We investigated the behavior of the first Chern class vectors of \( \mathfrak{sl}(2) \) conformal blocks associated with \( \overline{M}_{0,n} \). Najmuddin Fakhruddin conjectured that all of these vectors fall within a specific cone with 128 extremal rays and 20,516 facets, which we call the Fakhruddin cone. We tested Fakhruddin’s conjecture for conformal blocks of level \( \ell = 1, 2, \ldots, 10 \). We also investigated how conformal blocks with weights of the form \((\ell - j, \ell - j, \ell - j, \ell - j, \ell - j, \ell - j, \ell - j, \ell - j)\) behave in the nef cone.

### Conformal Blocks

The work of Deligne, Mumford, Mayer, and Knudsen demonstrates that \( \overline{M}_{0,n} \) (see below) can be described using polynomial equations. To each configuration of points in \( \overline{M}_{0,n} \) we may assign a vector space called a conformal block. Conformal blocks have three ingredients: a Lie algebra \( \mathfrak{g} \), a level \( \ell \), and a weight vector \( \lambda \). A conformal block is denoted \( \mathcal{V}(\mathfrak{g}, \ell , \lambda) \). We are working with the Lie algebra \( \mathfrak{sl}(2) \), the set of \( 2 \times 2 \) matrices with trace equal to zero. The level \( \ell \) is an integer. The coordinates of the weight vector \( \lambda \) are positive integers less than or equal to \( \ell \). \( \lambda \) has six coordinates because we are studying conformal blocks related to \( \overline{M}_{0,6} \).

### Computational Results

**Result.** Using Macaulay2 we tested the first Chern class vectors of \( \mathfrak{sl}(2) \) conformal blocks against the facets of the Fakhruddin cone, and found that for levels \( \ell = 1, 2, \ldots, 10 \) Fakhruddin’s conjecture is true.

### References