The Variance Risk Premium in Equilibrium Models

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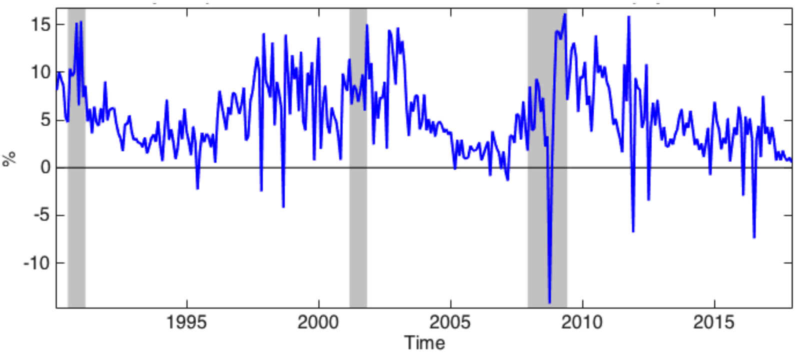
Motivation

- Use option markets phenomena, such as variance risk premium, to discipline consumption-based asset pricing literature
- Expand stylized facts $\Rightarrow$ existing models fail
- New model that does match key facts

Variance Risk Premium

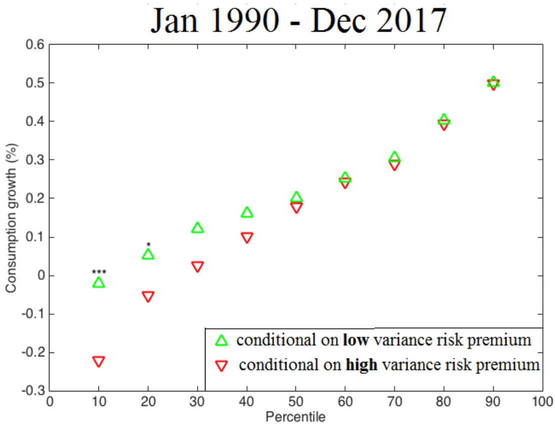
- **Variance risk premium**=risk-neutral variance - physical variance of aggregate equity claim return
- **Volatility risk premium**=risk-neutral volatility - physical volatility of aggregate equity claim return
- Many ways to compute physical variance/volatility, but our findings are robust

Monthly volatility risk premium



- On average positive: 5.36% annually ($\Rightarrow$ variance risk premium 0.0196 annually)
- Low autocorrelation: 0.54 (0.48 in the pre-Great Recession sample)

Next Month Consumption Growth Conditional on Variance Risk Premium



- High variance risk premium=above 80^{th} unconditional percentile
- Low variance risk premium=below 20^{th} unconditional percentile

Risk neutral entropy - risk neutral variance

- Martin (2017) shows that risk-neutral entropy of aggregate equity return is very close to, but still higher than its risk-neutral variance
- Informative moment: left-tail of risk-neutral distribution is only moderately heavier than right tail

Extant consumption-based models 1/2

- Time-varying volatility of consumption growth volatility (Bollerslev, Tauchen, and Zhou, 2009):
  - Fully conditionally log-normal models **can not simultaneously generate positive variance risk premium and equity premium**: for risk-neutral variance to be above physical variance, covariance between returns and pricing kernel must be positive, but then equity premium becomes negative
- Long-run risks with volatility jumps (Drechsler and Yaron, 2011):
  - Consumption growth is conditionally log-normal $\Rightarrow$ can not generate consumption growth shifts conditional on variance risk premium

Extant consumption-based models 2/2

- Rare disasters (Wachter, 2013):
  - Left tail of risk-neutral return distribution is much heavier than right tail $\Rightarrow$ **risk neutral entropy is much larger than risk-neutral variance**
  - Reducing size/probability of disasters helps, but then model has problems generating realistic equity premium
- Non-Gaussian habit (Bekaert and Engstrom, 2017):
  - Risk neutral entropy is almost equal to the risk-neutral variance
  - Very non-tractable: difficult to evaluate fit under alternative parametrisation

