirical results

- Economically intuitive characterization of macroeconomic shocks
- Implications for stock and bond return correlation:
  - macroeconomic shocks generate sizeable positive and negative correlations, although negative correlations smaller and less frequent than in data
  - historically, macroeconomic shocks are important in explaining high correlations from late 70's to early 90's and low correlations pre- and during the Great Recession

Overview of the model

- External habit utility:
  - realistic asset pricing moments: in particular, realistic term structure
- Macroeconomic dynamics from Bekaert, Engstrom, and Ermolov (2014c):
  - convenient for modeling time-varying bond risk: drives time-varying stock and bond return correlations

Consumption growth and inflation

- Consumption growth: $g_{t+1} = \bar{g} + \epsilon^g_{t+1}$
  - Constant mean $\bar{g}$
  - Heteroskedastic 0-mean shock ϵ^g_{t+1}
- Inflation: $\pi_{t+1} = \bar{\pi} + x^\pi_t + \epsilon^\pi_{t+1}$
  - Unconditional mean $\bar{\pi}$
  - Persistent 0-mean inflation expectations x^π_t
  - Heteroskedastic 0-mean shock ϵ^π_{t+1}

Macroeconomic shocks

$$\epsilon^g_{t+1} = \underbrace{\sigma^d_g}_{>0} u^d_{t+1} + \underbrace{\sigma^s_g}_{>0} u^s_{t+1},$$

$$\epsilon^\pi_{t+1} = \underbrace{\sigma^d_\pi}_{>0} u^d_{t+1} - \underbrace{\sigma^s_\pi}_{>0} u^s_{t+1},$$

$$Cov(u^d_{t+1}, u^s_{t+1}) = 0, Var(u^d_{t+1}) = Var(u^s_{t+1}) = 1.$$

- u^d_{t+1} - "demand shock": moves g_{t+1} and π_{t+1} in the same direction $\Rightarrow$ nominal bonds hedge well
- u^s_{t+1} - "supply shock": moves g_{t+1} and π_{t+1} in opposite directions $\Rightarrow$ nominal bonds hedge poorly

Macroeconomic environments

- If supply and demand shocks are heteroskedastic, $Cov_t(\epsilon^g_{t+1}, \epsilon^\pi_{t+1})$ will vary over time:

$$Cov_t(\epsilon^g_{t+1}, \epsilon^\pi_{t+1}) = \sigma^d_g \sigma^d_\pi Var_t(u^d_{t+1}) - \sigma^s_g \sigma^s_\pi Var_t(u^s_{t+1})$$

- Demand shock environment: $Cov_t(\epsilon^g_{t+1}, \epsilon^\pi_{t+1}) > 0 \Rightarrow$ stock and bond correlations relatively low

- Supply shock environment: $Cov_t(\epsilon^g_{t+1}, \epsilon^\pi_{t+1}) < 0 \Rightarrow$ stock and bond correlations relatively high

Modeling demand and supply shocks

- Demand and supply shocks modeled using Bad Environment-Good Environment (BEGE) structure (Bekaert and Engstrom, 2014): component models of two 0-mean shocks

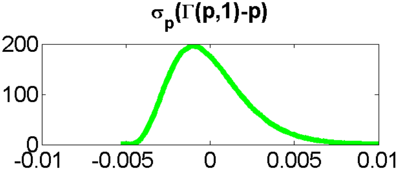
$$\left.\begin{aligned} u^d_{t+1} &= \sigma^d_p \omega^d_{p,t+1} - \sigma^d_n \omega^d_{n,t+1}, \\ u^s_{t+1} &= \sigma^s_p \omega^s_{p,t+1} - \sigma^s_n \omega^s_{n,t+1}, \end{aligned}\right\} \begin{aligned} &\omega_{p,t+1} \text{ - good shock} \\ &\omega_{n,t+1} \text{ - bad shock} \end{aligned}$$

- Shocks follow demeaned gamma distributions:

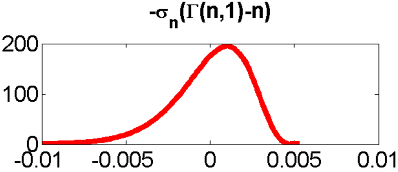
$$\left.\begin{aligned} \omega^d_{p,t+1} &\sim \Gamma(p^d_t, 1) - p^d_t, \\ \omega^d_{n,t+1} &\sim \Gamma(n^d_t, 1) - n^d_t, \\ \omega^s_{p,t+1} &\sim \Gamma(p^s_t, 1) - p^s_t, \\ \omega^s_{n,t+1} &\sim \Gamma(n^s_t, 1) - n^s_t. \end{aligned}\right\} \Gamma(x, y) - \text{gamma distribution with shape parameter x and scale parameter y}$$

