

EXISTENCE AND REFERENCE IN MEDIEVAL LOGIC

1. Introduction: Existential Assumptions in Modern vs. Medieval Logic

“The expression ‘free logic’ is an abbreviation for the phrase ‘free of existence assumptions with respect to its terms, general and *singular*’.”¹ Classical quantification theory is not a free logic in this sense, as its standard formulations commonly assume that every singular term in every model is assigned a referent, an element of the universe of discourse. Indeed, since singular terms include not only singular constants, but also variables², standard quantification theory may be regarded as involving even the assumption of the existence of the values of its variables, in accordance with Quine’s famous *dictum*: “to be is to be the value of a variable”.³

But according to some modern interpretations of Aristotelian syllogistic, Aristotle’s theory would involve an even stronger existential assumption, not shared by quantification theory, namely, the assumption of the non-emptiness of *common* terms.⁴ Indeed, the need for such an assumption seems to be supported not only by a number of syllogistic forms, which without this assumption appear to be invalid, but also by the doctrine of Aristotle’s *De Interpretatione* concerning the logical relationships among categorical propositions, commonly summarized in the *Square of Opposition*.

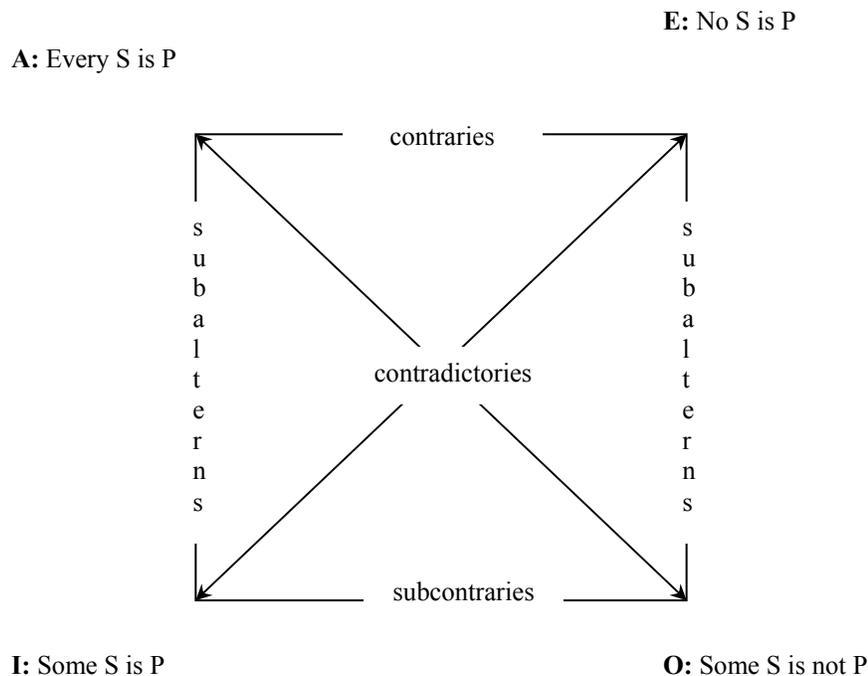
For example, Aristotle’s theory states that universal affirmative propositions imply particular affirmative propositions. But if we formalize such propositions in quantification theory, we get formulae between which the corresponding implication does not hold. So, evidently, there is some discrepancy between the ways Aristotle and quantification theory interpret these categoricals. One possible suggestion concerning the nature of this discrepancy could indeed be that Aristotelian logic contained the (tacit) existential assumption as to the non-emptiness of common terms. On the other hand, we know that this suggestion would definitely be rejected by the most ardent followers and expounders of Aristotle’s logic, namely, medieval logicians.⁵ Indeed, they would reject not only this suggestion (and for good reasons, too)⁶, but also the existential assumptions of standard quantification theory mentioned above.

In this paper I am going to give a brief, primarily systematic (as opposed to primarily historical) account of how it was possible for medieval logicians to maintain Aristotle’s theory of the four categoricals and to dispense with these existential assumptions in the framework of their theory of reference, the *theory of supposition*.⁷ Besides the informal account I will also indicate how these informal ideas can be put to work in a formal semantic system.⁸ By this I hope to show that the ideas of medieval logicians can provide us with valuable insights even in the apparently modern field of free logic.

2. Supposition Theory and the Square of Opposition

The main reason why medieval logicians did not need any extra (tacit or otherwise) assumptions “to save” the logical relationships of the *Square of Opposition* is that these logical relationships are direct consequences of their semantic analysis of the four types of categorical propositions. According to this analysis, in contrast with the analysis of quantification theory, the subject terms of these propositions have a referring function: they stand for (*supponunt pro*) the particulars falling under them, provided there are any such particulars. If, however, these subject terms are “empty”⁹, then they simply refer to nothing (*pro nullo supponunt*). But then the affirmative categoricals are false, whence their contradictories are true, that is to say, affirmative categoricals have existential import, while negative ones do not, which “automatically” yields the relationships of the *Square*. Namely, if A: universal affirmative; E: universal negative; I: particular affirmative; O: particular negative, then $A \Rightarrow I$; $E \Rightarrow O$; $A \Leftrightarrow \sim O$; $E \Leftrightarrow \sim I$; $A \Rightarrow \sim E$; $\sim I \Rightarrow O$.

Notice that from $A \Rightarrow I$, $E \Leftrightarrow \sim I$ and $A \Leftrightarrow \sim O$ the remaining are derivable, that is, if A is assigned existential import and the diametrically opposed propositions are construed as contradictories, then all relationships of the *Square* are valid.



There can be basically two types of justification for attributing this kind of existential import to affirmative categoricals (regardless of their quantity, i.e., whether they are singular, indefinite, universal or particular), on the basis of the two fundamental types of predication theories endorsed by various medieval authors.¹⁰

According to what historians of medieval logic dubbed the *inherence theory of predication*, an affirmative categorical proposition (in the present tense with no

ampliation, cf. n. 9 above) is true only if an individualized property (form, or nature) signified by the predicate term actually inheres in the thing(s) referred to by the subject term. So, for example, the proposition ‘Socrates is wise’ is true if and only if wisdom actually inheres in Socrates, that is, Socrates has wisdom, or, which is the same, Socrates’ wisdom actually exists. But of course Socrates’ wisdom, or for that matter any other inherent property of Socrates, can actually exist only if Socrates himself exists. So, it follows that if Socrates does not exist, then the proposition ‘Socrates is wise’ is false, and so are all affirmative categoricals the predicate term of which signifies some inherent property of Socrates.

On the other hand, according to the other basic type of medieval predication theories, the so-called *identity theory*, an affirmative categorical proposition is true only if its subject and predicate terms refer to the same thing or things. For example, on this analysis ‘Socrates is wise’ is true if and only if Socrates, the referent of ‘Socrates’, is one of the wise persons, the referents of the term ‘wise’. If any of the two terms of an affirmative categorical is “empty”, then the term in question refers to nothing. But then, since “nothing is identical with or diverse from a non-being”, as Buridan (the “arch-identity-theorist” of the 14th century) put it, “every affirmative proposition whose subject or predicate refers to nothing is false”.¹¹

In any case, as we can see, from the point of view of the doctrine of the *Square* it does not matter which predication theory a medieval author endorsed, as both of these theories imply that affirmative categoricals in general, including universal affirmatives, have existential import.¹²

3. Two Objections to the Medieval Analysis

Anyone trained in the modern Frege-Russell tradition in logic may have at least two immediate misgivings concerning attributing existential import to all affirmative categoricals, regardless of further philosophical worries concerning the above-mentioned theories of predication.¹³

First, if universal affirmatives have existential import, then their contradictories must be true when their subject terms are empty. But the contradictory of, say, ‘Every winged horse is a horse’ is ‘Some winged horse is not a horse’. The latter, however, cannot be true, both because it is contradictory and because it implies the existence of winged horses, while there are no winged horses.

Second, this position seems to undermine the very idea of the affirmation of universal laws concerning hypothetical, never actualized situations. For example, Newton’s law of inertia, referring as it does to bodies not acted upon by external forces, would not be true on this analysis as a categorical statement.

Of course, our medieval colleagues were quite aware of these possible objections, and worked out their theories accordingly.

4.1. Reply #1: Reference and Negation

The first type of objection was easily dismissed by a distinction between *negating* (what we would call *propositional* or *external*) and *infinitizing* (what we would call *term-* or *internal*) negation.¹⁴ To use Russell's famous example, the intended contrast is between

- [1] The King of France is not bald
[\Leftrightarrow It is not true that the King of France is bald]

which is true, because France presently has no king, and so it is not the case that the King of France is bald [*negating negation*], and

- [2] The King of France is non-bald

which is true when the King of France is a non-bald person, i.e., a person who is both King of France and has hair, whence the proposition is actually false, precisely because there is no such a person [*infinitizing negation*].