Consumption growth process

- Consumption growth has constant mean and heteroskedastic shock: $g_{t+1} = \bar{g} + \epsilon^{g}_{t+1}$
- Dividend shock=levered consumption shock: $d_{t+1} = \bar{g} + \gamma_g(\sigma_{cp}\omega_{p,t+1} - \sigma_{cn}\omega_{n,t+1})$
- Shock modeled using Bad Environment-Good Environment (BEGE) structure (Bekaert and Engstrom, 2017) - component models of two 0-mean shocks:

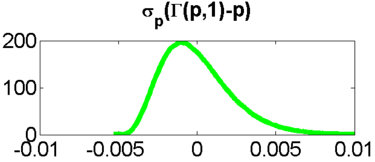
$$\epsilon^{g}_{t+1} = \underbrace{\sigma_{cp}}_{>0} \cdot \underbrace{\omega_{p,t+1}}_{\text{good shock}} - \underbrace{\sigma_{cn}}_{>0} \cdot \underbrace{\omega_{n,t+1}}_{\text{bad shock}}$$

- Good and bad shocks follow demeaned gamma distributions:

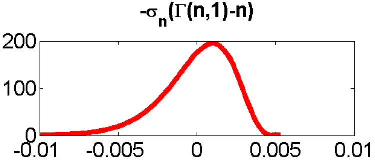
$$\left.\begin{array}{l} \omega_{p,t+1} \sim \Gamma(p_t, 1) - p_t, \\ \omega_{n,t+1} \sim \Gamma(n_t, 1) - n_t. \end{array}\right\} \Gamma(x, y) - \text{gamma distribution with shape parameter x, scale parameter y}$$

Bad Environment-Good Environment structure: Probability density function

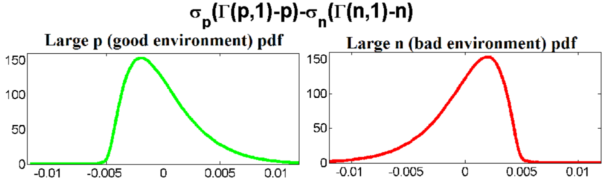
Good component pdf:



Bad component pdf:



Sum pdf:



Time-varying variances

- Shape parameters can be interpreted as variances:
    - p_t - **"good"** variance
    - n_t - **"bad"** variance
- Shape parameter driven by level shock (Gourieroux and Jasiak, 2006):

$$n_{t+1} = \bar{n} + \rho_n(n_t - \bar{n}) + \sigma_{nn}\omega_{n,t+1}$$

- For parsimony, p_t is constant: only bad shock distribution is time-varying

Utility function

- External habit utility: $E_t \sum_{j=t}^{\infty} \beta^{j-t} \frac{(C_j - H_j)^{1-\gamma} - 1}{1-\gamma}$
- β - discount rate, C_j - consumption, H_j habit stock ($C_j > H_j$)
- Inverse consumption surplus ratio, $Q_t = \frac{C_t}{C_t - H_t}$ ($q_t = \ln(Q_t)$), driven by consumption shocks: $q_{t+1} = \bar{q} + \rho_q(q_t - \bar{q}) + \underbrace{\sigma_{qp}}_{>0} \omega_{p,t+1} + \underbrace{\sigma_{qn}}_{<0} \omega_{n,t+1}$
- Log-stochastic discount factor: $m_{t+1} = m_0 + m_q q_t + \underbrace{m_{\omega,p}}_{<0} \omega_{p,t+1} + \underbrace{m_{\omega,n}}_{>0} \omega_{n,t+1}$
- Compared to Campbell and Cochrane (1999) and Bekaert and Engstrom (2017):
    - constant prices of risk
    - economically intuitive closed-form solutions: for example, variance risk premium$= \underbrace{r_p}_{<0} \cdot p_t + \underbrace{r_n}_{>0} \cdot n_t$

