A Unified Theory of Bond and Currency Markets

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Stylized Facts about Bond Markets

US Fact 1: Upward Sloping Real Yield Curve

In US, real long yields are on average higher than short yields (Gurkaynak et.al., 2009; Ang and Ulrich, 2012; Chernov and Mueller, 2012)

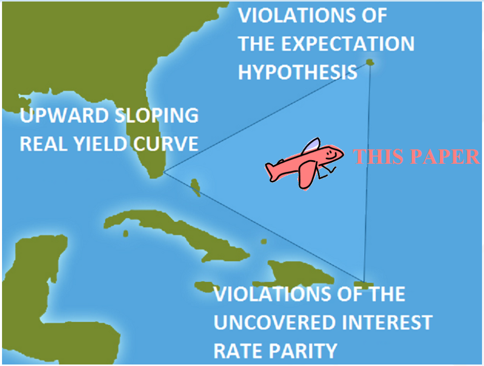
US Fact 2: Violations of the Expectation Hypothesis

High slope of the yield curve $\Rightarrow$ high return on long-term bonds over the life of short-term bonds (Fama and Bliss, 1987; Campbell and Shiller, 1991)

International Fact 1: Uncovered Interest Rate Parity Violations

High differential between foreign and domestic interest rates $\Rightarrow$ high return on borrowing in domestic bonds and investing in foreign bonds (Hansen and Hodrick, 1980; Fama, 1984)

Bermuda Triangle of The Theoretical Bond Markets Literature



- US and international bond markets closely integrated, but theoretically difficult to explain them jointly
- This paper tries to address this task

Agenda

- Model
- Solutions to the puzzles
- Empirical eividence
- Calibration

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Model: Overview

- Only real sector
- Key components:
  - Habit utility
  - Heteroskedastic consumption growth
- Contribution: applying model to jointly explain US and international term structure

Model: Utility

- Representative agent
- Habit utility: $E_0 \sum_{t=0}^{\infty} \beta^t \frac{(\frac{C_t}{H_t})^{1-\gamma}}{1-\gamma}$
- Risk-aversion γ (always assumed >1)
- C_t - consumption
- H_t - habit: exogeneous standard of living

Model: Consumption Growth

- Consumption growth as a mixture of two shocks:

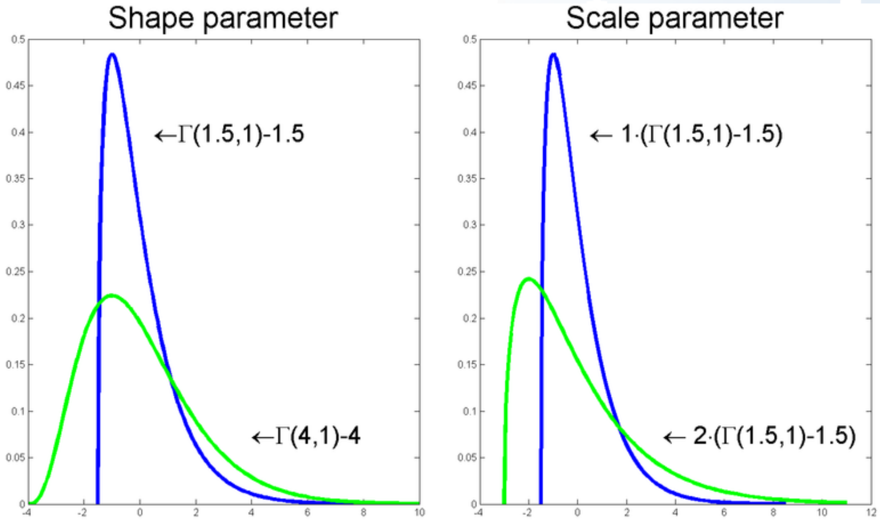
$$g_{t+1} = \bar{g} + \sigma_{cp}\omega_{p,t+1} - \sigma_{cn}\omega_{n,t+1}$$

- Demeaned gamma distributed shocks (Bekaert and Engstrom, 2009)
  - $\omega_{p,t+1} \sim \Gamma(\bar{p}, 1) - \bar{p}$,
  - $\omega_{n,t+1} \sim \Gamma(n_t, 1) - n_t$,

Model: Why Gamma Shocks?