The regimented Latin syntax of medieval logic could systematically express this distinction by placing the negation [*'non'*] either before [*negating negation*] or after [*infinitizing negation*] the copula, yielding

- [1_L] *Rex Franciae non est calvus*

and

- [2_L] *Rex Franciae est non calvus*

respectively.¹⁵

Of course, anyone familiar with Russell's treatment of this example would recognize the distinction between the scopes of the negation in [1] and [2] (or, sticking with Russell's original terminology, the distinction between *primary* and *secondary* occurrences of the description¹⁶), but they would reject in the same breath that this scope-distinction has anything to do with the strange claim concerning the truth of

- [3] Some winged horse is not a horse

implied by the medieval analysis. After all, Russell's distinction is based on the elimination of the *merely apparent reference* to the King of France in both [1] and [2] by paraphrases in which there is not even an appearance of such a reference. This is immediately evident if we consider the corresponding formulae of quantification theory:

- [1'] $\sim(\exists y)(Ky \ \& \ \forall x(Kx \supset x = y) \ \& \ By)$

- [2'] $(\exists y)(Ky \ \& \ \forall x(Kx \supset x = y) \ \& \ \sim By)$

In these formulae (where '*K*' represents '*... is present King of France*', and '*B*' represents '*... is bald*') there is not even a trace of the apparent referring phrase 'the King of France', and this is why there is not even an apparent reference here to a person who is presently King of France. So Russell's distinction boils down to the difference in the position of the negation in the logical form of [1] and [2], whereas in the case of [3] no such distinction seems to make sense. Indeed, [3] can be formalized *only in one way* with respect to the position of negation in it, namely:

[4'] $(\exists x)(Wx \ \& \ \sim Hx)$

(where '*W*' represents '... is a winged horse' and '*H*' represents '... is a horse', and the reason for the apparently inconsistent numbering of this formula should become clear soon). Placing the negation anywhere else in this formula would obviously yield a formula that is not a formalization of [3], but either of 'No winged horse is a horse' or of 'Something that is not a winged horse is a horse', which are clearly different from [3]. But then [3] evidently does not contain the kind of scope-ambiguity that sentences with definite descriptions do, which Russell's distinction is intended to handle.

However, if we recall what I said in the second section above about the medieval analysis of the categoricals, we can easily see what connects the cases of sentences with definite descriptions and [3]. According to this analysis *all* categorical propositions are instances of the following scheme, regardless of their quantity:

[Cat] [neg] [Q] S [neg] cop P

where bracketed parts of speech are optional, [neg] stands for negation (possibly even iterated), [Q] stands for some *signum quantitatis*, i.e., some determiner, cop stands for a copula (in any tense) and *S* and *P* stand for the (possibly *very* complex) subject and predicate term¹⁷, respectively. Accordingly, if we regard the definite article as one possible substitution in this scheme (to complicate matters, one that does not exist in Latin), we can easily see the required analogy:

[Cat]* [neg] [a/the/some/every/...] S [neg] cop P

where the English determiners in the place of [Q] are going to determine that, completing the scheme with appropriate English parts of speech, the resulting categorical sentences are going to be indefinite, definite, particular and universal,¹⁸ respectively. But then, the relative scope-relations concerning negation and the definite article in this scheme will apply to the other determiners occurring in this scheme as well. Hence, [Cat]* may be completed, for example, as

[C1] [neg] [a/the/some/every/...] winged horse is not a horse

or as

[C2] [neg] [a/the/some/every/...] winged horse is a non-horse

where the further rule is that the initial [neg] may be replaced by the phrase: 'It is not the case that', yielding the contradictory of the sentence to which it is prefixed. But then it is clear that no concrete instances of [C1] and [C2] are going to be equivalent, and that, in particular, there is a clear difference between

[3] Some winged horse is not a horse
and

[4] Some winged horse is a non-horse

To express the intuitive difference between [3] and [4] (matching that between [1] and [2]) in the logical forms of these sentences, we have to expand the language of standard quantification theory with *restricted variables*, representing general terms in their referring function, as they occur in the subject-positions of these sentences.¹⁹ A restricted

variable is a variable formed from an open sentence, which takes its values from the extension of the open sentence, if this extension is not the empty set, while it takes a zero-entity as its value otherwise. For example, let ' $x.Wx$ ' be the restricted variable formed from the open sentence ' Wx ', so that, for all value assignments, it takes an element of the extension of this open sentence in a model in which this extension is not empty, while it takes 0 as its value in all those models in which this extension is empty, where the only requirement concerning 0 is that it is not an element of the universe of discourse of that model. Also, to reflect the difference between 'is not a horse' and 'is a non-horse' let me introduce a term-negation of a predicate parameter simply by bracketing a negation sign together with the predicate parameter, along with the semantic rule that the extension of the resulting negated predicate will be the complement of the original relative to the universe of discourse. So, for example, if the extension of ' H ' in a model is some subset A of the universe of discourse U of that model, then the extension of ' $[\sim H]$ ' in that model is going to be U-A.

But then [3] may be formalized as

$$[3''] \quad (\exists x.Wx)\sim(H(x.Wx))$$

while [4] will become

$$[4''] \quad (\exists x.Wx)([\sim H](x.Wx))$$

which, in view of the above-sketched semantic rules will obviously have a different import than [3'']. Indeed, in a model in which the extension of ' Wx ' is empty (representing the actual situation, i.e., that there are no winged horses) it is easy to see that [3''] is true, while [4''] is false, in perfect parallelism with [1] and [2], which, using ι as the definite descriptor, now may be re-formalized as

$$[1''] \quad (\iota x.Kx)\sim(B(x.Kx))$$

and

$$[2''] \quad (\iota x.Kx)([\sim B](x.Kx)).$$

In a complete semantics for such and similar formulae (see *Appendix*) it is easy to see the equivalence between these and the Russellian formulae [1'] and [2'], and also between [3''] and [4''] and

$$[3'] \quad \sim(\exists x)(Wx) \vee (\exists x)(Wx \& \sim Hx)$$

and

$$[4'] \quad (\exists x)(Wx \& \sim Hx)$$

respectively. However, the advantage of the formulations with restricted variables lies in the fact that they reveal the structural analogy between Russell's distinction *concerning only sentences with definite descriptions* and the medieval distinction *concerning all types of categorical propositions*.

But then to the original objection we can say that it fails to distinguish between [3] and [4]. For the objection draws its conclusions from 'Some winged horse is not a horse' using it in the sense of [4] (i.e., as having the logical form [4''], i.e., [4']). However, in

that sense ‘Some winged horse is not a horse’ is not the contradictory of ‘Every winged horse is a horse’ (analyzed, of course, as another instance of the same scheme *[Cat]*). So we can hold without any absurdity that, precisely because there are no winged horses, ‘Every winged horse is a horse’ is false, and that its contradictory, [3] (having the logical form [3’], i.e., [3’]) is true. Of course, if anyone still feels that this analysis is in conflict with their linguistic intuition in that ‘Some winged horse is not a horse’ according to that intuition has to carry existential import, they may always use this sentence in the sense of [4], but then they would have to distinguish between ‘Some winged horse is not a horse’ and the contradictory of ‘Every winged horse is a horse’, i.e., ‘It is not the case that every winged horse is a horse’. Actually, this was the course taken by Abelard in the 12th century, but later medievals rather settled on not attributing existential import to particular negatives, treating them as the genuine contradictories of universal affirmatives.²⁰

In any case, as from the general scheme above it should be clear, the great advantage of this type of analysis is that it provides us with a uniform, systematic account of relative scope relations of negation and all sorts of determiners in categorical propositions. So, the formulae with restricted variables immediately point us in the direction of formulating a generalized quantification theory, i.e., one in which for any replacement of *[Q]* we can easily construct the corresponding formulae giving the correct true conditions for the resulting sentences, some of which will demonstrably have no equivalents in classical quantification theory (those formulated with the determiner ‘most’, for example). But instead of going into the details of formulating such a generalized quantification theory, let us see how the other objection can be handled in the medieval framework.²¹

4.2. Reply #2: Habitual Predication, Natural Supposition and Ampliation

The second objection received two different types of answer in medieval logic, both of which introduced further distinctions concerning the possible interpretations of categorical propositions. The one distinguished between modes of predication in a categorical proposition, while the other attributed a different type of reference to the subject terms of law-like statements.

According to the first type of answer, the predication expressed by the copula of an affirmative categorical proposition may be interpreted not only *actually*, or *according to real existence*, when it requires the existence of what it is about, but also *absolutely*, or *habitually*²², in which case the categorical proposition is equivalent to a hypothetical proposition, much in the way quantification theory analyzes *all* universal categoricals. So, accordingly, if the sentence

[5] Every winged horse is a horse

is put forth with the force of a universal truth, *absolutely* or *habitually*, concerning the analytic conceptual connection between the subject and predicate of this sentence regardless of the actual instantiation of its subject, then it may be handled as having the form:

[5’] $(\forall x)(Wx \rightarrow Hx)$

which is indeed going to be necessarily true in a formal semantics in which we stipulate that the extension of ‘*W*’ is a part of the extension of ‘*H*’ in every model, while it may well be the case that the extension of ‘*W*’ is empty in some models. On the other hand, as in the medieval framework this is not *the only* available analysis of a universal affirmative proposition, we are not stuck with the truth of such a claim when it is put forth with the force of a statement of fact, neither is the hypothetical analysis going to undermine the validity of the *Square of Opposition*, which concerns categorical propositions put forth with categorical force, having the form

$$[5''] \quad (\forall x.Wx) (Hx.) \quad [\Leftrightarrow (\exists x)(Wx) \& (\forall x)(Wx \rightarrow Hx)]$$

But more pertinent to our present concern with existence and reference was the second type of answer, according to which even law-like statements could be analyzed as *necessarily true categoricals* put forth with *categorical force*, despite the *actual* “emptiness” of their subjects, while the implication $A \Rightarrow I$ could still be regarded as valid.²³ The key to the possibility of this position is assigning the subject terms of such statements a different type of reference, commonly called *natural supposition* by medieval logicians.