Bad Environment-Good Environment structure: Probability density function

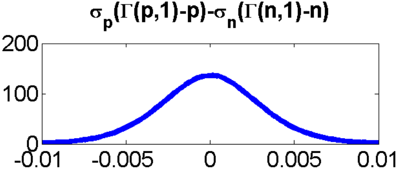
Good component pdf:



Bad component pdf:



Sum pdf:



Time-varying variances

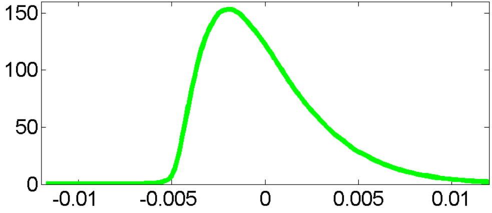
- p_t can be interpreted as good variance and n_t as bad variance
- Variances are persistent and driven by the realization shocks, capturing volatility clustering (Gourieroux and Jasiak, 2006):

$$p^d_{t+1} = \bar{p}^d + \rho^d_p(p^d_t - \bar{p}^d) + \sigma^d_{pp}\omega^d_{p,t+1},$$

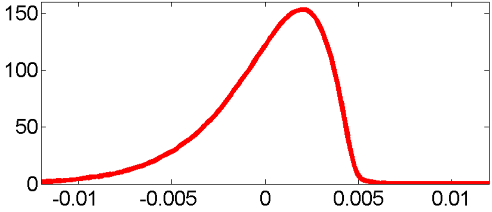
- Similar processes for n^d_{t+1}, p^s_{t+1}, n^s_{t+1}

Time-varying variances: Probability density functions

Large p_t
(Good environment):
pdf



Large n_t
(Bad environment):
pdf



Intuitive expressions for the moments

Model: Why gamma distributed shocks?

- Empirically supported to capture non-Gaussian features prevalent in consumption and inflation data (Bekaert and Engstrom, 2009; Bekaert, Engstrom, and Ermolov, 2014a,b)
- Non-Gaussian features facilitate theoretically matching risk-premia
- Intuitive closed form solutions
- Efficient estimation

Data

- US quarterly observations: 1969Q4-2012Q4
- Working (1960) adjusted consumption of non-durables and services
- Inflation: St.Louis Fed
- Inflation expectations: Survey of Professional Forecasters

Estimation

- Maximum likelihood estimation using only macroeconomic data (no financial data)
- Input: consumption growth and inflation time series
- Output 1: macroeconomic dynamics parameters estimates
- Output 2: expected p_t^d, n_t^d, p_t^s, n_t^s time series
- Methodology: sequentially computing likelihood over observations - in characteristic function domain formulas for computing likelihood available in closed form (Bates, 2006)

Detailed estimation overview Maximum likelihood estimation overview

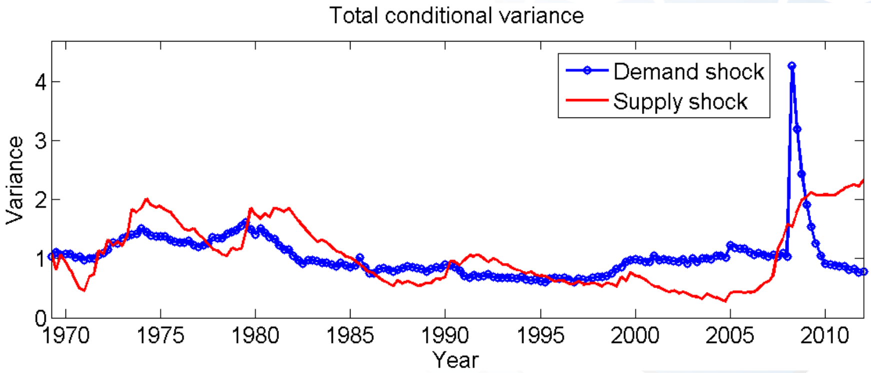
Consumption growth and inflation shocks

$$\epsilon^{g}_{t+1} = \underset{(0.0003)}{0.0015} u^{d}_{t+1} + \underset{(0.0003)}{0.0037} u^{s}_{t+1}$$