- Qualitatively: logic works with Gaussian shocks
- Quantitatively: calibration with Gaussian shocks challenging
- Gamma distribution has more tail mass: increases agent's sensitivity to shocks
- Unlike for rare disasters, there is strong empirical evidence of gamma shocks in US consumption (Bekaert and Engstrom, 2009)

Model: Gamma Shocks



Model: Volatility and Habit

- Time-varying volatility:
  $n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1},$
- Consumption-habit ratio: $s_t = \ln \frac{C_t}{H_t}$
- $s_{t+1} = \bar{s} + \rho_s(s_t - \bar{s}) + \underbrace{\sigma_{sp}\omega_{p,t+1} - \sigma_{sn}\omega_{n,t+1}}_{\text{constant sensitivity to consumption shocks}}$
- Habit=weighted average of past consumption shocks ($\omega_{n,t}$ and $\omega_{p,t}$)

	Here	Campbell and Cochrane (1999)
Price of risk	Constant	Time-varying
Amount of risk	Time-varying	Constant

Model: Pricing

- Stochastic discount factor:

$$M_{t+1} = \beta e^{-g_{t+1}+(1-\gamma)(s_{t+1}-s_t)}$$

- Innovations to stochastic discount factor:

$$m_{t+1} - E_t(m_{t+1}) = \underbrace{a_p}_{const<0} \omega_{p,t+1} + \underbrace{a_n}_{const>0} \omega_{n,t+1}$$

- Positive consumption shocks decrease marginal utility
- Negative consumption shocks increase marginal utility

Model: Risk-free rate

- Time t one period log risk-free rate:

$$y_{1,t} = constant + \overbrace{\underbrace{S_1}_{const<0} \quad s_t}^{\text{intertemporal smoothing}} + \overbrace{\underbrace{N_1}_{const<0} \quad n_t}^{\text{precautionary savings}}$$

- Intertemporal smoothing: interest rate decreasing in consumption-habit ratio
- Precautionary savings: interest rate decreasing in consumption growth volatility

Agenda

- Model
- **Solutions to the puzzles**
- Empirical evidence
- Calibration

Solutions to The Puzzles: Setup 1/3

- Time $t = 0, 1, 2$
- At time 0: 1 and 2 period zero-coupon bonds: prices $P_{1,t}$, $P_{2,t}$
- Yields: $y_{1,t} = -\ln P_{1,t}$ and $y_{2,t} = -\frac{1}{2}\ln P_{2,t}$
- Return on holding 2 period bond over 1 period: $R_{2,t\to t+1} = \frac{P_{1,t+1}}{P_{2,t}}$
- Taking logs: $r_{2,t\to t+1} = -y_{1,t+1} + 2y_{2,t}$

Solutions to The Puzzles: Setup 2/3

- Rearranging and taking expectations:

$$\underbrace{y_{2,t} - y_{1,t}}_{\text{yield curve slope}} = \frac{1}{2} \underbrace{E_t(y_{1,t+1} - y_{1,t})}_{\text{expected short rate change}} + \frac{1}{2} \underbrace{E_t(r_{2,t\to t+1} - y_{1,t})}_{\text{risk premium}}$$

- Holding 2 period bond over 1 period is not risk-free: subject to short rate risk $\Rightarrow$ risk premium

- $E_t(r_{2,t\to t+1} - y_{1,t}) \approx -cov_t(m_{t+1}, r_{2,t\to t+1}) = cov_t(m_{t+1}, y_{1,t+1})$

Solutions to The Puzzles: Setup 3/3

$$E_t(r_{2,t\to t+1} - y_{1,t}) \approx$$

$$\overbrace{\underbrace{-S_1\sigma_{sn}a_n n_t}_{>0}}^{\text{intertemporal smoothing, } \propto cov_t(m_{t+1}, s_{t+1})} + \overbrace{\underbrace{N_1\sigma_{nn}a_n n_t}_{<0}}^{\text{precautionary savings, } \propto cov_t(m_{t+1}, n_{t+1})}$$