Although natural supposition has an interesting early history²⁴, in this brief exposition I am going to rely on the account of Jean Buridan, who explicitly defends this type of reference (against those of his contemporaries who did not admit it) on the basis of what he recognizes as its use in science.²⁵

“Furthermore, fourthly, also the demonstrative sciences use this sort of supposition. For if we say in [a commentary on] the *Meteorologica* that every thunder is a sound made in the clouds, or that every rainbow is a reflection or refraction of light, then we do not intend to say these things only concerning the present ones; indeed, even if there were no thunder or rainbow at the present time, we would nevertheless state the same things. And if a geometer has a demonstration that every triangle has three angles equal to two right angles, we should not imagine that thereby he would have knowledge only of those triangles that actually exist, on the contrary, if this habitual knowledge of his remains for three years, and meanwhile many triangles are generated, he will have knowledge of those as well as of the others without a new demonstration. Aristotle explicitly states this in bk. 1. of the *Posterior Analytics*: ‘I call ‘[true] for all’ [‘*de omni*’] that which is not such that it holds for some and does not hold for another, nor such that sometimes it holds and sometimes it does not.’ [73a30-35] And he clarifies this by an example, saying: ‘just as ‘animal’ [is true] for all men, because if it is true to call someone a man, then it is true to call him an animal, and if the one is true now, then so is the other’. He also confirms this by a further evidence [*signum*], saying: ‘an evidence [*signum*] for this is that when we object to such a [true]-for-all-claim then we inquire whether it does not hold for some or sometimes’. Nevertheless, we can correctly say that in this case [putting forth] such a locution with this intention is not in accord with its proper meaning [*de proprietate sermonis*], but [it is put forth] for the sake of brevity. For when we want to speak demonstratively, and we say that every thunder is a sound made in the clouds, or that every lunar eclipse is due to the interposition of the Earth

between the Sun and the Moon, these propositions would not be true in virtue of their proper meaning [*de proprietate sermonis*], for the verb ‘is’ in virtue of its proper meaning was imposed to signify only the present time, while there may be no thunder or lunar eclipse at the present time. And so such propositions are put forward for the sake of brevity in place of ‘Every thunder, whenever it is, was, or will be is, was, or will be a sound made in the clouds’ and ‘Every lunar eclipse, whenever it is, was, or will be is, was, or will be due to the interposition of the Earth between the Sun and the Moon’. Such propositions therefore are not to be denied, since they are true as they are put forward, but they would be false if they were put forward and taken in their proper sense [*ad sensum proprium*]. And since sophists want to take propositions only in their proper sense [*secundum sensus proprios*], they do not use such supposition in the manner described in this case.”

In order to be able to appreciate Buridan’s point we have to know that *supposition*, or *reference*, was commonly regarded by medieval logicians as a property of terms *only in the context of a proposition*²⁶. Accordingly, on this view, the same term may refer in different ways to different things in different propositional contexts, or it may even refer to some thing(s) in one proposition, while it may refer to nothing at all in another. For a medieval logician it would not make much sense to say that the name ‘Bill Clinton’ has a referent while the term ‘King of France’²⁷ does not, without specifying a propositional context. For example, we can say that in the sentence ‘Bill Clinton is the President of the US’ the name ‘Bill Clinton’ refers to Bill Clinton, if this sentence is uttered in 1994, while the same name will refer to nothing, if the same sentence will be uttered, say, in 2194, when Clinton will not exist, and, similarly, it would have referred to nothing if the same sentence had been uttered in 1794, when Clinton did not exist. (To be sure, this case is not to be confused with the case when the name does have a referent, but the sentence is simply false, as the same sentence uttered, say, in 1990.) But, of course, in the sentence ‘Bill Clinton *was* the President of the US’, which *will be true* if uttered in 2194, the same name will refer to the same person, Bill Clinton, even if at the time of the utterance of this sentence the person referred to will not exist, and similarly in ‘Bill Clinton *will be* the President of the US’, which would have been a true sentence already in 1794, if it had been uttered then, by a soothsayer for example. Again, in ‘The King of France is bald’ the term ‘King of France’ actually refers to nothing, whence the sentence uttered now, in 1994, is false. But if the same sentence had been uttered in 844, for example, then it would have been true, as its subject would have referred to Charles the Bald, then King of France. Also, *now* we can truly say: ‘A King of France *was* bald’, precisely because France once had a bald king, who, among other past Kings of France, is referred to by the subject term of this sentence²⁸.

As can be seen, on this approach both singular and common terms are treated as referring phrases the actual reference of which is determined (besides their meaning, the speaker’s intention, and relevant circumstances of their formation and/or interpretation) by the propositional context in which they occur. Most notably, general terms in the context of a past tense proposition can refer to things to which they applied in the past (if there were any such things), which may be things that actually do not exist but existed, and similarly, in a future tense context these terms can refer to things to which they will apply

in the future (if there will be any such things), which may be things that do not exist now but will exist in the future. As medieval logicians put it, in these contexts the subject terms get *ampliated* (*ampliantur*), i.e., their range of reference extends beyond the domain of actually existing entities, and this is why affirmative categoricals about the past or the future may be true even if their subject terms are *actually* empty, i.e., even if in a present tense affirmation these subject terms apply to nothing. So, for example, the sentence ‘Every dinosaur *was* a reptile’ is true, because its subject term refers to things that either *are* or *were* dinosaurs, all of which were reptiles, even if there *is* nothing in the world *now* of which one could truly affirm that it *is* a dinosaur.²⁹

But such *ampliative contexts* were recognized also in present tense sentences. For example, in ‘Some man *is* dead’ the subject term has to refer to past men, i.e., things that *were* men, for certainly no actually existing human being is dead (*zombies* do not count as humans). In the same way, in ‘Every dinosaur is extinct’, ‘dinosaur’ has to refer to things that *were* dinosaurs, despite the fact that the copula of the sentence is in the present tense. Indeed, we can say that the *theory of ampliation* was designed to cover all contexts which modern logicians would recognize as *intensional contexts* in general. So, for example, *modal* and *intentional* verbs and their participles, as well as related adjectives and adverbs were also regarded as *ampliative*, and so terms were also regarded as *ampliated* in their contexts even in present tense sentences.

But then we can see why in this analysis of the categoricals acceptance of the subalternation $A \Rightarrow I$ does not necessarily mean attributing *existential* import, properly speaking, to universal affirmatives. For if their subject terms are *ampliated*, then neither the A nor the I propositions imply the *actual existence* of their referents, or *supposita*.

In view of these considerations, however, we can easily see why the *actual emptiness* of their subject terms will not falsify law-like categorical statements whose subject terms were interpreted as having *natural supposition*, i.e., a type of reference amplified to all times (or perhaps even to all [logical] possibilities), on account of the intention with which they were put forward, i.e., as concerning everything to which their subjects do or did or will [or perhaps only can or could]³⁰ apply. And this gives us the answer to the second objection above.

Now, clearly, if we construe general terms in their referring function as restricted variables along the lines sketched above, then accepting *ampliated* terms, i.e., terms referring to things that were, will be, or can be, but which actually do not exist means that here we have an analysis in which variables are allowed to range over *nonexistents*. So, again this medieval analysis takes us beyond the limitations of classical quantification theory, into the realm of free logics, in which we may, if we so choose, distinguish between existent and non-existent values of our variables.³¹

Technically, the simplest way to construct a formal semantics of *ampliated terms* is to combine the well-known techniques of possible worlds or intensional semantics with the idea of using restricted variables as representing general terms in their referring function. Instead of going into the technical details here, however, let me now turn to a brief, concluding discussion of the *philosophical significance* of the idea of a free logic we could glean from medieval logic.

5. Conclusion: Reference and Intentionality

The foregoing sketch of some of the main medieval ideas concerning reference and existential commitment is far from being a complete account of all the complexities of (the many varieties of) the theory of *supposition*. Still, I think even this sketchy account provides us with enough to reflect on the broader significance of the medieval approach. In view of the above discussion we can regard this approach as yielding a comprehensive and highly flexible system of many-sorted quantification in which the ranges of variables are determined differently in different types of propositional contexts.

Now this approach, aside from having the obvious advantage of “automatically” leading to a generalized quantification theory, also has the philosophical advantage that it is not going to leave us wondering about the “weird features” of “non-actualized entities” causing so much headache to philosophers exploring, gardening or uprooting (depending on their temper) the Meinongian Jungle. For on this approach we can refer to non-existents only in contexts in which no actual properties can be attributed to them. So when we are talking about them, we are not going to get the false impression of exploring *a different realm of entities*, where just *anything* can happen, well, even things that *cannot* happen. For, in the first place, by contradictory terms, such as ‘round square’, we cannot possibly refer to anything even in ampliative contexts, whence not only ‘A round square is round’ will be false, but also the modal propositions with amplified terms: ‘A round square is necessarily round’ and ‘A round square could be round’. Again, we are not going to be wondering about the mysteriously missing further properties of an “incomplete” merely possible entity, like a winged horse, for we are going to concede in the beginning that a winged horse is not winged, indeed, it is not even a horse, for it *is* nothing at all. Still, we might concede the truth of, say, ‘A winged horse *can be* pink’, in which we *refer to something that can be a winged horse*, which, after all, *can be* pink.

At this point, however, someone with a *strong* “taste for desert landscapes” might immediately decry this approach on the basis that despite its apparent capability of resolving many of the inconveniences of talking about entities in the Meinongian Jungle it *leaves them right where they are* and does nothing for *eliminating them*. For although now we cannot say anything truly about winged horses in a *non-ampliative* context, by conceding that we can *refer to something that can be a winged horse* we smuggle back winged horses into our universe of discourse. So, the medieval universe of discourse is as overpopulated as Meinong’s, and so the medieval approach does not provide any more peace of mind to a genuine nominalist than the Meinongian Jungle does.

Well, since it is hard to imagine any “more genuine” nominalists than, say, Ockham, Buridan, or Albert of Saxony, who did not have any qualms about referring to nonexistents in the appropriate contexts (while they would certainly not tolerate the “slum” of abstract entities endorsed by their contemporary counterparts in set theory), it is perhaps not entirely unjustified here to look at the bottom of the worries of our contemporary “nominalists”.