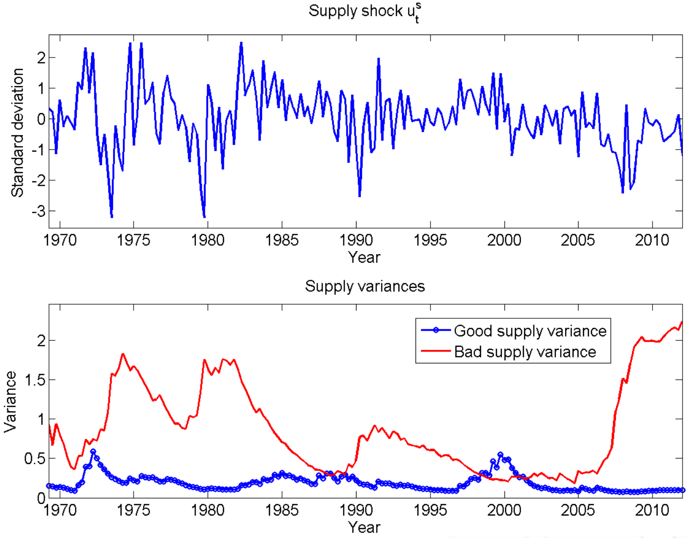
$$\epsilon^{\pi}_{t+1} = \underset{(0.0010)}{0.0055} u^{d}_{t+1} - \underset{(0.0006)}{0.0032} u^{s}_{t+1}$$

- Consumption growth shocks: supply driven
- Inflation shocks: demand driven

Demand and supply variances

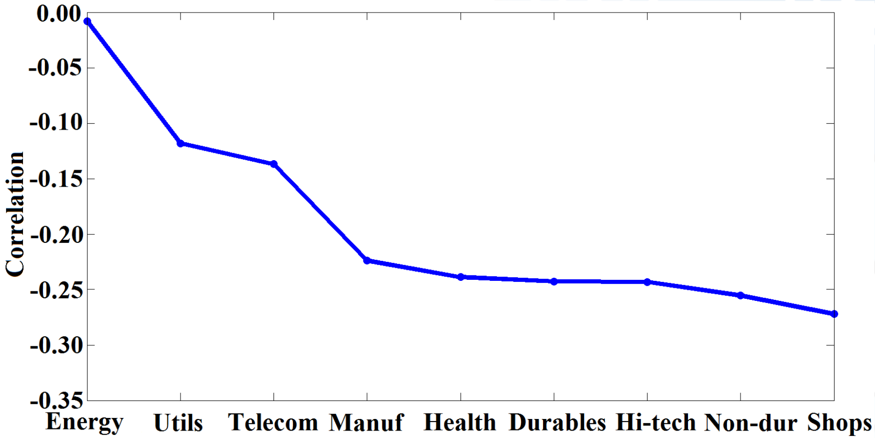


Supply shocks



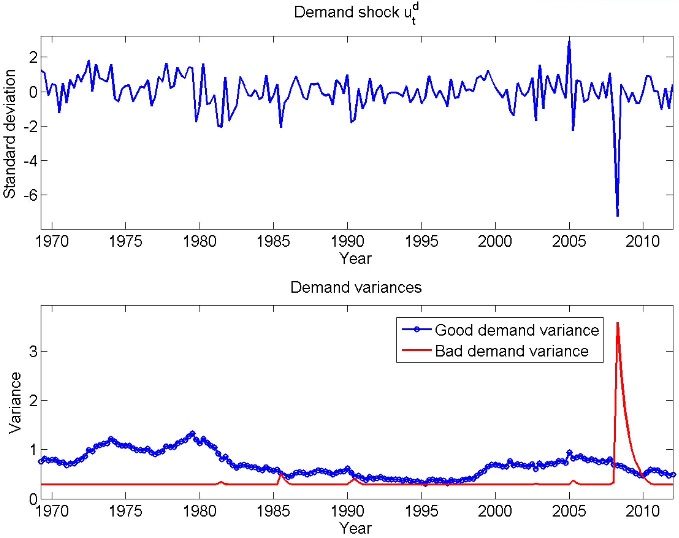
▸ Supply shock parameter estimates

Correlation between industry portfolio returns and bad supply shocks ($\omega^s_{n,t+1}$)