- Intertemporal smoothing: negative consumption shock at $t = 1$...
  - decreases consumption-habit ratio and thus...
  - increases short rate decreasing bond prices...
  - and consequently increasing risk premium on 2 period bond at $t = 0$
- Precautionary savings: negative consumption shock at $t = 1$...
  - increases consumption volatility and thus...
  - decreases short rate increasing bond prices...
  - and decreasing risk premium on 2 period bond at $t = 0$

Solutions to The Puzzles: Average slope of the yield curve

$$\underbrace{E_t(y_{2,t} - y_{1,t})}_{\text{yield curve slope}} = \frac{1}{2} \underbrace{E_t(y_{1,t+1} - y_{1,t})}_{\text{expected short rate change}} + \frac{1}{2} \underbrace{E_t(r_{2,t\to t+1} - y_{1,t})}_{\text{risk premium}}$$

$$E(y_{2,t} - y_{1,t}) = \frac{1}{2} E(r_{2,t\to t+1} - y_{1,t})$$

Dominant effect	Intertemporal smoothing	Precautionary savings
$E(r_{2,t\to t+1} - y_{1,t})$	>0	<0
Slope of the yield curve	$\Uparrow$	$\Downarrow$

Solutions to The Puzzles: Expectation hypothesis

$$\underbrace{E_t(y_{2,t} - y_{1,t})}_{\text{yield curve slope}} = \frac{1}{2}\underbrace{E_t(y_{1,t+1} - y_{1,t})}_{\text{expected short rate change}} + \frac{1}{2}\underbrace{E_t(r_{2,t\to t+1} - y_{1,t})}_{\text{risk premium}}$$

- Suppose volatility, n_t, is high
- $E_t(y_{1,t+1})$ is high as the next period volatility is expected to mean-revert and thus short rate is expected to be higher
- $|E(r_{2,t\to t+1} - y_{1,t})| \propto n_t$ is high

Dominant effect	Intertemporal smoothing	Precautionary savings
$E_t(y_{1,t+1} - y_{2,t})$	High	High
$E(r_{2,t\to t+1} - y_{1,t})$	High	Low
$E_t(y_{2,t} - y_{1,t})$	High	Ambiguous
$Corr(E_t(y_{2,t} - y_{1,t}), E(r_{2,t\to t+1} - y_{1,t}))$	High	Ambiguous

Solutions to the Puzzles: Average Slope of the Yield Curve and The Expectation hypothesis

Dominant effect	Intertemporal smoothing	Precautionary savings
Average Slope of the Yield Curve	$\Uparrow$	$\Downarrow$
Expectation Hypothesis	Violated	Ambiguous
Example	US	UK

Solutions to the Puzzles: Longer Horizons

- Depending on the parameters, intertemporal smoothing and precautionary savings will have different strengths at different horizons
- Consequently, the yield curve can be upward- or downward-sloping, hump- or U-shaped
- Similarly, expectation hypothesis can be violated at some horizons and not violated at others

Solutions to The Puzzles: International Setup 1/3

- 2 symmetric and independent countries: H and L
- Each country has its own good
- Complete markets=marginal utilities are equal across countries
- No trading frictions or arbitrage opportunities
- Exchange rate: $Q = \frac{\text{H country goods}}{\text{L country good}}$

Solutions to The Puzzles: International Setup 2/3

- Time $t = 0, 1$
- At $t = 0$:
  - country H has high conditional consumption growth volatility: n_t^H
  - country L has low conditional consumption growth volatility: $n_t^L < n_t^H$
- Stochastic discount factors: M_{t+1}^H and M_{t+1}^L
- 1 period risk-free bonds with returns (yields) R_{t+1}^H ($y_{1,t}^H$) and R_{t+1}^L ($y_{1,t}^L$)