In his comprehensive analysis of these worries, William Lycan characterizes the basis for the incapability of our “nominalists” to stomach Meinong’s Jungle in the following manner:

“In particular, what I am implicitly demanding is a *model-theoretic* semantics, done entirely in terms of actual objects and their properties — for what else *is there really*? I am allowing the Meinongian his funny operator [an “existential” quantifier ranging over *possibilia* — G. K.] only on the condition that he explain it to me in non-Meinongian terms. To this the Meinongian may reply that he will be happy to give us a model-theoretic semantics — one whose domains include nonactual objects, true enough, but that is all right, since *there are* nonactual objects after all. And so it seems we have arrived at another impasse.”³²

In a way, the issue is trivial. The only difference between the Meinongian and the anti-Meinongian seems to be that while the anti-Meinongian regards the notion of *being, existence, reality, actuality*, etc., whichever words we use, as co-extensive with the range of *quantification* (whence he is more than willing to call a *particular* — as opposed to a universal or a singular — proposition *existentially* quantified), the Meinongian, on the contrary, views the notion of *actuality*, or *real existence*, etc., as covering only a part of the range of quantification, and hence he is prepared to make distinctions between items within that range that are *actual, really existing*, etc., and items that are not such, but which, still, somehow, *there are*. However, what seems to be the common assumption of both parties, in my view not shared by the medievals (at least the ones I am discussing here — see n. 33 below), is that for quantification and reference things somehow already *have to be there*, they must be somehow *given* in order to be referred to or to be quantified over. The only disagreement seems to be that when we come across an apparent instance of reference to or quantification over something that is agreed on all hands not to exist (*really and genuinely*), then the Meinongian happily admits this item in his bloated universe of discourse, i.e., within the range of his quantifiers, although outside the domain of real existents, while his opponent would try everything within his ken by which he could *analyze away* what he regards as an instance of a *merely apparent* reference to or quantification over something that does not exist, in terms of phrases in which his quantifiers will range only over admittedly existing things.

Now the medieval approach sketched here certainly has a greater superficial resemblance to the Meinongian than to the anti-Meinongian position. (And this is of course not without historical reasons. In fact, I suspect that Meinong himself was much closer to the medieval conception than contemporary American “Meinongians”, but I will not pursue this point here. My basis of comparison here are the “relentless Meinongians”, characterized as such by Lycan in his above-mentioned paper.) Nevertheless, the fundamental difference between both antagonistic modern positions and the medieval approach seems to be that from the point of view of the medieval approach discussed here it is just sheer nonsense to talk about non-actual entities somehow *being out there*, awaiting our reference to and quantification over them. Non-actual entities, mere *possibilia*, are *literally nothing* for this approach.³³ But then how can we say we can *refer to them* (in the appropriate, *ampliative contexts*, of course)?

The answer is that ‘refer’ (*supponit*), just as other *intentional verbs*, also creates an ampliative context, wherefore a term construed with it will also be amplified to things that possibly do not exist, but which did, will, or can exist. For *referring* is not something that words, i.e., inscriptions or utterances do *per se*, but it is *something that we humans*

do by means of our words. For words mean and refer to what *we mean and refer to* by them.³⁴ And we mean and refer to the things we *think of*. But of course we can think of whatever can be an object of our consciousness, which of course need not be an actually existing thing, as anyone who ever had dreams, memories, fantasies, wishes, expectations, let alone abstract, universal thoughts, can easily attest.

As Buridan put it in his questions-commentary on Aristotle's *De Interpretatione*:

“... a name signifies what is understood by it when it is put in an expression, for to signify is to give rise to some understanding of a thing [*intellectum rei constituere*]. But by the name ‘rose’ we understand a rose and by the name ‘roses’ we understand roses. For example, [suppose] last year we, you and I, saw many red roses together. If I ask you: ‘The roses we saw were red, weren’t they?’, then you say: ‘Indeed’. And this you know to be true. But you wouldn’t know this, unless you thought of those roses. Therefore, by the name ‘roses’, when I say ‘We saw roses’, you understand those things that we saw. But we saw red roses. So you think of roses. [...] the name ‘rose’ refers to [*supponit pro*] roses, although nothing is a rose, for according to the above-mentioned case, namely, that last year we saw many red roses, you concede the proposition ‘There were many red roses last year’, and you know that this is true. And since this is an affirmative [proposition], it would not be true, unless its subject, which is the name ‘roses’, referred to some thing or some things. But it does not refer to [any] other thing or other things, but roses. [...] we should note that we can think of things without any difference of time and think of past or future things as well as present ones. And for this reason we can also impose words to signify without any difference of time. For this is the way names [as opposed to verbs — G. K.] signify. Therefore, by the specific concept of ‘man’ I conceive indifferently all men, present, past and future. And by the name ‘man’ all [men] are signified indifferently, present, past and future [ones alike]. So we truly say that every man who was was an animal, and every man who will be will be an animal. And for this reason it follows that the [verbs] ‘think/understand’ [*intelligere*], ‘know’, ‘mean/signify’ [*significare*] and the like, and the participles deriving from them, *ampliate* the terms with which they are construed to refer indifferently to present, past and future and possible [things] which perhaps neither are, nor will be, nor ever were. Therefore, even if no rose exists, I think of a rose, not one that is, but one which was, or will be, or can be. And then, when it is said: the name ‘rose’ signifies something, I concede it. And when you say: that [thing] is not, I concede that; but it was. If, then, you conclude: therefore, something is nothing, I deny the consequence, for in the major premise the term ‘something’ was amplified to past and future [things], and in the conclusion it is restricted to present ones.³⁵”

But then, from this point of view, there should be nothing mysterious about objects of reference (and hence of quantification, in appropriate contexts), i.e., objects of human thought, which do not exist. At least, there are no separate *ontological* mysteries here, other than those involved in the nature and workings of the human mind. But, again, from the point of view of the theory of reference, all we need is the recognition of such, mysterious or not, but certainly familiar, simple facts of human existence that we refer by

our words to what we mean by them, and we mean by them things we think of when we use these words in communication, and that we certainly can think of things other than just those that are actually present in our broader or narrower physical environment. Indeed, the recognition of such simple facts about the relationship between human thought and reference will even yield such further useful conceptual tools in semantic theory as Buridan's distinction between *suppositio propria vs. impropria*, rediscovered in Kripke's distinction between *linguistic reference vs. speaker's reference*,³⁶ or Buridan's theory of *appellatio rationis* for handling the problem of reference in intentional contexts, still awaiting rediscovery by contemporary semanticists³⁷.

In any case, whether the particular answers provided by medieval philosophers to questions of the field are *the right ones* or not, I hope it is clear even from this sketchy overview that the greatest merit of the medieval approach to these questions in general is its placing the theory of reference in the framework of a comprehensive philosophical theory of mind, language and communication, based on a firm metaphysical view of human nature. If none else, at least this general idea of the medieval approach can certainly be regarded as pointing us in a promising direction also in our contemporary research in the field.

Appendix

The Syntax and Semantics of a Theory of Ampliation

Syntax

The language AMPL* is defined as follows³⁸:

$$\text{AMPL}^* := \langle C, P, V, T, F \rangle,$$

where $C := \{\sim, \&, =, Q, \exists, \mathbf{E}, \iota, ., \alpha, (,), [,]\}$, P is a set of parameters, V is a set of *proper variables*, as opposed to the set of *restricted variables*, V_{res} , a subset of T , the set of terms, and F is the set of formulae of AMPL*. P detailed: $P := P_{\text{ind}} \cup P_{\text{pred}}$, where P_{ind} is the set individual parameters, while P_{pred} is the set of predicate parameters of AMPL*. *Note*: $P_{\text{pred}} \cap C := \{\mathbf{E}, =\}$. (That is, \mathbf{E} , the existence-predicate of AMPL*, is a distinguished predicate in the same way as identity is.) Q is a “generalized” quantifier, i.e., one that may represent various different natural language determiners depending on its actual intended interpretation. \exists is the familiar existential quantifier with its usual interpretation. ι is the *descriptor*, and α is the *ampliator* of AMPL*, which is used here in place of the familiar tense operators as well as the possibility operator (the intended intuitive interpretation may be indicated by subscripts to α).

The sets of terms and formulae of AMPL* are defined by the following simultaneous recursive definition:

- (1) If $a \in P_{\text{ind}}$, then $a \in T$
- (2) If $x \in V$, then $x \in \text{Var}$, where $\text{Var} := V \cup V_{\text{res}}$
- (3) If $x \in V$ and $A \in F$, then $'x.A' \in V_{\text{res}}$
- (4) If $t_1, \dots, t_n \in T$ and $P^n \in P_{\text{pred}}$, then $'P^n(t_1) \dots (t_n)' \in F$, $'[\sim P^n](t_1) \dots (t_n)' \in F$, $'\mathbf{E}(t_i)' \in F$ and $'(t_i = t_j)' \in F$
- (5) If $A, B \in F$ and $v \in \text{Var}$, then $'\sim(A)' \in F$, $'\alpha(A)' \in F$, $'(Qv)(A)' \in F$, $'(\exists v)(A)' \in F$, $'(\iota v)(A)' \in F$ and $'(A \& B)' \in F$

For the sake of convenience we may apply the following abbreviations:

- (Abbr1) The matrix of a restricted variable may be omitted in all of its occurrences following its first occurrence in a formula, provided different restricted variables have different operator variables.
- (Abbr2) Further connectives are to be regarded as abbreviations of their usual definients.
- (Abbr3) When from their omission no confusion arises parentheses may be omitted.

Semantics

The definition of a model for AMPL* is the following:

$$M := \langle W, S, a_s, D, R, 0 \rangle$$

where W and S are nonempty sets, a_s is a distinguished element of S , D is a function from S to the set of all subsets of W , i.e., if $s \in S$, then $D(s) \subseteq W$, R is a function assigning

semantic values to the parameters of AMPL* and 0 is the zero-entity, the semantic value of empty terms, which falls outside W , i.e., $0 \notin W$.

Intuitively, W is the *universe of discourse* of M , S is a set of *situations*, or states of affairs, which may be thought of as past, present or possible with respect to the *actual situation* a_s (in accordance with the intended intuitive interpretation of α), and D is the *domain assignment of situations*, so that $D(s)$ is *the domain of the situation* s .