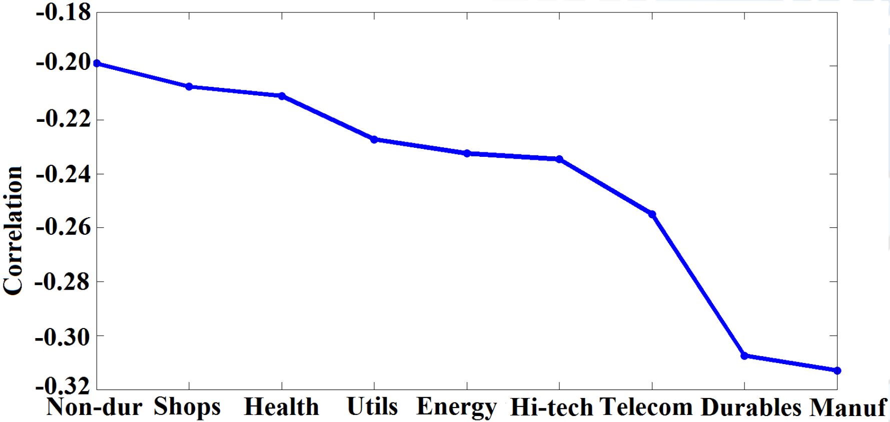
▸ More correlations

Demand shocks



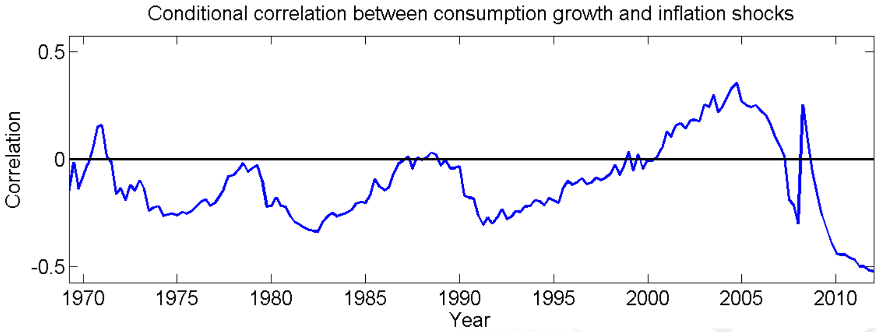
▸ Demand shock parameter estimates

Correlation between industry portfolio returns and bad demand shocks ($\omega^d_{n,t+1}$)



More correlations

Conditional correlation between consumption growth and inflation



Utility

- Representative agent
- Habit utility: $E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma}}{1-\gamma}$
- Discount factor β
- "Risk-aversion" coefficient γ (always assumed >1)
- C_t - consumption
- H_t - external habit: e.g., exogeneous standard of living

Habit

- Inverse surplus ratio: $q_t = \ln \frac{C_t}{C_t - H_t}$
- $q_{t+1} = \bar{q} + \rho_q(q_t - \bar{q}) - \underbrace{\gamma_q}_{const>0} \epsilon^g_{t+1}$
- Habit = weighted average of past consumption shocks

	Here	Campbell and Cochrane (1999)
"Price of risk"	Constant	Time-varying
"Amount of risk"	Time-varying	Constant

- Ermolov (2014a) shows that the time-varying "amount of risk" specification has advantages in term structure modeling (+asset prices in closed-form!)

Financial Assets

- Risk-free 0-coupon nominal bonds
- Aggregate equity = claim to the aggregate dividends

Dividends and expected inflation

- Real dividend growth: $d_{t+1} = \bar{g} + \epsilon^{d}_{t+1}$
- ϵ^{d}_{t+1} heteroskedastic 0-mean shock, $0 < Corr(\epsilon^{d}_{t+1}, \epsilon^{g}_{t+1}) < 1$
- Persistent inflation expectations x^{π}_{t}, $0 < Corr(x^{\pi}_{t}, \epsilon^{\pi}_{t}) < 1$
- $\bar{g}$ - consumption growth mean, ϵ^{g}_{t+1} - consumption growth shock, ϵ^{π}_{t+1} - inflation shock

More details

Pricing

- Stochastic discount factor (SDF):

$$M_{t+1} = \beta e^{-\gamma g_{t+1} + \gamma (q_{t+1} - q_t)}$$

- Innovations to SDF:

$$m_{t+1} - E_t(m_{t+1}) = \underbrace{a_p}_{const<0} \omega^d_{p,t+1} + \underbrace{a_n}_{const>0} \omega^d_{n,t+1} + \underbrace{a_p}_{const<0} \omega^s_{p,t+1} + \underbrace{a_n}_{const>0} \omega^s_{n,t+1}$$

- Positive consumption shocks decrease marginal utility
- Negative consumption shocks increase marginal utility
- Nominal SDF: $m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}$

Asset prices

- Time t n-period nominal bond prices:
  $P^{\$}_{n,t} =$
  $exp(C^{\$}_n + Q^{\$}_n q_t + X^{\pi}_n x^{\pi}_t + P^{d\$}_n p^d_t + N^{d\$}_n n^d_t + P^{s\$}_n p^s_t + N^{s\$}_n n^s_t)$

