Problem 1: Prove that the set $\mathbb{Q} \setminus \mathbb{Z}$ is dense in $\mathbb{R}$.

Problem 2: Let $\{a_n\}, \{b_n\}, \{c_n\}$ be three sequences of real numbers such that $a_n \leq b_n \leq c_n$ for all $n \in \mathbb{N}$. Suppose that $\{a_n\}$ and $\{c_n\}$ are convergent and that they have the same limit, $\ell$. Prove that $\{b_n\}$ is convergent with limit $\ell$ as well.

Problem 3: Prove that the function $f(x) = \frac{1}{1+x}$ is uniformly continuous on the domain $(0, \infty)$ but not on the domain $(-1, \infty)$.

Problem 4: Prove that if $f : (0, 1) \to \mathbb{R}$ is uniformly continuous, then $\lim_{x \to 0} f(x)$ exists.