R is defined by the following clauses:

- (R1) If $a \in P_{\text{ind}}$, then $R(a) \in W$
- (R2) If $P^n \in P_{\text{pred}}$, then $R(P^n)(s) \subseteq W^n$
- (R3) $R(\mathbf{E})(s) = W$

Let us define further *the extension of a predicate* P^n *in the situation* s , $\text{Ext}_s(P^n)$, in the following manner: $\text{Ext}_s(P^n) := R(P^n)(s) \cap D(s)^n$. (Whence, $\text{Ext}_s(\mathbf{E})=D(s)$.)

An *assignment in the situation* s , f_s (a function from $T \cup F$ to $W \cup \{1,0\}$)³⁹, is defined in the following manner (for the sake of simplicity, henceforth I omit quasi-quotes):

- (f_s 1) If $x \in V$, then $f_s(x) \in W$
- (f_s 2) If $a \in P_{\text{ind}}$, then $f_s(a) = R(a)$ (i.e., individual parameters are “rigid designators”)
- (f_s 3) If $v.A \in V_{\text{res}}$, then
 - (i) $f_s(v.A) = f_s(v)$, if $f_s(A) = 1$
 - (ii) $f_s(v.A) = 0$ otherwise
- (f_s 4) $f_s(P^n(t_1) \dots (t_n)) = 1$ iff $\langle f_s(t_1), \dots, f_s(t_n) \rangle \in \text{Ext}_s(P^n)$
- (f_s 5) $f_s([\sim P^n](t_1) \dots (t_n)) = 1$ iff $\langle f_s(t_1), \dots, f_s(t_n) \rangle \in D(s) - \text{Ext}_s(P^n)$
- (f_s 6) $f_s((t_1 = t_2)) = 1$ iff $f_s(t_1) = f_s(t_2) \in D(s)$
- (f_s 7) $f_s(\sim(A)) = 1$ iff $f_s(A) = 0$
- (f_s 8) $f_s((A \& B)) = 1$ iff $f_s(A) = f_s(B) = 1$
- (f_s 9) $f_s((\exists v)(A)) = 1$ iff for *some* $u \in \text{Rg}(v)(f_s)$, $f_s[v:u](A) = 1$; where *the range of* v *in respect of* f_s , $\text{Rg}(v)(f_s)$, is defined: $\text{Rg}(v)(f_s) = \{u \in W: \text{for some assignment } g_s \text{ differing from } f_s \text{ at most in the value assigned to } v, g_s(v) = u\}$, if this set is not empty, otherwise $\text{Rg}(v)(f_s) = \{0\}$; and $f_s[v:u]$ is the same as f_s except that it assigns u to v (i.e., $f_s[v:u](w) = f_s(w)$, if $v \neq w$, otherwise $f_s[v:u](w) = u$).
- (f_s 10) $f_s((\forall v)(A)) = 1$ iff for *the* $u \in \text{Rg}(v)(f_s)$, $f_s[v:u](A) = 1$, i.e., iff there is exactly one $u \in \text{Rg}(v)(f_s)$, such that $f_s[v:u](A) = 1$
- (f_s Q) $f_s((Qv)(A)) = 1$ iff for $Q'u \in \text{Rg}(v)(f_s)$, $f_s[v:u](A) = 1$, where Q' holds the place of any English determiner that is intended to be represented by Q
- (f_s 11) $f_s(\alpha(A)) = 1$ iff for some $s' \in S$, $f_{s'}(A) = 1$
- (f_s 12) $f_s(A) = 1$ iff $f_s(A) \neq 0$.

Truth in a model M is defined as follows:

- (T) $|A|_M = T$ iff for some f_{a_s} , $f_{a_s}(A) = 1$.

(That is to say, A is true in M , iff for some assignment in the actual situation, A is true according to, or is satisfied by, that assignment.)

As usual, a formula is *satisfiable* if there is a model in which it is true, *valid*, if its negation is not satisfiable, and *an inference is valid*, if the conjunction of the premises with the negation of the conclusion is not satisfiable.

In this system we can prove the following metatheorem:

(EQ) If $v \in V$ and $A \in F$, then for every model M ,

$$|(\exists v)(A)|_M = |\mathbf{E}(v.A)|_M \text{ iff } \text{EXT}(v, M, a_s)(A) = \emptyset \text{ or } \text{EXT}(v, M, a_s)(A) \cap D(a_s) \neq \emptyset$$

where $\text{EXT}(v, M, a_s)(A)$, *the extension of A in respect of v in M in the actual situation*, is defined as follows:

$$\text{EXT}(v, M, a_s)(A) := \{u \in W : \text{for some } f_{a_s}, f_{a_s}[v:u](A) = 1\}.$$

Proof

To simplify the proof I first prove two lemmas.

Lemma 1.

$$|(\exists v)(A)|_M = T \text{ iff } \text{EXT}(v, M, a_s)(A) \neq \emptyset$$

Proof of Lemma 1.

Suppose $\text{EXT}(v, M, a_s)(A) \neq \emptyset$. Then for some f_{a_s} , and for some $u \in W$, i.e., for some $u \in \text{Rgf}_{a_s}(v) \text{ --- since for any } v \in V \text{ and for any } f_s, \text{Rg}(v)(f_s) = W \text{ ---, } f_{a_s}[v:u](A) = 1$, and so, $|(\exists v)(A)|_M = T$; and conversely. *Q.e.d.*

Lemma 2.

$$|\mathbf{E}(v.A)|_M = T \text{ iff } \text{EXT}(v, M, a_s)(A) \cap D(a_s) \neq \emptyset$$

Proof of Lemma 2.

Suppose $\text{EXT}(v, M, a_s)(A) \cap D(a_s) \neq \emptyset$. Then for some f_{a_s} , and for some $u \in \text{Rgf}_{a_s}(v) \cap D(a_s)$, $f_{a_s}[v:u](A) = 1$, and so, $f_{a_s}(v.A) = u$; whence for some f_{a_s} , $f_{a_s}(v.A) \in D(a_s) = \text{R}(\mathbf{E})(a_s)$, that is, $|\mathbf{E}(v.A)|_M = T$; and conversely. *Q.e.d.*

Proof of metatheorem (EQ)

Suppose that (1) $\text{EXT}(v, M, a_s)(A) \neq \emptyset$ and (2) $\text{EXT}(v, M, a_s)(A) \cap D(a_s) = \emptyset$.

Then, by (1) and Lemma 1, $|(\exists v)(A)|_M = T$, while from (2) and Lemma 2 it follows that $|\mathbf{E}(v.A)|_M \neq T$.

On the other hand, suppose that

(1) $\text{EXT}(v, M, a_s)(A) = \emptyset$ or (2) $\text{EXT}(v, M, a_s)(A) \cap D(a_s) \neq \emptyset$.

Then, from (1) by Lemma 1 it follows that $|(\exists v)(A)|_M \neq T$. From (1) it also follows that $\text{EXT}(v, M, a_s)(A) \cap D(a_s) = \emptyset$; whence, by Lemma 2, it follows that $|\mathbf{E}(v.A)|_M \neq T$. But from (2) and Lemma 2 it follows that $|\mathbf{E}(v.A)|_M = T$, while from (2) it also follows that $\text{EXT}(v, M, a_s)(A) \neq \emptyset$, whence also $|(\exists v)(A)|_M = T$. And this completes the proof. \square

The significance of this metatheorem is that it shows the close connection, but without blurring the distinction between an existential statement, and an existential, or, perhaps better to say, *particular* quantification.

What metatheorem **(EQ)** states is that if the actual extension, $\text{EXT}(v, M, a_s)(A)$, of the open sentence involved in the quantification and in the restricted variable of the existential statement is not amplified to non-actual individuals, then these two forms of statement are equivalent, but if it *is* amplified to non-actual individuals and does not contain actual ones, then they are not equivalent. For example, in virtue of metatheorem **(EQ)** the following formulae are equivalent:

$$[6'] \quad (\exists x)(Cx)$$

$$[7'] \quad E(x.Cx) \\ [\Leftrightarrow (\exists x.Cx)E(x.)]$$

And this is how it should be. Clearly, the sentence

$$[6] \quad \text{Something is a centaur}$$

is equivalent to

$$[7] \quad \text{A centaur exists} \\ [\Leftrightarrow \text{Some[thing that is a] centaur exists}]$$

This is why we are entitled to use the types of statements represented by (6') and (7') interchangeably.

But let us take the following two sentences:

$$[8] \quad \text{Something is destroyed}^{40}$$

$$[9] \quad \text{Something that is destroyed exists}$$

These are clearly not equivalent. For if we define 'is destroyed' as 'existed and does not exist' then we get the true

$$[10] \quad \text{Something existed and does not exist}$$

$$[10'] \quad (\exists x)(\alpha_p(\mathbf{E}x) \ \& \ \sim(\mathbf{E}x))$$

and the false, indeed, inconsistent

$$[11] \quad \text{Something that existed and does not exist exists} \\ [\Leftrightarrow \text{A thing that existed and does not exist exists}]$$

$$[11'] \quad (\exists x. \alpha_p(\mathbf{E}x) \ \& \ \sim(\mathbf{E}x))(\mathbf{E}x.) \\ [\Leftrightarrow \mathbf{E}(x. \alpha_p(\mathbf{E}x) \ \& \ \sim(\mathbf{E}x))]$$

And this is why we cannot use [8] and [9] interchangeably.

