Bad Environments, Good Environments: A Non-Gaussian Asymmetric Volatility Model

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Motivation and Contribution 1/4

Hi!

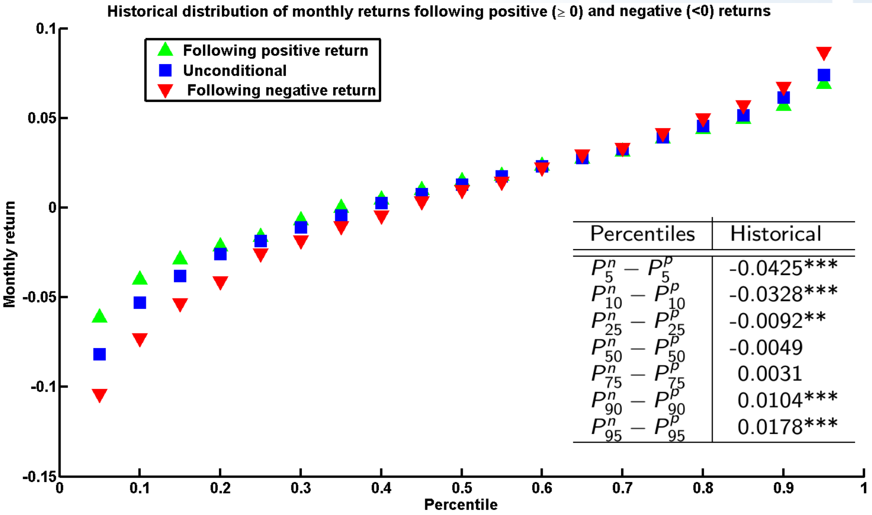
I am abundant and, unfortunately, do not fit this slide.

Best Wishes,
GARCH literature

Motivation and Contribution 2/4

- Given the size of the GARCH-literature, the barriers to entry are high:
  - Better empirical fit
  - Ease of estimation
  - Tractability for risk management applications

Motivation and Contribution 3/4



Percentiles	Historical
$P^n_5 - P^p_5$	-0.0425***
$P^n_{10} - P^p_{10}$	-0.0328***
$P^n_{25} - P^p_{25}$	-0.0092**
$P^n_{50} - P^p_{50}$	-0.0049
$P^n_{75} - P^p_{75}$	0.0031
$P^n_{90} - P^p_{90}$	0.0104***
$P^n_{95} - P^p_{95}$	0.0178***

Motivation and Contribution 4/4

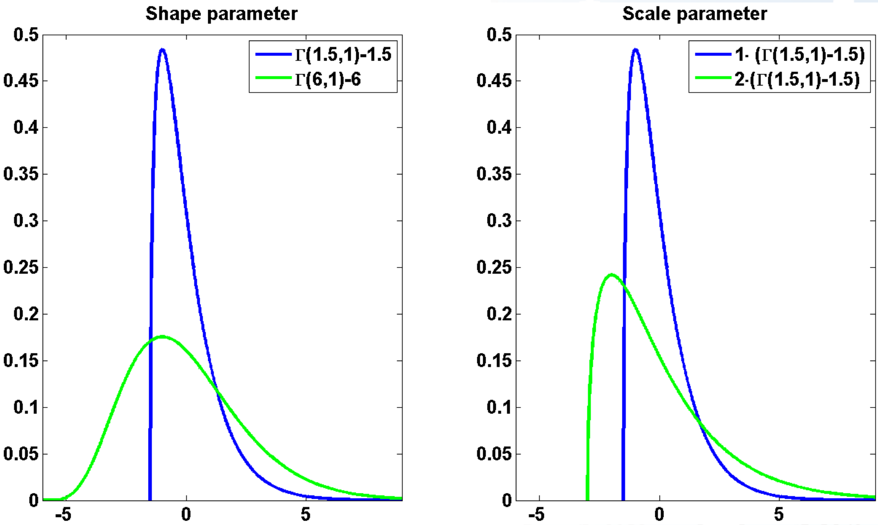
- Propose a novel GARCH model with non-Gaussianities:
    - Better empirical fit: superior fit of unconditional and especially **CONDITIONAL** return distribution!
    - Ease of estimation: Fast maximum likelihood estimation!
    - Tractability for risk management applications: Intuitive closed form expressions for conditional volatility, skewness, kurtosis and higher order moments!