Solutions to The Puzzles: International Setup 3/3

- Country L Euler $E_t(M_{t+1}^L R_{t+1}^L) = 1$
- Country H Euler for investing in country L bond $E_t(M_{t+1}^H \frac{Q_{t+1}}{Q_t} R_{t+1}^L) = 1$
- Complete markets $\Rightarrow$ unique SDF $\Rightarrow$ $M_{t+1}^L = M_{t+1}^H \frac{Q_{t+1}}{Q_t}$
- Change in log-exchange rate: $q_{t+1} - q_t = m_{t+1}^L - m_{t+1}^H$

Solutions to The Puzzles: Uncovered Interest Rate Parity Violations 1/2

- Return from borrowing in H and lending in L: $r^{FX}_{t+1} = -y^H_{1,t} + y^L_{1,t} + q_{t+1} - q_t$

$$Cov_t(m^H_{t+1}, r^{FX}_{t+1}) = Cov_t(m^H_{t+1}, -y^H_{1,t} + y^L_{1,t} + m^L_{t+1} - m^H_{t+1}) = -Var_t(m^H_{t+1}) < 0$$

- Interpretation: a negative consumption shock in H at $t = 1$ simultaneously increases marginal utility and strengthens the exchange rate

Solutions to The Puzzles: Uncovered Interest Rate Parity Violations 2/2

- In the model: $E_t r_{t+1}^{FX} = \underbrace{N_1}_{>0} (n_t^H - n_t^L)$

- Consumption volatility in H is high:
  - $E_t r_{t+1}^{FX}$ is high because r_{t+1}^{FX} is a poor hedge against consumption shocks and magnitude (volatility) of these shocks is high
  - interest rate in H are low due to precautionary savings

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Empirical evidence: US bonds 1/3

$$y_{1,t} = constant + \overbrace{\underbrace{S_1}_{const<0} \quad s_t}^{\text{intertemporal smoothing}} + \overbrace{\underbrace{N_1}_{const<0} \quad n_t}^{\text{precautionary savings}}$$

- Predictions and assumptions of the model:
  - Interest rate should be decreasing in consumption-habit ratio (intertemporal smoothing)
  - Interest rate should be decreasing in consumption growth volatility (precautionary savings)
  - Positive and negative consumption shocks can affect consumption-habit ratio differently

Empirical evidence: US bonds 2/3

Quarterly US data 1969-2013

Theoretical variable	Empirical proxy
real yield	nominal yield - expected inflation
consumption-habit ratio	$\sum_{i=0}^{40} \rho_s^i (g_{t-i} - \bar{g})$
consumption-habit ratio: negative shocks	$\sum_{i=0}^{40} \rho_s^i (g_{t-i} - \bar{g}) \mathbb{1}_{(g_{t-i} - \bar{g})<0}$
conditional volatility	from consumption GARCH models

Empirical evidence: US bonds 3/3

- Positive and negative consumption shocks seem to affect habit differently
- Evidence of precautionary savings and intertemporal smoothing for negative shocks

$y^{\$}_{t,t+1} - E_t\pi_{t+1} = \alpha_0 + \alpha_1 \cdot \textit{consumption-habit} + \alpha_2 \cdot \textit{consumption-habit}^{-} + \alpha_3 \cdot \textit{conditional volatility} + \epsilon_{t+1}$

	Setup 1	Setup 2	Setup 3	Setup 4
Constant	0.0421***	-0.0084	-0.0001	0.0000
	(0.0091)	(0.0146)	(0.0154)	(0.0184)
Consumption-habit	0.1720	0.5715***	0.2270	0.8010***
	(0.1674)	(0.2012)	(0.1812)	(0.2414)
Consumption-habit^{-}		-0.8452***		-1.4544***
		(0.2736)		(0.2411)
Conditional volatility			6.2180**	-6.6084*
			(2.6823)	(3.9976)
R^2	0.0673	0.3057	0.2098	0.3341

Empirical evidence: International bonds

- Low-interest rate countries have higher consumption growth volatilities

Historical consumption growth volatilities		
Low-interest rate	Mid-interest rate	High-interest rate
		