*Department of Philosophy
Fordham University*

REFERENCES

- Abaelardus, Petrus (1956): *Dialectica*, ed. by Lambertus Marie De Rijk, Assen: Van Gorcum & Comp. N.V.–G.A.Hak & Dr. H. J. Prakke. 2nd, revised edition: Assen: Van Gorcum & Comp. N.V.–Dr. H.J. Prakke & H.M.G. Prakke, 1970.
- Albert of Saxony (1974): *Perutilis Logica*, Hildesheim-New York: Georg Olms Verlag.
- Ashworth, Earline Jennifer (1973): “Existential Assumptions in Late Medieval Logic”, in *American Philosophical Quarterly* **10**, 141–147.
- Ashworth, Earline Jennifer (1978): *The Tradition of Medieval Logic and Speculative Grammar*, Toronto: Pontifical Institute of Medieval Studies.
- Broadie, Alexander (1987): *Introduction to Medieval Logic*, Oxford: Oxford University Press.
- Buridan, John (1966): *Sophisms on Meaning and Truth*, transl. and intr. by Theodore Kermit Scott, New York: Appleton-Century-Crofts.
- Buridan, John (1983): *Quaestiones Longe super Librum Perihermeneias*, ed. by Ria van der Lecq, Nijmegen: Ingenium Publishers.
- Cajetan, Thomas De Vio (1939): *Scripta Philosophica: Commentaria in Praedicamenta Aristotelis*, ed. by M.H. Laurent, Rome: Angelicum.
- Cronin, Timothy J. (1987): *Objective Being in Descartes and in Suarez*, New York-London: Garland Publishing.
- De Rijk, Lambertus Marie (1956): “Introduction”, in Abaelardus (1956), VII–XCVIII. Extended version in 2nd, revised edition (1970), VII–CIII.
- De Rijk, Lambertus Marie, ed. (1967): *Logica Modernorum*, Assen: van Gorcum.
- De Rijk, Lambertus Marie (1971): “The Development of *Suppositio Naturalis* in Medieval Logic”, in *Vivarium* **9**, 71–107.
- Ebbesen, Sten (1981): “‘The Present King of France Wears Hypothetical Shoes with Categorical Laces’: Twelfth Century Writers on Well-Formedness”, in *Medioevo* **7**, 91–113.
- Geach, Peter T. (1964): “Nominalism”, in *Sophia* **3** [2]. Reprinted in Peter T. Geach: *Logic Matters*, Oxford: Blackwell, 1972, 289–301.
- Henry, Desmond Paul (1972): *Medieval Logic and Metaphysics*, London: Hutchinson University Library.
- Henry, Desmond Paul (1984): *That Most Subtle Question*, Manchester: Manchester University Press.
- King, Peter (1985): *Jean Buridan’s Logic: The Treatise on Supposition, The Treatise on Consequences*, Dordrecht: Reidel.

- Klima, Gyula (1988): *Ars Artium: Essays in Philosophical Semantics, Medieval and Modern*, Budapest: Institute of Philosophy of the Hungarian Academy of Sciences.
- Klima, Gyula (1991): “Ontological Alternatives vs. Alternative Semantics in Medieval Philosophy”, in Jeff Bernard, ed., *Logical Semiotics, S – European Journal for Semiotic Studies* **3**, 587–618.
- Klima, Gyula (1993a): “The Changing Role of Entia Rationis in Mediaeval Philosophy: A Comparative Study with a Reconstruction”, in *Synthese* **96**, 25–59.
- Klima, Gyula (1993b): “‘Debeo tibi equum’: A Reconstruction of Buridan’s Treatment of The Sophism”, in Stephen Read, ed., *Sophisms in Mediaeval Logic and Grammar: Acts of the Ninth European Symposium for Medieval Logic and Semantics held at St. Andrews, June 1990*, Dordrecht: Kluwer, 333–347.
- Kneale, William/Kneale, Martha (1962): *The Development of Logic*, Oxford: Clarendon Press.
- Kretzmann, Norman, ed. (1989): *Meaning and Inference in Medieval Philosophy*, Dordrecht: Kluwer.
- Kretzmann, Norman/Pinborg, Jan/Kenny, Anthony, eds. (1982): *The Cambridge History of Later Medieval Philosophy*, Cambridge: Cambridge University Press.
- Kretzmann, Norman/Stump, Eleonore, eds. (1988): *The Cambridge Translations of Medieval Philosophical Texts*, Cambridge: Cambridge University Press.
- Kripke, Saul (1977): “Speaker’s Reference and Semantic Reference”, in Peter A. French, Theodore F. Uehling, Jr., Howard K. Wettstein, eds., *Contemporary Perspectives in the Philosophy of Language*, Minneapolis: University of Minnesota Press, 6–27. Reprinted in Jay L. Garfield, Murray Kiteley, eds., *Meaning and Truth: Essential Readings in Modern Semantics*, New York: Paragon House, 1991, 162–188.
- Lambert, Karel (1983): *Meinong and the Principle of Independence*, Cambridge: Cambridge University Press.
- Lambert, Karel (1991): “The Nature of Free Logic”, in Karel Lambert, ed., *Philosophical Applications of Free Logic*, Oxford: Oxford University Press, 3–13.
- Lycan, William (1979): “The Trouble with Possible Worlds”, in Michael J. Loux, ed., *The Possible and the Actual*, Ithaca/NY: Cornell University Press, 1979, 274–316. Reprinted in Jay L. Garfield, Murray Kiteley, eds., *Meaning and Truth: Essential Readings in Modern Semantics*, New York: Paragon House, 503–539.
- Ockham, William (1974): *Summa Logicae*, St. Bonaventure/NY: Franciscan Institute.
- Peter of Spain (1972): *Tractatus*, ed. by Lambertus Marie de Rijk, Assen: van Gorcum.
- Priest, Graham/Read, Stephen (1981): “Ockham’s Rejection of Ampliation”, in *Mind* **90**, 274–279.
- Quine, Willard Van Orman (1953), “On What There Is”, in Willard Van Orman Quine, *From A Logical Point of View*, Cambridge/MA: Harvard University Press, 1–19.

William of Sherwood (1937): *Introductiones in Logicam*, ed. by Martin Grabmann,
Sitzungsberichte der Bayerischen Akademie der Wissenschaften **10**.

Notes

¹ Lambert, K.: *Meinong and the Principle of Independence*, Cambridge University Press, Cambridge, 1984, p. 104. (emphasis in the original) Cf.: Lambert, K.: “The Nature of Free Logic”, in: Lambert, K. (ed.): *Philosophical Applications of Free Logic*, Oxford University Press, New York-Oxford, 1991. pp. 3-13.

² See Lambert: *Meinong and the Principle of Independence*, Cambridge University Press, Cambridge, 1984, p. 105. n. 9.

³ W.V.O. Quine: “On What There Is”, in: *From A Logical Point of View*, Harvard University Press, Cambridge MA, 1980, p.15. Concerning the real import of Quine’s *dictum* and its relationship to free logics see Lambert: *Meinong and the Principle of Independence*, Cambridge University Press, Cambridge, 1984, pp. 110-112.

⁴ Perhaps, the most authoritative account of this interpretation is found in: W. Kneale-M. Kneale: *The Development of Logic*, The Clarendon Press, Oxford, 1962, II.5., pp. 54-67.

⁵ To be sure, not by all of them. Cf.: S. Ebbesen: “‘The Present King of France Wears Hypothetical Shoes with Categorical Laces’: Twelfth Century Writers on Well-Formedness”, *Medioevo*, 7(1981), pp. 91-113. But the authors mentioned here represent the exception, rather than the rule.

⁶ Cf. E.J. Ashworth: “Existential Assumptions in Late Medieval Logic”, *American Philosophical Quarterly*, 10(1973), pp.141-147. For good arguments that we cannot afford to ignore the medievals’ treatment of empty terms see A. Broadie: *Introduction to Medieval Logic*, Oxford, 1987, p.120.

⁷ For good bibliographies on the vast recent literature on supposition theory see e.g.: E. J. Ashworth: *The Tradition of Mediaeval Logic and Speculative Grammar*, Toronto, 1978; N. Kretzmann-J. Pinborg-A. Kenny (eds.): *The Cambridge History of Later Mediaeval Philosophy*, Cambridge, 1982. For more recent references see: N. Kretzmann (ed.): *Meaning and Inference in Mediaeval Philosophy*, Kluwer Academic Publishers, 1989.

⁸ For a full-fledged formal semantics constructed along the lines presented here see the *Appendix* of this paper. Closely related formal semantic systems with thorough discussions of their applications in natural language semantics can be found in G. Klima: *Ars Artium: Essays in Philosophical Semantics, Mediaeval and Modern*, Institute of Philosophy of the Hungarian Academy of Sciences, Budapest, 1988.

⁹ ‘Empty’, in the case of *natural supposition* and of other *intensional contexts*, created by tenses, modalities and what medieval logicians would call *ampliative* verbs and their participles, will have to be interpreted more strongly than ‘*actually* not true of anything’. As we shall see, A propositions with empty subject terms in such contexts do not have ‘*existential import*’ properly speaking (i.e., ‘Every S is/was/will/can/could be P’ does not imply ‘Something *is* an S’ in these cases), still, the relation of subalternation, i.e., $A \Rightarrow I$, is valid even with such propositions. For more on this see section 4.2. below. Nevertheless, in sections 2, 3. and 4.1. I am considering only extensional (non-tensed, non-modal, non-*ampliative*, etc.) contexts, and, accordingly, throughout these sections I am going to use the phrases ‘empty’ and ‘existential import’ in their usual extensional senses, i.e., according to which the subject term of an A proposition is *empty* iff it is not true of anything and the proposition has *existential import* iff its truth implies that its subject is not empty.

¹⁰ Cf. e.g. L. M. de Rijk’s *Introduction* to his edition of Abaelard’s *Dialectica*, Assen, 1956, pp. 37-38; D. P. Henry: *Medieval Logic and Metaphysics*, London, 1972, pp. 55-56, P. T. Geach, “Nominalism”, in his: *God and the Soul*, London, 1969.