Model 1/4: Setup

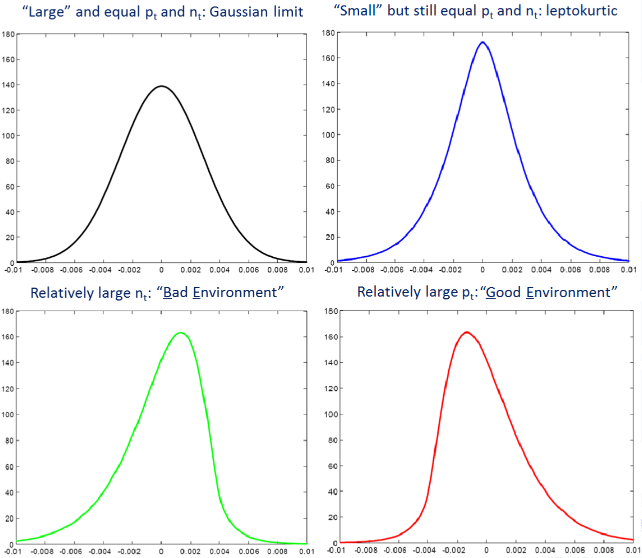
$$
\begin{aligned}
r_{t+1} &= \mu + u_{t+1} \\
u_{t+1} &= \omega_{p,t+1} - \omega_{n,t+1}, \\
\omega_{p,t+1} &\sim \sigma_p(\Gamma(p_t, 1) - p_t), \\
\omega_{n,t+1} &\sim \sigma_n(\Gamma(n_t, 1) - n_t), \\
p_t &= p_0 + \rho_p p_{t-1} + \phi_p^+ \frac{u_{t-1}^2}{2\sigma_p^2} \mathbb{1}_{u_{t-1}>0} + \phi_p^- \frac{u_{t-1}^2}{2\sigma_p^2} \mathbb{1}_{u_{t-1}\leq 0}, \\
n_t &= n_0 + \rho_n n_{t-1} + \phi_n^+ \frac{u_{t-1}^2}{2\sigma_n^2} \mathbb{1}_{u_{t-1}>0} + \phi_n^- \frac{u_{t-1}^2}{2\sigma_n^2} \mathbb{1}_{u_{t-1}\leq 0}.
\end{aligned}
$$

- Shock: demeaned gamma distributions (Bad Environments - Good Environments, BEGE, density): Bekaert and Engstrom (2010)
- Asymmetric impact of positive and negative innovations: Glosten, Jagannathan, Runkle (1993) (GJR)

Model 2/4: Gamma PDF



Model 3/4: BEGE PDF



Model 4/4: Conditional Moments

- Intuitive theoretical expressions for (unscaled) moments:
  - $Var_t(r_{t+1}) = \sigma_p^2 p_t + \sigma_n^2 n_t$
  - $Skw_t(r_{t+1}) = 2(\sigma_p^3 p_t - \sigma_n^3 n_t)$
  - $Ex.Kur_t(r_{t+1}) = 6(\sigma_p^4 p_t + \sigma_n^4 n_t)$

Estimation 1/4: Data and Methodology

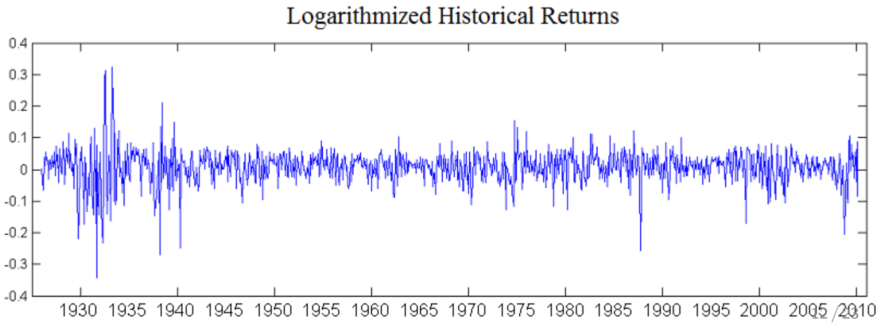
- Data: logarithmized US monthly aggregate equity returns 1926-2010
- BEGE density can be evaluated:
  - theoretically using Whittaker W function
  - numerically integrating over gamma distributions
- Fast estimation via MLE