- Time t aggregate equity $\frac{P}{D}$-ratio:
  $\frac{P_t}{D_t} =$
  $\sum_{n=1}^{\infty} exp(C^e_n + Q^e_n q_t + P^{de}_n p^d_t + N^{de}_n n^d_t + P^{se}_n p^s_t + N^{se}_n n^s_t)$

- Coefficients recursively defined

Price impact of demand shocks

- Suppose a positive demand shock occurs

Channel	$r^e_{t+1} - E_t r^e_{t+1}$	$r^b_{t+1} - E_t r^b_{t+1}$
Intertemporal smoothing	+	+
Precautionary savings	+	+
Dividend growth	+	
Expected inflation		-

Price impact of demand shocks

- Suppose a positive demand shock occurs

Channel	$r^e_{t+1} - E_t r^e_{t+1}$	$r^b_{t+1} - E_t r^b_{t+1}$
Intertemporal smoothing	+	
Precautionary savings		
Dividend growth	+	
Expected inflation		-

Demand shocks move stock and bond returns in opposite directions

Price impact of supply shocks

- Suppose a positive supply shock occurs

Channel	$r^e_{t+1} - E_t r^e_{t+1}$	$r^b_{t+1} - E_t r^b_{t+1}$
Intertemporal smoothing	+	+
Precautionary savings	+	+
Dividend growth	+	
Expected inflation		+

Price impact of supply shocks

- Suppose a positive supply shock occurs

Channel	$r^e_{t+1} - E_t r^e_{t+1}$	$r^b_{t+1} - E_t r^b_{t+1}$
Intertemporal smoothing	+	+
Precautionary savings		
Dividend growth	+	
Expected inflation		+

Supply shocks move stock and bond returns in the same direction

Conditional return comovements

- In the model: $Cov_t(r^e_{t+1}, r^b_{t+1}) \approx$

$$\underbrace{a^e_{dp}a^b_{dp}}_{<0} p^d_t + \underbrace{a^e_{dn}a^b_{dn}}_{<0} n^d_t + \underbrace{a^e_{sp}a^b_{sp}}_{>0} p^s_t + \underbrace{a^e_{sn}a^b_{sn}}_{>0} n^s_t$$

- Demand shock environment: $Cov_t(r^e_{t+1}, r^b_{t+1}) < 0$ - nominal bonds hedge well
- Supply shock environment: $Cov_t(r^e_{t+1}, r^b_{t+1}) > 0$ - nominal bonds hedge poorly

Data

- US quarterly observations: 1969Q4-2012Q4
- Corporate earnings payout (Longstaff and Piazzesi, 2004): NIPA
- Aggregate stock returns: CRSP
- Treasury yields: Gürkaynak et.al. (2006)

Estimation

- Macroeconomic dynamics already estimated from consumption and inflation data
- Generalized method of moments (GMM) estimation
- 5 preference parameters to estimate: β, γ, $\bar{q}$, ρ_q, γ_q
- 9 unconditional moments to match:
  - 1 quarter nominal interest rate and its variance
  - 5 year bond excess return and its variance
  - price-dividend ratio and its variance
  - equity premium and its variance
  - unconditional 5 year bond and stock return covariance

Estimated preference parameters

β	0.99 fixed
γ	4.12 (0.51)
$\bar{q}$	1.00 fixed
ρ_q	0.99 (0.02)
γ_q	-9.51 (0.84)

GMM moments match

Moment	Data	Model
$E(y_{1q}^{\$})$	1.33%	1.53%
	(0.18%)	
$Var(y_{1q}^{\$})$	6.48E-05	7.74E-05
	(2.00E-05)	
$E(r_{5y}^{bx})$	0.49%	0.62%
	(0.24%)	
$Var(r_{5y}^{bx})$	0.0011	0.0008
	(0.0003)	
$E(pd)$	5.01	5.09
	(0.10)	
$Var(pd)$	0.18	0.12
	(0.04)	
$E(r^{ex})$	1.08%	0.90%
	(0.58%)	
$Var(r^{ex})$	0.0085	0.0074
	(0.0013)	
$Cov(r^{ex}, r^{bx})$	0.0002	0.0007
	(0.0005)	
Overidentification test p-value	0.2406	