¹¹ “et tamen sequitur ‘b est aliud ab a; ergo utrumque est, scilicet b et a’, ut patet per Aristotilem decimo Metaphysicae - non enti enim nihil est idem vel diversum”, *Tractatus Sophismatum* Johannis Buridani, Primum Capitulum *De Significatione*, 4um sophisma: ‘Hoc Nomen ‘Chimaera’ Nihil Significat’. The reference to Aristotle is *Met* 1054b20. And somewhat later: “Unde haec est regula de qua postea magis intendimus dicere quod omnis propositio affirmativa cujus subjectum vel praedicatum pro nullo supponit est falsa.” *ibid.* Quinta conclusio. (John Buridan: *Sophisms on Meaning and Truth*, tr. intr. by T. K. Scott, Appleton-Century-Crofts, New York, 1966, p. 66. and p.72.) Cf. also: “sicut dicitur decimo Metaphysicae, ‘idem’ vel ‘diversum’ numquam dicitur nisi ens enti, licet ens non enti dicatur non idem aut non diversum.” *Questiones in Analytica Priora*, lb. I. q. 22.: *Utrum ex puris negativis sequatur aliqua conclusio*. Translations in this paper, if not otherwise indicated are mine. Quotations from Buridan are based on Prof.

Hubert Hubien's unpublished editions of Buridan's *Lectura de Summa Logicae* and *Questiones in Analytica Priora et Posteriora*. These texts will serve as the basis for my translation of Buridan's works in the series *The Yale Library of Medieval Philosophy*. (Where applicable, I will also add references to existing English translations.) The critical Latin text is being prepared by an international team with the coordination of Prof. Sten Ebbesen.

¹² For Aristotle's authority on this point, an important factor for the medievals' approach, see e.g. *Cat.* 13a38-13b35; *Periherm.* 19b5-20b13; *Anal. I.*, 51b6-52b25. (Cf. also Boethius's comments on these texts.)

¹³ To help dispel such further worries let me refer the reader to my book, *Ars Artium*, referred to in n. 8 above, in which both predication theories receive rigorous, formal semantic treatment. For the philosophical significance of these different medieval predication theories see G. Klima: "Ontological Alternatives vs. Alternative Semantics in Medieval Philosophy", in: J. Bernard: *Logical Semiotics, S - European Journal for Semiotic Studies*, 3(1991) No. 4, pp. 587-618. and G. Klima: "The Changing Role of Entia Rationis in Mediaeval Philosophy: A Comparative Study with a Reconstruction", *Synthese* 96(1993) No. 1., pp. 25-59.

¹⁴ The medieval distinction goes back, of course, through the comments of Boethius, to Aristotle's remarks in *On Interpretation* 16a30 and 16b13.

¹⁵ Cf.: "Again, some of the propositions are affirmative, some negative. An affirmative proposition is one in which its formal part is left affirmative, while a negative proposition is one in which its formal part is negated, and by the formal part of a categorical proposition I mean its verbal copula." Albert of Saxony: *Perutilis Logica*, Georg Olms Verlag, Hildesheim-New York, 1974, fol. 17.vc.

¹⁶ Note that the description has what Russell calls a *primary occurrence* (wider scope than the negation) in [2], and what he calls a *secondary occurrence* in [1].

¹⁷ It should be noted here that besides the obvious complexities obtained by genitives, relative clauses, adjectival and adverbial constructions, the "Boolean" operations (i.e., term-negation/conjunction/disjunction), and those caused by participles of transitive verbs, according to Buridan even a determiner possibly added to the predicate term is a part of that term (whereas that added to the subject is a functor operating on the whole proposition). This is why in the scheme above I did not have to add an optional occurrence of another determiner constructed with the predicate term.

¹⁸ Of course, the scheme [*Cat*] may be completed by any other determiner besides the "classical quantifier words". This is what is indicated by the *unfinished* list in the place of [*Q*].

¹⁹ Of course, "identity-theorists" would regard also the predicate term as a referring expression on a par with the subject term, but I need not pursue this point here. For technical details and philosophical significance of this point see my *Ars Artium*.

²⁰ For Abaelard's solution see Abaelard, *Dialectica*, ed. L. M. de Rijk, Assen, 1956, pp. 175-177. Cf. W. and M. Kneale: *The Development of Logic*, Oxford, 1971, p.210-211

²¹ A full-fledged semantic theory constructed along these lines with a thorough discussion of its applications in natural language semantics can be found in Essay III. of my *Ars Artium*. The formal semantics presented in the *Appendix* of this paper will also show how naturally we can move in this direction within this framework.

²² Cf.: "it is in two ways that something can be said of something: in the first way absolutely, and for the truth of this the connection of the terms suffices; and in this way 'animal' can be said of man although of no men, just as when no rose exists 'substance' is said of rose [absolutely], though of no roses; and this is what Porphyry's authority concludes to, nor Aristotle thought its contrary. But in the second way something is said of something according to real existence. And I call saying something of something according to real existence, when the predicate inheres in the subject that exists in the nature of things." Cajetan, Thomas de Vio: *Scripta Philosophica: Commentaria in Praedicamenta Aristotelis*, ed. M. H. Laurent, Angelicum, 1939, Romae, pp.: 50-51. Cf. also William of Sherwood: "When I say 'Every man is an animal', here an habitual 'is' is predicated. And insofar as it is necessary, this proposition is equivalent to the following conditional 'If it is a man, then it is an animal'." William of Sherwood: *Introductiones in Logicam*, ed. M. Grabmann, Sitzungsberichte der Bayerischen Akademie der Wissenschaften, (10)1937, p. 83. (For a text to the same effect from Garland the Computist (11th century) see D. P. Henry, *That Most Subtle Question*, ed. cit., pp. 85-86. Cf. also L. M. de Rijk (ed.), *Logica Modernorum*, Assen, 1967, II-2,

p.730.) I think it is instructive to quote here at some length from Ockham's criticism of this approach: "... supposing that there are no donkeys they reject this syllogism: 'Every animal is a man; every donkey is an animal, so every donkey is a man', saying that 'is' equivocates here, for in the major premise 'is' is taken as the operation of being, in the minor premise, however, 'is' is taken as the 'is' of habitude, or consequence, as in: 'if it is white, then it is colored'. And this is entirely absurd, for this leads to the destruction of all syllogistic forms. For whenever I liked, I would say that 'is' equivocates in the propositions and reject at caprice any syllogism on account of this equivocation. Similarly, just as a syllogism is valid with any terms, so it is valid however the things may change [...] So even if [...] all donkeys were destroyed, this would be a valid syllogism. [...] And so such distinctions as 'is' is either the operation of being or it is the 'is' of consequence are frivolous and are posited by those who are unable to distinguish between a categorical and a hypothetical proposition. So these propositions are to be distinguished: 'A donkey is an animal' and 'If it is a donkey, then it is an animal', because the one is categorical and the other is conditional or hypothetical; and they are not equivalent for the one may be true the other being false. As this is now false: 'God not-creating is God', but this conditional is true: 'If this is God not-creating, then this is God'." W. Ockham: *Summa Logicae*, St. Bonaventure N.Y., 1974, pp. 263-264. (See the whole chapter.)

²³ Cf. n. 9 above.

²⁴ L. M. de Rijk: "The Development of *Suppositio Naturalis* in Mediaeval Logic", *Vivarium*, 9 (1971), pp. 71-107.

²⁵ Buridan: *Tractatus de Suppositionibus*, c. 3. 4. (Cf. P. King: *Jean Buridan's Logic: The Treatise on Supposition, The Treatise on Consequences*, D. Reidel Publishing Company, Dordrecht Holland, 1985, pp. 126-127.)

²⁶ Although Peter of Spain, e.g., regards natural supposition as the kind of supposition a term has absolutely [*per se*], apparently even outside of the context of a proposition. See: Peter of Spain: *Tractatus*, ed. L. M. de Rijk, Van Gorcum, Assen, 1972, p. 81.

²⁷ Note the omission of the definite article here, which on this approach is just one of the possible determiners one can add to the *common* term 'King of France'.

²⁸ Of course, we cannot truly say: 'The King of France was bald', but not because the *common* subject term 'King of France' (*not*: 'the King of France!') refers to nothing, but because there were many Kings of France, while a categorical determined by a definite article is true only if its subject refers only to one thing. But for the same reason it *is* true to say: 'The King of France in 844 was bald'.

²⁹ Note in this analysis the disjunctive subject term. The present tense disjunct is required to take care of cases like 'A white thing was black'. This proposition certainly cannot be analyzed as 'A thing that was white was black', for in a possible situation (*in casu possibili*, as Buridan would put it) the one may be true and the other false; namely, if everything in the world that is now white has never been white before, but some of them were black earlier. For more on this, see Buridan's *Sophismata*, cc. 4-5. (John Buridan: *Sophisms on Meaning and Truth*, tr. intr. by T. K. Scott, Appleton-Century-Crofts, New York, 1966, pp. 109-121, 144-157.) It is also worth mentioning here that medieval logicians, like Ockham, who rejected the theory of ampliation also recognized reference to non-actual, past, future, or even merely possible entities. Their rejection was not based on any qualms about this issue, but on their analysis of such propositions as being systematically ambiguous. Accordingly, they distinguished between two senses of such propositions, one in which the subject is taken to refer to what it actually applies to, and another in which it is taken to refer *only* to what it did/will/can apply, as the context requires. Cf.: G. Priest-S. Read: "Ockham's Rejection of Ampliation", *Mind* 90(1981), pp. 274-279. Cf. also: W. Ockham: *Summa Logicae*, St. Bonaventure N.Y., 1974, P. I. c. 72, pp. 215-216.

³⁰ In medieval works natural supposition sometimes is interpreted as covering all times, sometimes as even all, perhaps absolutely unrealized, logical possibilities. Hence the need for the bracketed addition.