Whole time period: 1971-2012		
1.94% (0.22%)	1.42% (0.14%)	1.18% (0.13%)
Modern time period: 1988-2012		
1.42% (0.22%)	1.24% (0.21%)	1.21% (0.22%)

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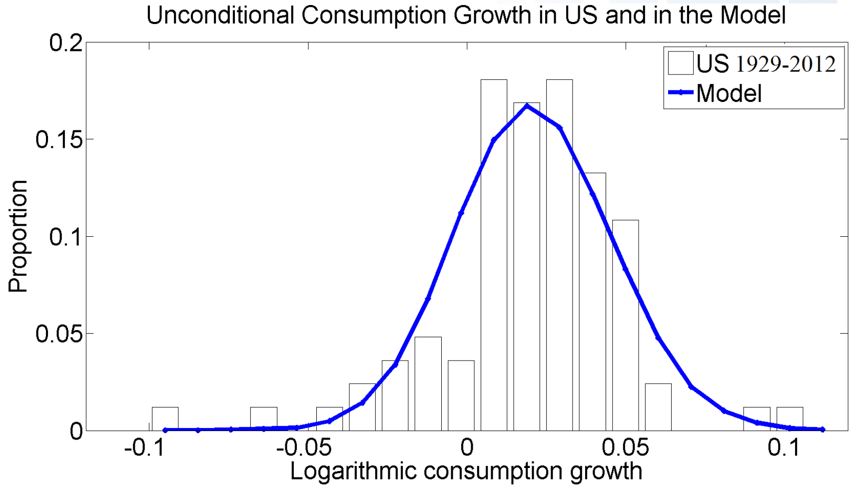
Calibration: Parameters

- Annual frequency

	Preferences	
β	discount factor	0.98
γ	**risk-aversion**	**6.69**
$\bar{s}$	average consumption-habit ratio	1.00
ρ_s	persistence of the consumption-habit ratio	0.79
σ_{sp}	sensitivity of the consumption-habit ratio to positive shocks	10^{-4}
σ_{sn}	sensitivity of the consumption-habit ratio to negative shocks	-0.16
	Consumption dynamics	
$\bar{g}$	average consumption growth	0.02
$\bar{p}$	shape parameter of positive shocks	202.25
σ_{cp}	impact of positive shocks on the consumption growth	$0.17 \cdot 10^{-4}$
$\bar{n}$	average shape parameter of negative shocks	0.02
σ_{cn}	impact of negative shocks on the consumption growth	0.05
ρ_n	volatility persistence	0.88
σ_{nn}	scale of the volatility shock	0.08