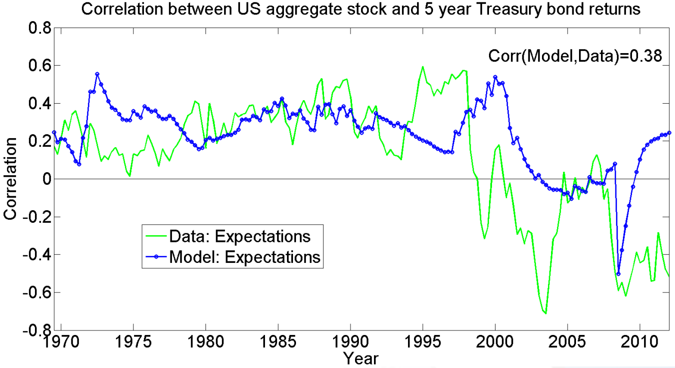
▸ Implied macro moments ▸ Implied local risk-aversion ▸ Implied financial moments

Implied stock and bond return correlations

	Unconditional correlation	
	Data	Model
	0.05	0.30
	(0.13)	
	Conditional correlations	
	Data (expectations)	Model
Min	-0.71	-0.48
Max	0.60	0.55
1^{st} percentile	-0.68	-0.19
	(0.05)	
2.5^{th} percentile	-0.60	-0.10
	(0.04)	
97.5^{th} percentile	0.55	0.56
	(0.02)	
99^{th} percentile	0.57	0.62
	(0.03)	

- Macroeconomic shocks generate sizeable positive and negative stock and bond return correlations
- Negative correlations less extreme and frequent than in data

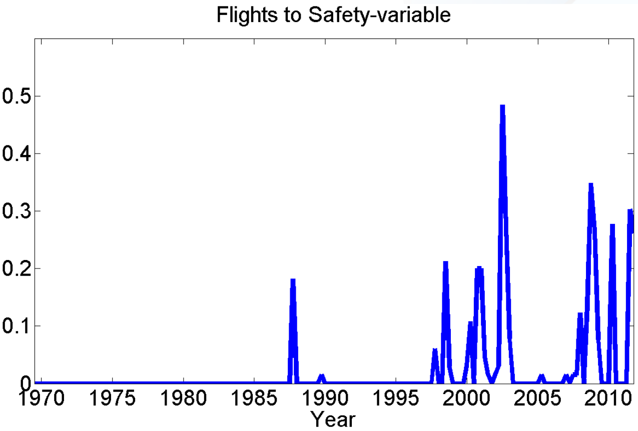
Historical stock and bond return correlations



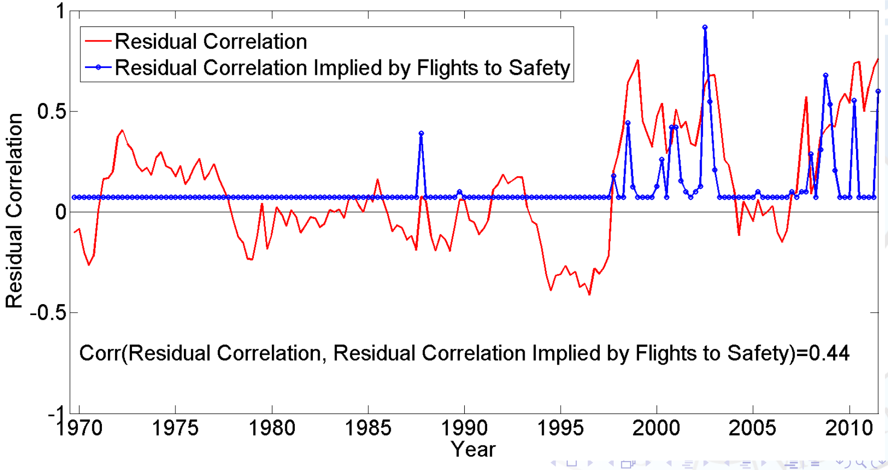
- Macroeconomic shocks important from late 70's until early 90's and pre- and during Great Recession
- Excluding 1997-2003 and 2010-2012: Corr(Model, Data)=0.58, Corr(r^{ex},r^{bx})=0.27 ▸ Additional results

Defining flights to safety episodes

- High-frequency episodes of simultaneous extreme positive bond and negative stock returns unlikely to be related to macroeconomic factors (Baele et.al. 2014)



Explaining residual stock and bond return correlations with flights to safety episodes



Comparision to the literature

- Studies finding weak links between risk of nominal assets and macroeconomy: restrictive macroeconomic dynamics (difficult to incorporate realistic dynamics into asset pricing frameworks in a tractable manner)
- Studies finding strong links between risk of nominal assets and macroeconomy: rely on financial data to estimate macroeconomic shocks