Estimation 2/4: Parameter Estimates

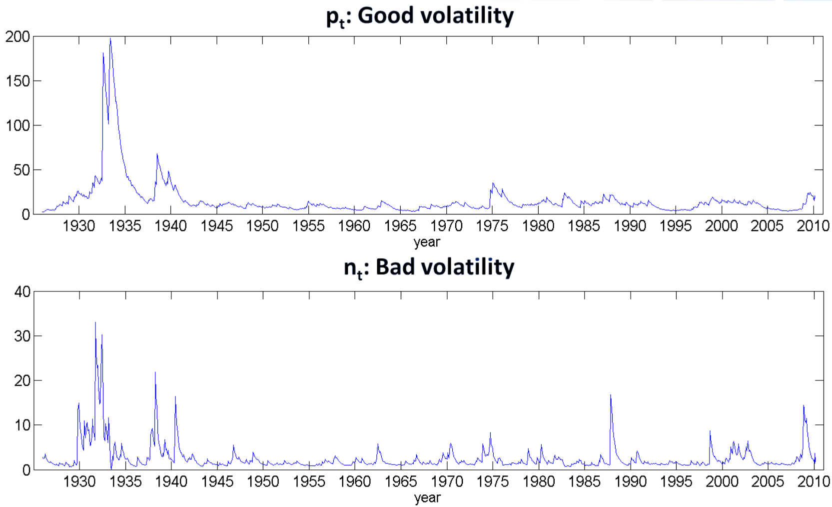
μ	0.0100		
	(0.0015)		
σ_p	0.0072	σ_n	0.0282
	(0.0008)		(0.0087)
p_0	0.0890	n_0	0.2204
	(0.0070)		(0.0111)
ρ_p	0.9099	ρ_n	0.7822
	(0.0009)		(0.0086)
ϕ_p^+	0.0964	ϕ_n^+	-0.0789
	(0.0426)		(0.0524)
ϕ_p^-	0.0128	ϕ_n^-	0.3548
	(0.0139)		(0.0901)

Estimation 3/4: Input

- Two types of volatility clustering:
  - large positive and negative returns: Great Depression, WWII, oil shocks in 70's
  - only large negative returns: Asia and Russia in late 90's, Dot-com, Great Recession



Estimation 4/4: Results



Empirical fit 1/6: Overall fit

Model	Log-likelihood	BIC
BEGE	1724.26	-3372.32
BEGE, constant good volatility	1711.82	-3368.22
t-GJR-GARCH	1703.60	-3365.63
Multifractal RS model	1697.56	-3360.48
Symmetric BEGE	1695.50	-3349.43
2-regime w jump	1692.74	-3330.06
GJR-GARCH	1671.77	-3308.90

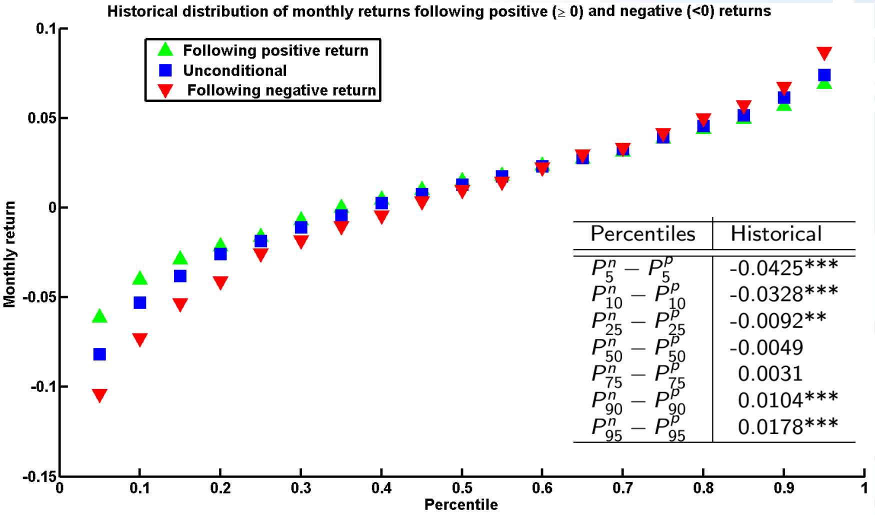
- Asymmetric right and left tails important, but regime-switching models do not seem to be a proper way to model that asymmetry
- Also time-varying good volatility is important

Empirical fit 2/6: Unconditional moments

- Generate time series of historical length from different models: 100,000 replications
- Compute proportion of replications where unconditional moment is less than or equal to its historical value

Model	Standard deviation	Skewness	Ex. Kurtosis
Historical value	0.0545	-0.5742	6.6134
		Simulated *p*-values	
GJR-GARCH	0.6296	0.0120**	0.9580*
t-GJR-GARCH	0.8728	0.0326*	0.9101
2-regime w jump	0.5114	0.3021	0.8368
Multifractal	0.5763	0.0208**	0.9594*
BEGE	0.6127	0.5925	0.9319