Calibration: Consumption Growth 1/2

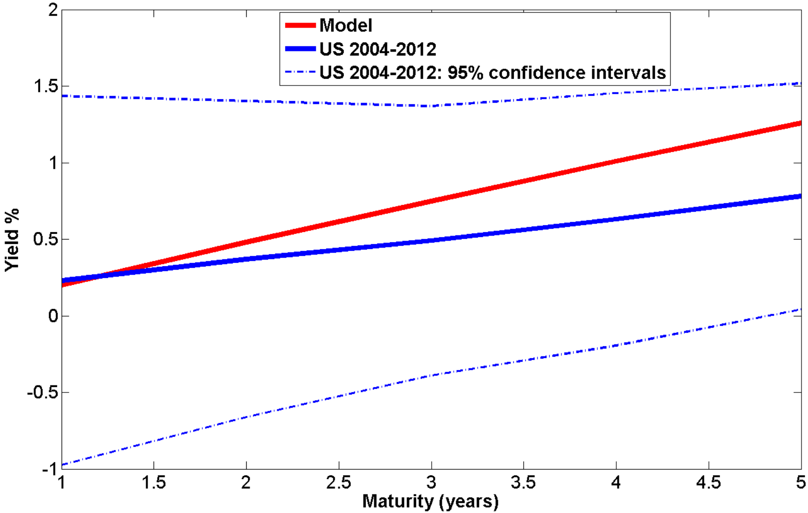


Calibration: Consumption Growth 2/2

- Unconditional consumption growth more Gaussian than in data
- No disasters

	Model	US 1929-2012
Mean	2.10%	2.00%
	(0.16%)	(0.36%)
Standard deviation	2.52%	2.98%
	(0.25%)	(0.45%)
Skewness	-0.20	-0.83
	(0.27)	(0.64)
Excess kurtosis	2.02	3.52
	(0.64)	(1.32)
$P(< \bar{g} - 2\sigma_g)$	1.93%	4.96%
	(1.13%)	(1.58%)
$P(< \bar{g} - 4\sigma_g)$	0.11%	1.20%
	(0.01%)	(0.53%)

Calibration: Real Yields 1/2



Calibration: Real Yields 2/2

- Model matches:
  - Upward sloping yield curve
  - Realistically low volatility of interest rates: intertemporal smoothing and precautionary savings effects offset each other

Moment	Model	US 2004-2012
$\sigma(y_1)$	1.65%	1.58%
	(0.51%)	(0.42%)

Calibration: Violations of the Expectation Hypothesis

- Model replicates expectation hypothesis violations
- $y_{n-1,t+1} - y_{n,t} = \beta_0 + \beta_n \frac{1}{n-1}(y_{n,t} - y_{1,t}) + \epsilon_t$

Moment	Description	Model	US nominal 1961-2012
β_2	n=2 years	-1.18	-0.71
		(0.57)	(0.42)
β_3	n=3 years	-1.21	-1.04
		(0.52)	(0.51)
β_4	n=4 years	-1.27	-1.29
		(0.47)	(0.53)
β_5	n=5 years	-1.31	-1.48
		(0.42)	(0.58)

Calibration: International

- Adequate fit of international moments
- α_{FX} is from the regression $r^{FX}_{t+1} = \alpha_0 + \alpha_{FX}(y_{1,t} - y^*_{1,t}) + \epsilon_t$
- Δq_{t+1} is the real exchange rate change

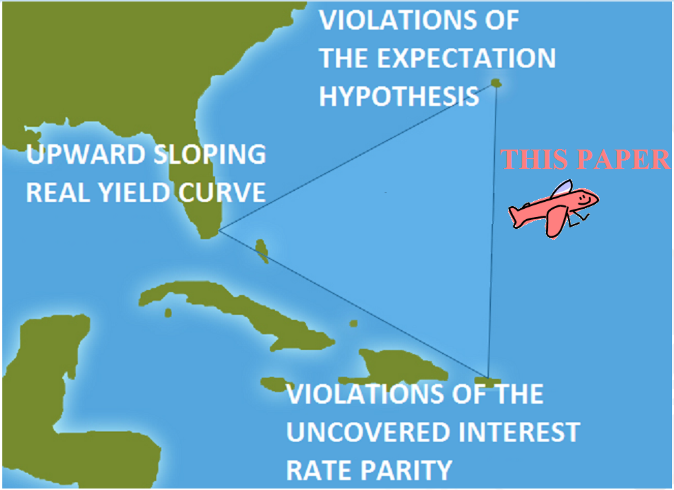
Moment	Model	G-10 countries, 1970-2000 (Backus et.al. 2001, and Benigno and Thoenissen, 2008)
α_{FX}	-1.92 (0.32)	[-0.74,-1.84]
$\sigma(\Delta q_{t+1})$	20.12% (12.54%)	[6.23%,17.54%]
$Corr(\Delta q_{t+1}, g^*_{t+1} - g_{t+1})$	-0.49 (0.10)	[-0.55,0.53]

Calibration: Equity

- Equity = claim to aggregate consumption
- Key equity moments replicated

Moment	Model	US 1929-2012
$r_{mkt} - y_1$	4.45%	5.67%
	(2.84%)	(2.11%)
Sharpe-ratio	0.36	0.29
	(0.15)	(0.13)
pd	3.66	3.40
	(0.12)	(0.09)
$Corr(pd_{t-1}, pd_t)$	0.81	0.85
	(0.08)	(0.12)

Conclusion



- A joint explanation of key US and international bond markets phenomena