Conclusions

- Tractable structural framework for understanding macroeconomic risk of nominal assets: tons of applications!
- Economically characterizing macroeconomic shocks
- Macroeconomic shocks:
  - produce sizeable positive and negative stock and bond return correlations, although negative correlations smaller and less frequent than in data
  - historically most important for correlations from late 70's to early 90's and pre- and during the Great Recession

Appendix 1: BEGE conditional moments

- Intuitive theoretical expressions for (unscaled) moments:
  - $Var_t(u_{t+1}) = \sigma_p^2 p_t + \sigma_n^2 n_t$
  - $Skw_t(u_{t+1}) = 2(\sigma_p^3 p_t - \sigma_n^3 n_t)$
  - $Ex.Kur_t(u_{t+1}) = 6(\sigma_p^4 p_t + \sigma_n^4 n_t)$

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Appendix 2: Macroeconomic dynamics estimation procedure

- **Stage 1:** Filter ϵ^g_{t+1} and ϵ^π_{t+1} using OLS
- **Stage 2:** Estimate σ^d_g, σ^s_g, σ^d_π, σ^s_π to invert ϵ^g_{t+1} and ϵ^π_{t+1} to u^d_{t+1} and u^s_{t+1} using GMM (based on unconditional second and third moments, including cross-moments)
- **Stage 3:** From u^d_{t+1} and u^s_{t+1}, estimate macroeconomic volatility parameters ($\bar{p}^d$, $\bar{n}^d$, $\bar{p}^s$, $\bar{n}^s$, ρ^d_p, ρ^d_n, ρ^s_p, ρ^s_n, σ^d_{pp}, σ^d_{nn}, σ^s_{pp}, σ^s_{nn}) using the characteristic function domain approximate maximum likelihood (Bates, 2006)
- **Stage 4:** Estimate inflation expectations and dividend dynamics by regressing them on u^d_{t+1} and u^s_{t+1}

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Appendix 3: Maximum likelihood estimation procedure

- Below is the algorithm for u_t^d, algorithm for u_t^s is identical
- Sequentially computing likelihood over $\{u_t^d = \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d\}_{t=1}^T$
  - Step 1: Computing likelihood of u_{t+1}^d given p_t^d and n_t^d distributions
  - Step 2: Updating p_t^d and n_t^d distributions given u_{t+1}^d
  - Step 3: Computing conditional distribution of p_{t+1}^d and n_{t+1}^d given u_{t+1}^d
- In characteristic function domain (approximate) Steps 1-3 formulas available in closed form (Bates, 2006)

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Appendix 4: Supply shocks parameters

Good variance		Bad variance	
σ_p^s	0.15	σ_s^n	0.26
	(0.03)		(0.07)
$\bar{p}^s$	7.69	$\bar{n}^s$	18.17
	(0.71)		(1.12)
ρ_p^s	0.92	ρ_n^s	0.99
	(0.09)		(0.14)
σ_{pp}^s	0.92	σ_{nn}^s	0.40
	(0.30)		(0.21)

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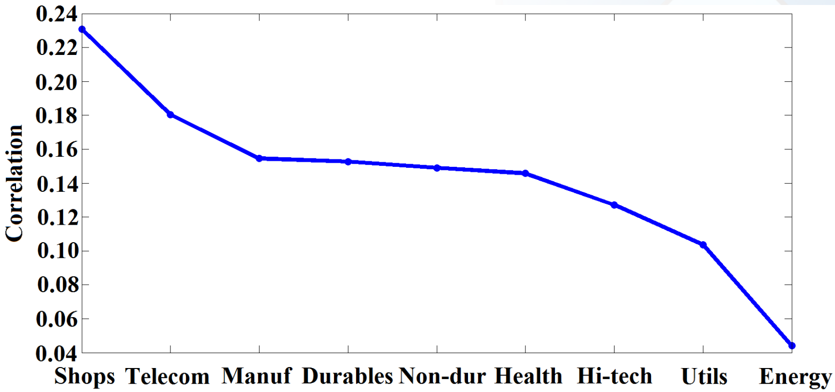
Appendix 5: Correlation between industry portfolio returns and good supply shocks $(\omega^s_{p,t+1})$



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Appendix 6: Demand shocks parameters

Good variance		Bad variance	
σ_p^d	0.07	σ_d^n	5.39
	(0.03)		(1.32)
$\bar{p}^d$	139.84	$\bar{n}^d$	0.01
	(7.17)		(0.01)
ρ_p^d	0.96	ρ_n^d	0.75
	(0.03)		(0.20)
σ_{pp}^d	0.96	σ_{nn}^d	0.08
	(0.14)		(0.04)