Empirical fit 3/6: Conditional moments



Percentiles	Historical
$P_5^n - P_5^p$	-0.0425***
$P_{10}^n - P_{10}^p$	-0.0328***
$P_{25}^n - P_{25}^p$	-0.0092**
$P_{50}^n - P_{50}^p$	-0.0049
$P_{75}^n - P_{75}^p$	0.0031
$P_{90}^n - P_{90}^p$	0.0104***
$P_{95}^n - P_{95}^p$	0.0178***

Empirical fit 4/6: Conditional moments

- Generate time series of historical length from different models: 100,000 replications
- Compute proportion of replications where the difference between conditional percentiles is less than or equal to its historical value

	Simulated p-values			
Percentiles	t-GJR-GARCH	2-regime w jump	Multifractal	BEGE
$P_5^n - P_5^p$	0.0120**	0.0196**	0.0009***	0.0527
$P_{10}^n - P_{10}^p$	0.0081**	0.0017***	0.0001***	0.0350*
$P_{25}^n - P_{25}^p$	0.0474*	0.0248**	0.0478*	0.1761
$P_{50}^n - P_{50}^p$	0.0585	0.0578	0.0497*	0.0535
$P_{75}^n - P_{75}^p$	0.5493	0.6260	0.5740	0.2887
$P_{90}^n - P_{90}^p$	0.7978	0.8268	0.8151	0.6038
$P_{95}^n - P_{95}^p$	0.8698	0.8090	0.8830	0.7527

Empirical fit 5/6: Conditional moments

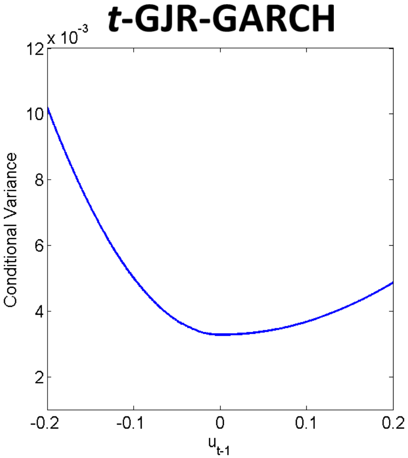
- Modified Jarque-Bera test:
  - at each time point compute CDF of the observation under the particular model
  - Inverse CDF to standard Gaussian
  - Test for the Gaussianity of the inverse CDF with Jarque-Bera test

Model	p-value
t-GJR-GARCH	0.0011***
2-regime w jump	0.0538*
Multifractal	0.0009***
BEGE	0.3756

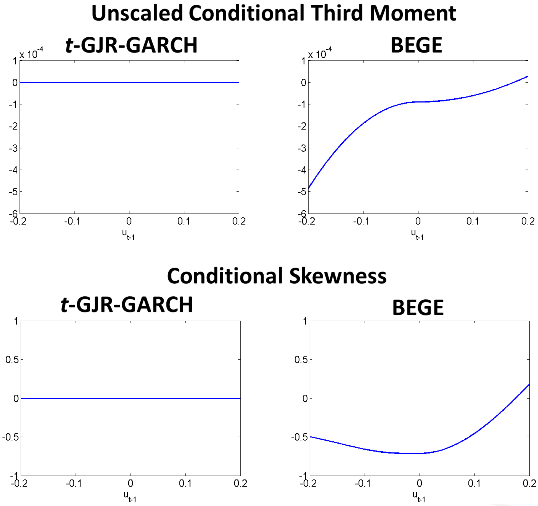
Empirical fit 6/6: Other tests

Test	Winner	
Likelihood ratio tests	BEGE	✓
Engle-Manganelli "hit"-tests	BEGE	✓
Out-of-sample performance	BEGE	✓

Implications 1/3: Volatility News Impact Curves



Implications 2/3: Skewness News Impact Curves



Implications 3/3: Risk-return trade-off

- Adding conditional mean:

$$r_{t+1} = \underset{(0.0025)}{0.0096} - \underset{(0.8587)}{0.9665}\sigma_p^2 p_t + (\underset{(0.8587)}{-0.9665} + \underset{(1.4333)}{1.9427})\sigma_n^2 n_t + u_{t+1}$$

- Excess returns:

$$er_{t+1} = \underset{(0.0011)}{0.0067} + \underset{(1.4900)}{3.6378}\sigma_p^2 p_t + (\underset{(1.4900)}{3.6378} - \underset{(2.0741)}{4.4059})\sigma_n^2 n_t + u_{t+1}$$

- Inconclusive evidence

Conclusions

- New GARCH model with tractable non-Gaussianities
- Beats standard GARCH and regime-switching models along several dimensions
- Many applications and extensions (e.g., realized variance models) $\Rightarrow$ we provide the code!