Data and Estimation

- US monthly data 1990:M1-2017:M12
- Classical minimum distance estimation - match unconditional moments of:
  - **Consumption growth**: mean, variance, skewness, kurtosis
  - **Risk-free rate**: mean, variance, autocorrelation
  - **Equity**: average equity premium, physical return variance, mean log-price-dividend ratio, log-price-dividend ratio variance, log-price-dividend ratio autocorrelation
  - **Options**: mean variance risk premium, variance risk premium variance, difference between risk-neutral entropy and variance

Parameter estimates

Preferences						
β	γ	$\bar{q}$	ρ_q	σ_{qp}	σ_{qn}	
1.0000	1.9870	1.0000	0.9904	$-2.64 \cdot 10^{-5}$	0.1140	
(fixed)	(0.5972)	(fixed)	(0.0121)	(0.0011)	(0.0327)	
Macro dynamics						
$\bar{g}$	σ_{cp}	σ_{cn}	$\bar{p}$	$\bar{n}$	ρ_n	σ_{nn}
0.0017	0.0007	0.0035	11.0848	0.0621	0.9954	0.0327
(0.0002)	(0.0002)	(0.0005)	(4.8705)	(0.0211)	(0.0164)	(0.0159)

standard errors in parentheses

- Good shock essentially Gauassian
- Bad shock very non-Gaussian

Key moments fit

Moment	Model	Data	Data standard error
Consumption growth			
Mean	0.0017	0.0020	0.0002
Standard deviation	0.0024	0.0024	0.0002
Skewness	0.1170	0.1163	0.3141
Kurtosis	2.0166	2.0186	0.7741
Equity			
Equity premium	0.0020	0.0041	0.0023
Physical standard deviation of equity return	0.0462	0.0426	0.0039
Options			
Variance risk premium	0.0015	0.0016	0.0003
Variance risk premium standard deviation	0.0020	0.0019	0.0003
Risk-neutral entropy - risk-neutral variance	0.0007	0.0006	0.0001

Consumption growth percentiles conditional on variance risk premium

Panel A: US data 1990M1-2017M2			
	High variance premium	Low variance premium	High-Low difference
10^{th} percentile	-0.23%	-0.02%	-0.21%***
			(0.07%)
50^{th} percentile	0.18%	0.20%	-0.02%
			(0.05%)
90^{th} percentile	0.50%	0.50%	0.00%
			(0.07%)
Panel B: Model			
	High variance premium	Low variance premium	High-Low difference
10^{th} percentile	-0.17%	-0.10%	-0.07%
50^{th} percentile	0.16%	0.15%	0.01%
90^{th} percentile	0.50%	0.48%	0.02%

bootstrap standard errors in parentheses
high variance risk premium=above 80^{th} unconditional percentile
low variance risk premium=below 20^{th} unconditional percentile

Limitations

- Monthly variance risk premium autocorrelation is 0.99 vs 0.52 (standard error 0.09) in data
- Problem in all consumption-based models: state variables driving asset prices are very persistent, so that realistically small shocks to state variables generate realistic asset pricing implications
- Can be resolved by adding a preference shock with less persistent variance (ρ_s) to inverse surplus ratio:

$$q_{t+1} = \bar{q} + \rho_q(q_t - \bar{q}) + \sigma_{qp}\omega_{p,t+1} + \sigma_{qn}\omega_{n,t+1} + \sigma_{qq}\omega_{q,t+1},$$
$$\omega_{q,t+1} \sim \Gamma(s_t, 1) - s_t,$$
$$s_{t+1} = \bar{s} + \rho_s(s_t - \bar{s}) + \sigma_{sq}\omega_{q,t+1}$$

Conclusions

- Use variance risk premium properties to discipline and refute existing consumption-based asset pricing models
- Extant models struggle
- Introduce tractable non-Gaussian habit model which does well