- Gaussian good component
- Rare-disaster type bad component

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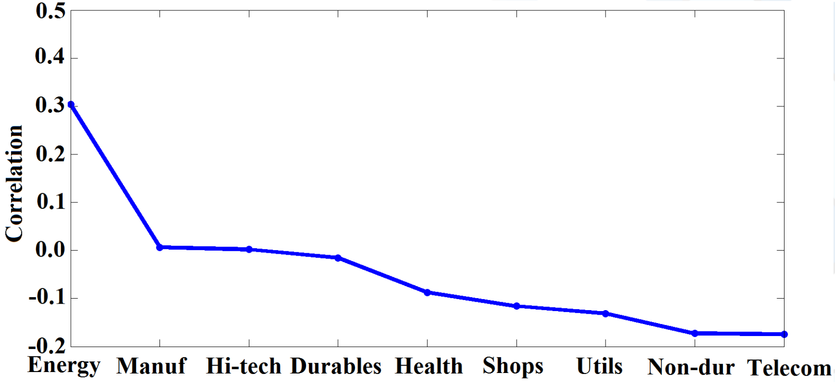
Appendix 7: Correlation between industry portfolio returns and good demand shocks ($\omega^d_{p,t+1}$)



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Appendix 8: Dividends and expected inflation specifications

- Real dividend growth: $d_{t+1} = \bar{g} + \gamma_d \epsilon^g_{t+1} + \gamma_{dd} u^d_{t+1} + \epsilon^{div}_{t+1}$, $\epsilon^{div}_{t+1} \sim \mathcal{N}(0, \sigma_d)$
- Inflation expectations: $x^\pi_{t+1} = \rho_{x^\pi} x^\pi_t + \gamma_{x^\pi} \epsilon^\pi_{t+1} + \gamma_{x^\pi d} u^d_{t+1} + \epsilon^{x^\pi}_{t+1}$, $\epsilon^{x^\pi}_{t+1} \sim \mathcal{N}(0, \sigma^\pi_x)$

Parameter	Estimate	Standard error
$\bar{g}$	0.42%	0.04%
$\bar{\pi}$	1.06%	0.07%
γ_d	1.35	1.73
γ_{d^d}	4.24	5.83
σ_d	0.06	0.03
ρ_{x^π}	0.93	0.02
γ_{x^π}	0.22	0.03
$\gamma_{x^\pi d}$	0.09	0.04
σ_{x^π}	0.0011	0.0007

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Appendix 9: Implied local risk-aversion

Percentile	1%	5%	25%	50%	75%	95%	99%
Value	6.33	7.30	8.99	10.58	13.02	19.85	29.23

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Appendix 10: Unconditional consumption growth and inflation dynamics

	Consumption growth		Inflation	
	Data	Model	Data	Model
Mean	0.42%	0.42%	1.06%	1.06%
	(0.04%)		(0.07%)	
Standard deviation	0.41%	0.44%	0.86%	0.86%
	(0.03%)		(0.08%)	
Skewness	-0.41	-0.37	0.11	-0.55
	(0.26)		(0.78)	
Excess kurtosis	1.24	1.75	4.68	7.17
	(0.56)		(2.53)	
Pr(<mean-2·Standard deviation)	2.91%	3.11%	0.58%	1.62%
	(0.97%)		(0.60%)	
Pr(<mean-4·Standard deviation)	0.00%	0.00%	0.58%	0.19%
	(0.12%)		(0.60%)	
Pr(>mean+2·Standard deviation)	2.91%	2.05%	5.54%	2.71%
	(1.04%)		(1.64%)	
Pr(>mean+4·Standard deviation)	(0.00%)	(0.03%)	0.00%	0.03%
	(0.00%)		(0.14%)	
$Corr(g_t, \pi_t)$	-0.14	-0.22		
	(0.11)	(0.18)		

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Appendix 11: Implied financial moments

	Data	Model
$y^{\$}_{5y} - y^{\$}_{1y}$	0.18%	0.12%
	(0.04%)	
$y_{5y} - y_{1y}$	0.11%	0.09%
	(0.02%)	
Fama-Bliss (1987) slope: 5 years vs 1 year	0.77	0.14
	(0.36)	
$AC_1(pd)$	0.98	0.99
	(0.03)	
Slope r^{ex}_{t+1} wrt pd_t	-0.0204	-0.0056
	(0.0171)	

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Appendix 12: Time pattern in stock and bond return correlations

	1970-2000	2001-2012	Difference
Data: expectations	0.27	-0.32	-0.59***
	(0.17)	(0.22)	
Model	0.30	0.06	-0.23***
	(0.09)	(0.15)	

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