³¹ On this point see again Lambert: *Meinong and the Principle of Independence*, Cambridge University Press, Cambridge, 1984, pp. 110-112.

³² William Lycan: "The Trouble with Possible Worlds", in: J. L. Garfield-M. Kiteley: *The Essential Readings in Modern Semantics*, Paragon Issues in Philosophy, Paragon House, New York, 1991, pp. 503-539. p. 511.

³³ Well, perhaps, Duns Scotus' conception of possibility provides an interesting medieval counterinstance to what I say here. But of course his *theological* reasons for this conception were (possible as well as actual) worlds apart from the modern secular belief in the sanctity of existential quantification. Cf.: *Opus Oxoniense* I. d.36, n.1. As in many other matters, Henry of Ghent seems to have had some influence on Scotus' thought also on this point. For a thorough discussion of the issue and the possible impact of the medieval discussions on Descartes' conception of eternal truths see: Timothy J. Cronin: *Objective Being in Descartes and in Suarez*, Garland Publishing, Inc. New York&London, 1987. esp. Appendix II. and III. pp. 167-207.

³⁴ Cf.: "The third opinion, with which I agree, is that an utterance does not have any proper import [*virtus propria*] in signifying and suppositing, except from us. So by an agreement of the disputing parties, as in obligational disputes, we can impose on it a new signification, and not use it according to its common signification, and we can also speak metaphorically and ironically, according to a different signification. But we call a locution 'proper', when we use it according to the signification commonly and principally given to it, and we call a locution 'improper', when we use it otherwise, although we can legitimately use it otherwise. So it is absurd to say that a proposition of an author is false, absolutely speaking, if he puts it forth according to an improper locution, according to which it is true. But we have to say that it is true, because it is put forth according to the sense in which it is true, in conformity with what Aristotle says in book 1. of the *Ethics*, namely, that locutions are to be taken and understood according to their subject matter. But we should correctly say that the proposition would not be true, if it were put forward and taken as a proper locution." Buridan: *Tractatus de Suppositionibus*, c. 3. 2. (Cf. P. King: *Jean Buridan's Logic: The Treatise on Supposition, The Treatise on Consequences*, D. Reidel Publishing Company, Dordrecht Holland, 1985, pp. 121-122.)

³⁵ Johannes Buridanus: *Questiones Longe super Librum Perihermeneias*, ed. Ria van der Lecq, Utrecht, 1983, pp.12-14. Cf.: "All verbs, even in the present tense, which of their very nature can concern future, past and possible things as well as present ones such as 'think', 'know', 'mean' and the like amplify their terms to all times, future, past and present. And what accounts for this is that a thing can be thought of without any difference of time, namely, abstracted from any place and time. And so, when a thing is thought of in this way, then a thing which was, or will be, or can be may be thought of as well as a thing which [actually] is. Therefore, if I have the common concept from which we take this name 'man', then I can think indifferently of all men, past, present and future. And this is why these verbs can concern past or future things as well as present ones." Albert of Saxony: *Perutilis Logica*, Georg Olms Verlag, Hildesheim-New York, 1974, Tr. 2.Tr.2. c.10. 8a regula. For an earlier example of the same explanation of ampliation see the selection from Lambert of Auxerre's *Logica* in: N. Kretzmann-E. Stump (eds.): *The Cambridge Translations of Medieval Philosophical Texts*, Cambridge University Press, Cambridge, 1988. pp. 104-163, esp. pp. 116-118.

³⁶ For Buridan's distinction see Buridan: *Tractatus de Suppositionibus*, c. 3. 1. (Cf. P. King: *Jean Buridan's Logic: The Treatise on Supposition, The Treatise on Consequences*, D. Reidel Publishing Company, Dordrecht Holland, 1985, pp. 117-118.) For Kripke's see S. Kripke: "Speaker's Reference and Semantic Reference", in: J. L. Garfield-M. Kiteley (eds.): *Meaning and Truth: The Essential Readings in Modern Semantics, Paragon Issues in Philosophy*, New York: Paragon House, 1991.

³⁷ For a detailed discussion and formal reconstruction of Buridan's theory see my "'Debeo tibi equum': A Reconstruction of Buridan's Treatment of the Sophisma", in: S. Read (ed.): *Sophisms in Mediaeval Logic and Grammar: Acts of the Ninth European Symposium for Mediaeval Logic and Semantics held at St Andrews, June 1990*, Kluwer Academic Publishers, Dordrecht-Boston-London, 1993. pp. 333-347.

³⁸ The system presented here is a slightly modified version of the system AMPL, presented in Essay 4. of my *Ars Artium*. The proof of metatheorem (EQ) and the illustrations of its significance are taken over with slight modifications from the same essay.

³⁹ Notice that 0 serves as the semantic value of empty terms as well as the semantic value of false formulae. This is a mere technical device with no philosophical significance attached to it.

⁴⁰ In this sentence the word 'Something' is to be thought of as analyzable into 'Some thing' *only if* we regard the term 'thing' in this analysis as amplified to past things, and which, therefore, could be formalized as: $(\exists x. \alpha_p(x = x))(\alpha_p(\mathbf{E}x.) \ \& \ \sim(\mathbf{E}x.))$. In fact, it would be much more in line with the medieval

conception to regard ‘something’ and ‘everything’ as *always* analyzable into ‘some thing’ and ‘every thing’. Formally, this could be represented by allowing *only* restricted variables to be bound by the quantifiers. Proper variables, on the other hand, then would have to be regarded as the formal counterparts of demonstrative pronouns, while their value-assignments might represent acts of pointing at things, at least *in thought*. For this point see Buridan: *Tractatus de Suppositionibus*, c. 1. 2. (Cf. P. King: *Jean Buridan’s Logic: The Treatise on Supposition, The Treatise on Consequences*, D. Reidel Publishing Company, Dordrecht Holland, 1985, p. 88.)

I wonder, however, about Gumb's reason for thinking that only a positive free logic is appropriate for dealing with non-strict functions. Gumb seems to believe that since something like an inner domain/outer domain model structure is needed to accommodate error objects – they fall outside the range of the quantifiers and hence in the outer domain – a positive free logic is required because such model structures are perhaps most commonly used in developing positive free logics. Nevertheless, such a model structure does not by itself guarantee that one's free logic will be positive. In fact, the most general treatment of negative free logic, the treatment of Ronald Scales in Scales (1969), utilizes inner domain/outer domain model structures. Of course, if one wishes statements such as ' $1/0 = 1/0$ ' to turn out true, then a positive free logic is required. But that depends on how one defines truth, and not on the kind of model structure employed. Thus, in a positive free logic the clause in the truth-definition for identity will be ' $a = b$ ' is true when and only when the referent of ' a ' is the same as the referent of ' b ', where the referents of the singular terms are chosen from the union of the inner and outer domains. But in a negative free logic, the clause in question will have the restriction that the referents of the singular terms must be in the inner domain (intuitively, the universe of existents) and thus exclude error objects. Hence ' $1/0 = 1/0$ ' will be true given the unrestricted clause for identity in the truth-condition, but false in the restricted reading despite the fact that the model structure is of the inner domain/outer domain stripe and that the referent of 'Vulcan' is an error object. This reinforces the fact that whether a free logic is positive or negative has more to do with the definition of truth in a model rather than with the character of the model structure itself.

4. COMMENT ON KLIMA

Klima's essay is a model of erudition and provocative argument. Contra a view of the relationship between free logic, classical predicate logic and medieval logic holding free logic to be but the completion of a task begun by the tradition beginning with Frege and Russell, namely, the ridding of logic of existence assumptions with respect to its terms both singular and general – indeed, a view I have promoted from time to time – Klima argues that the majority of medieval logicians espoused, *via* the doctrine of supposition, something like free logic, and only a minority sponsored the doctrine of existential import with respect to terms. The picture which evolves is that classical logic undid the balance which prevailed in medieval logic

by supporting only the admission of general terms free of existence assumptions, and that free logic goes at least part way in restoring the medieval balance by permitting singular terms that also may fail to refer to existents. Klima's expertise – historical, philosophical and technical – is hard to resist, and I shall make no such attempt here. But I do want to make a remark concerning his defense of the medieval logic against the charge that it undermines the affirmation of universal laws concerning unactualized entities, for example, Newton's law of inertia.

According to Klima, the dominant account in medieval logic can be preserved in the face of objections that it cannot accommodate laws like Newton's law of inertia, but only at the expense of allowing non-existents in the range of the variables. He stoutly defends this position against those who might characterize it as relentlessly Meinongian, that is, as requiring there to be non-actual entities "out there" awaiting reference to, and quantification over, them. His view is that such objects are intentional objects, that such objects are what we think of, and as such are things to which we refer. Nevertheless, I still feel uncomfortable. What disturbs non-Meinongians is not the ontological independence of non-existent entities, like bodies on which there are no external forces acting or round squares, but rather that there are any such objects at all serving as the objects of reference, even if always tethered to some mind. Nor is it any real resolution to say that what is referred to is the thought of round square or the thought of body on which no external force is acting because these objects exist whereas round squares and bodies upon which there is no external force acting do not. If the effort to accommodate Newton's law of inertia requires the variables to range over non-existents, they had better be non-existent.

5. COMMENT ON LEHMANN

Lehmann's Fregeian inspired semantics for free logic is new, and is a departure from the similarly inspired essays of Smiley and Skyrms, both in semantical details and in the treatment of statements of the form 'Vulcan exists'. Indeed, I find much of what Lehmann says appealing and clever. It is no secret that despite my own use of supervaluations, and despite a deep appreciation of the ingenuity explicit in their various manifestations by van Fraassen and Bencivenga, I am less than enamored of them as a foundation for positive free logic. So, I find Lehmann's view that interpretations should represent the possibilities given by reference failure intuitive, his treatment of existence as an object language notion to be just